SET-2

Series JBB/5

Code No. **30/5/2**

Roll No. | | | | | | | |

Note :

(I) Please check that this question paper contains **19** printed pages.

(II) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(III) Please check that this question paper contains **40** questions.

(IV) Please write down the Serial Number of the questions in the answer -book before attempting it.

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS (STANDARD)-Theory
HINTS & SOLUTIONS

Time allowed : 3 hours **Maximum Marks : 80**

General Instructions:

Read the following instructions very carefully and strictly follow them :

(i) This question paper comprises **Four** Sections — **A, B, C and D** There are 40 questions in the question paper. All questions are compulsory.

(ii) **Section A —** Questions no. **1** to **20** comprises of **20** questions of **one** mark each.

(iii) **Section B —** Questions no. **21** to **26** comprises of **6** questions of **two** mark each.

(iv) **Section C —** Questions no**. 27** to **34** comprises of **8** questions of **three** mark each.

(v) **Section D —** Questions no**. 35** to **40** comprises of **6** questions of **four** mark each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark, 2 questions of two marks, 3 questions of three marks and 3 questions of four marks. You have to attempt only one of the choices in such questions.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

SECTION-A

Question numbers 1 to 20 carry 1 mark each.
Question numbers 1 to 10 are multiple choice questions.
Choose the correct option.

1. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is

(A) 4 (B) ± 4 (C) – 4 (D) 0

Sol. given $2x^2 + Kx + 2 = 0$ has equal roots

So D = 0 a = 2, b = k. C = 2

$b^2 - 4\,ac = 0$

$(k^2) - 4\,(2)\,(2) = 0$

$k^2 - 16 = 0$

$k^2 = 16$

$k = \pm 4$

Option (B)

2. Which of the following is not an A.P. ?

(A) – 1.2, 0.8, 2.8,....... (B) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}$

(C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, ...$ (D) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, ...$

Sol. By checking options, option (C) is not in A.P

$\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}.....$

$d = a_2 - a_1 = \frac{7}{3} - \frac{4}{3} = \frac{3}{3} = 1$

$d = a_3 - a_2 = \frac{9}{3} - \frac{7}{3} = \frac{2}{3}$

Difference is not same so this is not an A.P.

3. In figure - 3, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90°$, then length of PQ is

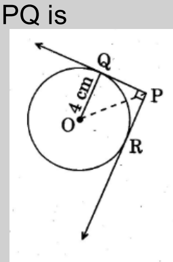


Figure-3

(A) 3 cm (B) 4 cm (C) 2 cm (D) $2\sqrt{2}$ cm

Sol. In the given figure QP and PR are the tangent so $\angle OQP$ and $\angle ORP = 90$

In quadrilateral $\angle P = \angle Q = \angle R = 90°$

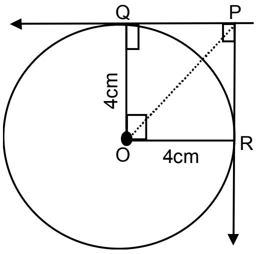
By ASP

$\angle P + \angle Q + \angle R + \angle Q = 360°$

$90 + 90 + 90 + \angle O = 360°$

$\angle O = 90°$

All angles of quadrilateral PQR are of 90º so it is rectangle and rectangle OPQR having adjacent sides equal to it is a square



4. The distance between the points (m, – n) and (– m, n) is

(A) $\sqrt{m^2+n^2}$ (B) m + n (C) $2\sqrt{m^2+n^2}$ (D) $\sqrt{2m^2+2n^2}$

Sol. Let point P(m – n) and Q(–m n)

So $PQ=\sqrt{(-m-m)^2+(n+n)^2}$

$PQ=\sqrt{(-2m)^2+(2n)^2}$

$=\sqrt{4m^2+4n^2}$

$PQ=2\sqrt{m^2+n^2}$

Option (C)

5. The degree of polynomial having zeroes -3 and 4 only is

(A) 2 (B) 1 (C) more than 3 (D) 3

Sol. A polynomial having only two zeros is a quadratic polynomial degree is 2 (Option A)

6. In figure - 1, ABC is an isosceles triangle, right angled at C. Therefore

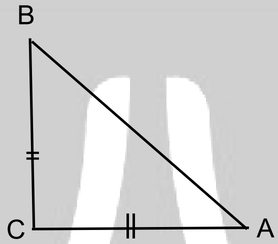


figure - 1

(A) $AB^2 = 2AC^2$ (B) $BC^2 = 2AB^2$ (C) $AC^2 = 2AB^2$ (D) $AB^2 = 4AC^2$

Sol. Given that Δ ABC is an isosceles right angled triangle with AC = BC

$\therefore$ $AB^2 = AC^2 + BC^2$ {by Pythagoras theorem}

$AB^2 = AC^2 + AC^2$ {given AC = BC}

$AB^2 = 2AC^2$

Option (A)

7. The point on the x-axis which is equidistant from (– 4, 0) and (10, 0) is

(A) (7, 0) (B) (5, 0) (C) (0, 0) (D) (3, 0)

Sol. Let the point be P(x, 0) which in equidistant from point A(–4, 0) & point B (10,0)

$\therefore$ PA = PB by distance formula

$\sqrt{(x+4)^2+(0-0)^2}=\sqrt{(x-10)^2+(0-0)^2}$

Squaring on the both sides

$(x + 4)^2 = (x – 10)^2$

$\Rightarrow$ $x^2 + 2x + 16 = x^2 - 20x + 100$

$22x = 84$

$x = 3$

required point is (3, 0)

Option (D)

OR

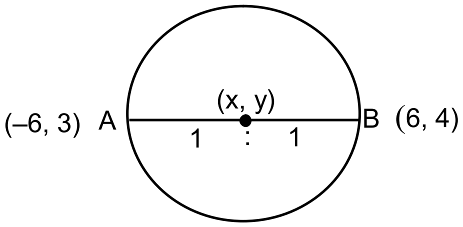
The centre of a circle whose end points of a diameter are (– 6, 3) and (6, 4) is

(A) (8, - 1) (B) (4, 7) (C) $\left(0,\frac{7}{2}\right)$ (D) $\left(4,\frac{7}{2}\right)$

Sol. Let AB in diameter and O is the centre

We know that

Centre O is the mid point of diameter so



By section formula

$x = \frac{6-6}{1+1}$ $\quad y = \frac{4+3}{1+1}$ $\quad \left\{x = \frac{mx_2 + nx_1}{m+n}\right\}$ $y = \left\{y = \frac{my_2 + ny_1}{m+n}\right\}$

x $= 0 \quad y = \frac{7}{2}$

$O\ (0, \frac{7}{2})$

Option (C)

8. The pair of linear equations $\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is

(A) consistent
(B) inconsistent
(C) consistent with one solution
(D) consistent with many solutions.

Sol. $\frac{3x}{2} + \frac{5y}{3} - 7 = 0$ … (1)

$9x + 10y - 14 = 0$ … (2)

$a_1 = \frac{3}{2}$ $\quad b_1 = \frac{5}{3}$ $\quad c_1 = -7$

$a_2 = 9$ $\quad b_2 = 10$ $\quad c_2 = -14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}$ … (A)

$\frac{b_1}{b_2} = \frac{\frac{5}{3}}{10} = \frac{1}{6}$ … (B)

$\frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$ … (C)

Equation (A), (B) & (C)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So the system is inconsistent Option (B)

9. In figure - 2, PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100°$, then $\angle ABP$ is equal to

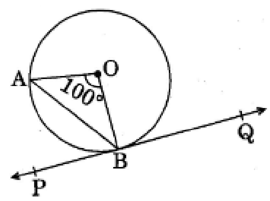


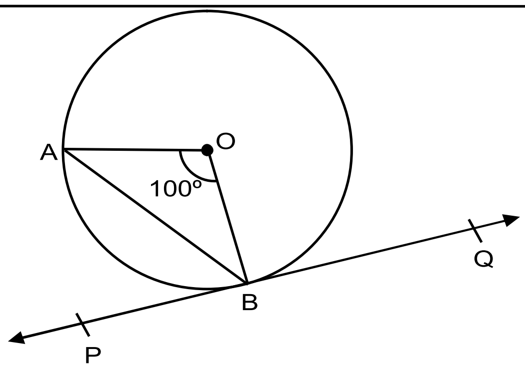
Figure-2

(A) 50° (B) 40° (C) 60° (D) 80°

Sol. In ΔAOB

OA = OB = radius

So $\angle OAB = \angle OBA$ … (1)



Now

$\angle AOB + \angle OAB + \angle OBA = 180°$ [By ASP]

$100 + 2\angle OBA = 180°$ [By equation (1)]

$2\angle OBA = 80°$

$\angle OBA = 40$ … (2)

$\Rightarrow$ $\angle OBP = 90°$ [$\because$ A tangent to a circle is perpendicular to the radius through the point of contact]

$\angle OBA + \angle ABP = 90°$

$40 + \angle ABP = 90°$

$\angle ABP = 90 - 40$

$\angle ABP = 50°$ Option (A)

10. The radius of a sphere (in cm) whose volume is 12π cm^3, is

(A) 3 (B) $3\sqrt{3}$ (C) $3^{\frac{2}{3}}$ (D) $3^{\frac{1}{3}}$

Sol. Let the radius of sphere be r cm given that

Volume of sphere = $12\ \pi$ cm^3

$\Rightarrow$ $\frac{4}{3}\pi r^3 = 12\pi$

$r^3 = 9$

$r^3 = 3^2$

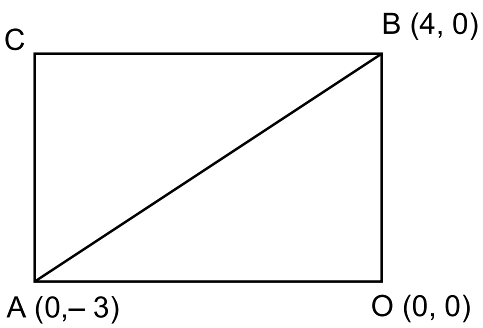
$r = 3^{\frac{2}{3}}$

Option (C)

Fill in the blanks in questions numbers 11 to 15.

11. AOBC is a rectangle whose three vertices are A(0, – 3), O(0, 0) and B(4, 0). The length of its diagonals is ________________.

Sol.



AB = $\sqrt{(4-0)^3 + (0+3)^2}$

$\sqrt{16+9} = \sqrt{25}$

AB = 5 units

12. In the formula $\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h_i, u_i$ = ___________.

Sol. $U_i = \frac{x_i - a}{h}$

13. All concentric circles are ________________ to each other.
Sol. Similar

14. The probability of an event that is sure to happen, is _______________
Sol. 1

15. Simplest from of (1 – $\cos^2$ A) (1 + $\cot^2$ A) is ____________.
Sol. (1 – $\cos^2$ A) (1 + $\cot^2$ A)

$\Rightarrow$ $\sin^2 A \times \operatorname{cosec}^2 A$

$\Rightarrow$ $\sin^2 A \times \frac{1}{\sin^2 A}$

$$\left\{\begin{array}{l} \therefore \sin^2 A + \cos^2 A = 1 \\ \therefore \sin^2 A = 1 - \cos^2 A \\ \therefore \operatorname{cosec}^2 A = 1 + \cot^2 A \end{array}\right\}$$

$$\left(\therefore \operatorname{cosec} A = \frac{1}{\sin A}\right)$$

= 1 Ans.

Answer the following question numbers 16 to 20

16. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.
Sol. Let the other number be a.

LCM (a, b) × HCF(a, b) = a × b

182 × 13 = a × 26

$a = \frac{182 \times 13}{26}$

a = 91

17. Form a quadratic polynomial, the sum and product of whose zeroes are (– 3) and 2 respectively.
Sol. Given that sum of zeros (–3) and product of zeros is 2.

Quadratic polynomial

P(x) = k [x^2 – (Sum of zeros) x + Product of zeros]

P(x) = k [x^2 – (– 3) x + 2]

P(x) = k [x^2 + 3x + 2]

OR

Can (x^2 – 1) be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by (x^2 + 3) ?

Sol.

$$\begin{array}{r|l} x^2+3 & x^4 - 3x^2 + 5x - 9 \;\big(\; x^2 - 6 \\ & \underline{\underset{-}{x^4} \underset{-}{+ 3x^2}} \\ & -6x^2 + 5x - 9 \\ & \underline{\underset{+}{-6x^2} \quad \underset{+}{-18}} \\ & 5x + 9 \end{array}$$

No, x^2 – 1 in not the remainder when divided by x^2 + 3.

18. Find the sum of the first 100 natural numbers.
Sol. First 100 natural numbers one

1, 2, 3, 4………………99, 100

So this should be can AP

$a = 1$ $d = 1$ $\ell = 100$ $n = 100$

$S_{100} = \frac{m}{2}(a+\ell)$

$S_{100} = \frac{100}{2}(1+100)$

$S_{100} = 5050$

19. Evaluate :

2 sec 30º × tan 60º

Sol. 2 sec 30º × tan 60º

$\Rightarrow 2 \times \frac{2}{\sqrt{3}} \times \sqrt{3}$

$$\begin{Bmatrix} \therefore \sec 30^\circ = \frac{2}{\sqrt{3}} \\ \therefore \tan 60^\circ = \sqrt{3} \end{Bmatrix}$$

$\Rightarrow$ 4. Ans.

20. In figure - 4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

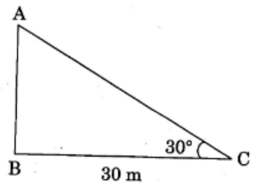


Figure-4

Sol. $\tan 30^\circ = \frac{AB}{30}$

$\frac{1}{\sqrt{3}} = \frac{AB}{30} \Rightarrow AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ m

Length of lower = $10\sqrt{3}$ m

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. Find the mode of the following distribution.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	4	6	7	12	5	6

Sol. Model Class = 30 – 40

Mode = $\ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 30 + \frac{12-7}{2(12)-7-5} \times 10$

$= 30 + \frac{5}{12} \times 10$

$= 30 + 4.17 = 34.17$

22. In figure - 6, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = BC + AD.

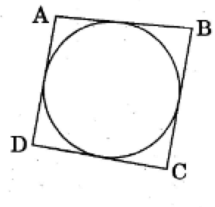
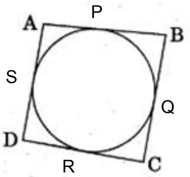


Figure-6

Sol. Sides AB, BC, CD and DA of a quadrilateral ABCD touch a circle at P, Q, R and S respectively.
To prove : AB + CD = AD + BC.



Proof :
AP = AS ....(i)
BP = BQ ....(ii)
CR = CQ ....(iii)
DR = DS ....(iv)
[Tangents drawn from an external point to a circle are equal]
Adding (1), (2), (3) and (4), we get
$\Rightarrow$ AP + BP + CR + DR = AS + BQ + CQ + DS
$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
$\Rightarrow$ AB + CD = AD + BC.

OR

In figure - 7, find the perimeter of ΔABC, if AP = 12 cm.

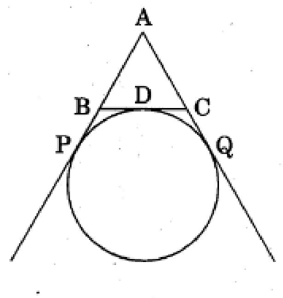


Figure-7

Sol. AP = AQ [$\because$ Length of tangent drawn from external point are equal]
AB + BP = AC + CQ
AB + BD = AC + CD [$\because$ BP = BD, CQ = CD]
So, perimeter of ΔABC = AB + BD + DC + CA = 2AP = 2 (12) = 24 cm

23. How many cubes of side 2 cm can be made from a solid cube of side 10 cm?

Sol. Number of small cubes = $\dfrac{\text{volume of bigger cube}}{\text{volume of one smaller cube}}$

$= \dfrac{(\text{side})^3}{(\text{side})^3} = \dfrac{10 \times 10 \times 10}{2 \times 2 \times 2}$

= 125 cubes.

24. In the figure - 5, DE || AC and DF || AE.

Prove that $\frac{BF}{FE} = \frac{BE}{EC}$

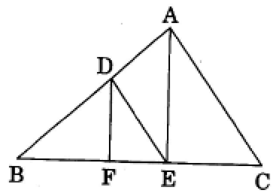
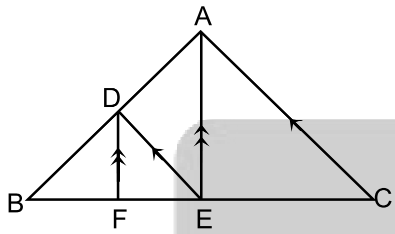


Figure-5

Sol.



In $\triangle BEA$

DF||AE

By BPT $\frac{BF}{FE} = \frac{BD}{AD}$ ..(1)

In $\triangle ABC$

DE || AC

By BPT $\frac{BE}{EC} = \frac{BD}{AD}$ ....(2)

From (1) and (2)

$\frac{BF}{FE} = \frac{BE}{EC}$ Hence proved.

25. Show that $5 + 2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

Sol. Let $5 + 2\sqrt{7}$ is a rational number

$\therefore\ 5 + 2\sqrt{7} = \frac{P}{q}$ Where P, q are integer ,$q \neq 0$

$$2\sqrt{7} = \frac{P}{q} - 5$$

$$\sqrt{7} = \frac{1}{2}\left[\frac{P}{q} - 5\right]$$

IN LHS we have $\sqrt{7}$ which is an irrational number and in RHS we have rational number. And we know a rational number is not equal to irrational number.

$\therefore$ LHS $\neq$ RHS

So our assumption is not correct

$\therefore\ 5 + 2\sqrt{7}$ is irrational number

OR

Check whether 12^n can end with the digit 0 for any natural number n.

Sol. $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$

= (2 × 2 × 2 × ---------- up to 2 n times) (3 × 3 × ----- up to n times)

to get zero at unit place we required a pair of 2 & 5. but here we not get a pair of 2 × 5

So it never ends with digit 0.

26. If A, B and C are interior angles of a ΔABC, then show that $\cos\left(\frac{B+C}{2}\right)=\sin\left(\frac{A}{2}\right)$.

Sol. m.d

LHS : $\cos\left(\frac{B+C}{2}\right)$

$= \cos\left(\frac{180-A}{2}\right)$

$= \left\{\begin{matrix} \text{In}\,\Delta ABC \\ \angle A+\angle B+\angle C=180 \\ \angle B+\angle C=180-\angle A \end{matrix}\right\}$

$= \cos\left(90-\frac{A}{2}\right)$

$= \sin\left(\frac{A}{2}\right)$ $\{\cos(90-\theta)=\sin\theta\}$

SECTION-C

Question numbera 27 to 34 carry 3 marks each

27. In figure-9, a square OPQR is inscribed in a quadrant OAQB of a circle. If the radius of circle is $6\sqrt{2}$ cm, find the area of the shaded region.

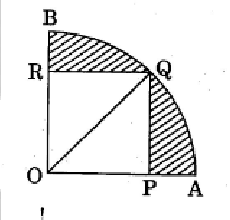
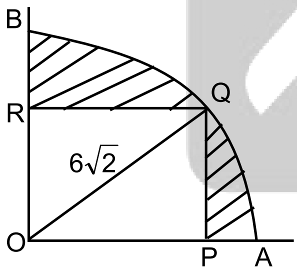


Figure-9

Sol. So area of OAQB = $\frac{1}{4}\pi r^2$



$= \frac{1}{4}\times\frac{22}{7}\times\left(6\sqrt{2}\right)^2$ $\left(r=6\sqrt{2}\right)$

$= \frac{1}{4}\times\frac{22}{7}\times 36\times 2 = 56.57\ cm^2$ …(i)

OR = OP = PQ = RQ (square)

So $(OP)^2 + (PQ)^2 = \left(6\sqrt{2}\right)^2 = 36 \times 2$

$a^2 + a^2 = 72$

$2a^2 = 72$

$a = 6$

area of (OPQR) = $a^2 = (6)^2 = 36\ cm^2$ …(ii)

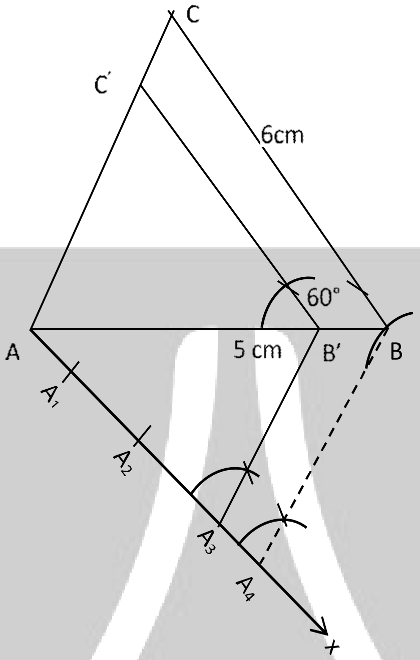
area of shaded region = area of quadrant – area of square [by equation (i) & (ii)]

= 56.57 – 36

= $20.57\ cm^2$

28. Construct a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

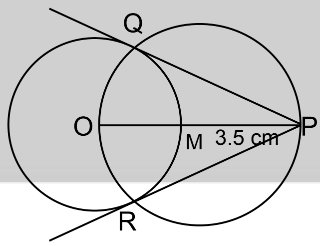
Sol. Step1. Construct a ΔABC with given data
Step.2 draw a angle $\angle BAX$.
Step.3 put on 4 equal parts on AX such as .
$AA_1 = A_1A_2 = A_2A_3 = A_3A_4$
Step.4 join B to A_4, and draw line segment from A_3 such as $A_3B' \parallel A_4B$.
Step.5 draw a line segment $B'C \parallel BC$ thus $AB'C'$ is required triangle



OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

Sol.



Steps of construction

(1) Take any point O of given plane as centre draw a circle of 3.5 cm radius, locate a point P 7 cm away from O join OP.
(2) Bisect OP, let M be the mid point of OP
(3) Taking M as centre and MO as radius draw a circle
(4) Let this circle intersect the previous circle at Q and R.
(5) Join PQ and PR, PQ and PR are required tangents,

29. Prove that :

$$\frac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta} = \cot\theta$$

Sol. LHS

$$\Rightarrow \quad \frac{2\cos^3\theta - \cos\theta}{\sin\theta - 2\sin^3\theta}$$

$\Rightarrow \quad \frac{\cos\theta\,(2\cos^2\theta-1)}{\sin\theta\,(1-2\sin^2\theta)}$

$\Rightarrow \quad \{\therefore \cos^2\theta = 1-\sin^2\theta\}$

$\Rightarrow \quad \frac{\cos\theta[2(1-\sin^2\theta)-1]}{\sin\theta[1-2\sin^2\theta]}$

$\Rightarrow \quad \frac{\cos\theta\,[2-\sin^2\theta-1]}{\sin\theta\,[1-2\sin^2\theta]}$

$\Rightarrow \quad \frac{\cos\theta\,[1-2\sin^2\theta]}{\sin\theta\,[1-2\sin^2\theta]}$

$\Rightarrow \quad \frac{\cos\theta}{\sin\theta} \qquad \left\{\because \frac{\cos\theta}{\sin\theta} = \cot\theta\right\}$

$\cot\theta$ = RHS

30. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Sol. Let the fraction be $\frac{x}{y}$

Case-I $\quad \frac{x-1}{y} = \frac{1}{3}$

$\Rightarrow \quad 3x - 3 = y$

$\Rightarrow \quad 3x - y = 3 \quad$ …………… (i)

Case-II $\quad \frac{x}{y+8} = \frac{1}{4}$

$\Rightarrow \quad 4x = y + 8$

$\Rightarrow \quad 4x - y = 8 \quad$ …………… (ii)

By solving eq^n (i) & (ii)

$3x - y = 3$
$4x - y = 8$
$- \quad + \quad -$

$-x = -5$

$x = 5 \quad$ Put x = 5 in eq^n (i) then
$y = 12$

$\therefore \quad$ Fraction = $\frac{x}{y} = \frac{5}{12}$

OR

The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

Sol. Let Father's age = x
Son's age = y

Case-I $x = 3y + 3$

$\Rightarrow \quad x - 3y = 3 \quad$ …………… (i)

Case-II $(x + 3) = 2(y + 3) + 10$

$\Rightarrow \quad x + 3 = 2y + 6 + 10$

$\Rightarrow \quad x - 2y = 13 \quad$ …………… (ii)

By solving eq^n (i) & (ii)

$x - 3y = 3$

$x - 2y = 13$
$- \quad + \quad -$

$-y = -10$
$y = 10$ years   Put $y = 10$ in eq^n (i) then
$x = 33$ years
Father's present age = x = 33 years
Son's present age = y = 10 years.

31. Using Euclid's Algorithm, find the largest number which divides 870 and 258 leaving remainder 3 in each case.

Sol. The largest number which divides 870 and 258 leaving remainder 3 is nothing but HCF of 870 – 3 and 258 – 3.
So required number is HCF of 867 & 255
By Eucid's algorithm.
$867 = 255 \times 3 + 102$
$255 = 102 \times 2 + 51$
$102 = 51 \times 2 + 0$
HCF = 51
remainder no. is 51 Ans.

32. Find the ratio in which the y-axis divides the line segment joining the points (6, – 4) and (–2, – 7). Also find the point of intersection.

Sol. Let the ratio be k : 1 and the point of intersection R(0, y)

3 : 1
k : 1
P ———————————— Q
(6, –4)   R (0, y)   (–2, –7)

By section formula

$x = \frac{mx_2 + nx_1}{m+n} = \frac{k(-2)+(1)(6)}{k+1} = 0$

$0 = -2k + 6$
$2k = 6$
$k = 3$

$y = \frac{my_2 + ny_1}{m+n} = \frac{k(-7)+(1)(-4)}{k+1} = \frac{-7k-4}{k+1}$

Put k = 3

$y = \frac{-21-4}{4} = \frac{-25}{4}$

$R(x, y) = \left(0, \frac{-25}{4}\right)$

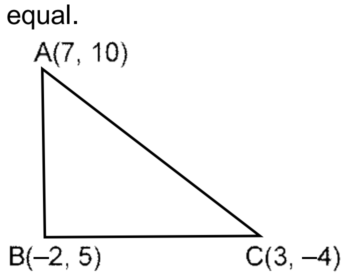
Ratio is k : 1 or 3 : 1

and point of intersection $R\left(0, \frac{-25}{4}\right)$

OR

Show that the points (7, 10), (–2, 5) and (3, – 4) are vertices of an isoceles right triangle.

Sol. In isosceles right triangle sum of square of two sides is equal to sphere of third side and two side are equal.



By distance formula

Now, AB = $\sqrt{(7-(-2))^2+(10-5)^2}$

= $\sqrt{9^2+5^2}$

= $\sqrt{81+25}=\sqrt{106}$

BC = $\sqrt{(-2-3)^2+[5-(-4)]^2}$

= $\sqrt{5^2+9^2}$

= $\sqrt{25+81}=\sqrt{106}$

AC = $\sqrt{(7-3)^2+(-10-(-4))^2}$

= $\sqrt{4^2+14^2}$

= $\sqrt{16+196}=\sqrt{212}$

Hence, AB = BC = $\sqrt{106}$

and AB^2 + BC^2 = $\left(\sqrt{106}\right)^2+\left(\sqrt{106}\right)^2$

= 106 + 106 = 212 = AC^2

If the sum of the squares of two sides is equal to the square of the third side then by triangle is right angled triangle.

So (7, 10), (–2, 5), (3, 4) are coordinates of isosceles right triangle.

33. In an A.P. given that the first term (a) = 54, the common difference (d) = – 3 and the nth term (a_n) = 0, find n and the sum of first n terms (S_n) of the A.P.

Sol. ATQ

A = 54, d = –3 $a_n = 0$

$a_n = a + (n-1)d$

$0 = 54 + (n-1)(-3)$

$(n-1)(-3) = -54$

$(n-1) = \frac{-54}{-3}$

$n - 1 = 18$

$n = 19$

Now

$S_n = \frac{n}{2}(a + l)$

$S_{19} = \frac{19}{2}(54 + 0)$

$S_{19} = 513$

so n = 19 and s_n = 513

34. Read the following passage and answer the questions given at the end :

Diwali Fair

A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bage are respresented in Figure - 8.

Prizes are given, when a black marbles is picked. Shweta plays the same once.

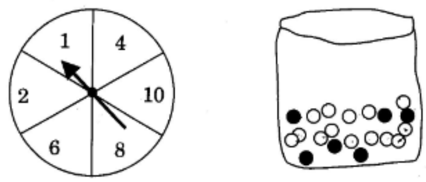


Figure-8

(i) What is the probability that she will be allowed to pick a marble from the bag ?

(ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black ?

Sol. Total numbers = 6

(i) Favourable case = 5, {4,10,8,6,2}, P(to pick marble from bag) = $\frac{5}{6}$

(ii) Favourable case = 6, Total case = 20, P(of getting prize) = $\frac{6}{20} = \frac{3}{10}$

SECTION-D

Question number 35 to 40 carry 4 marks each.

35. Sum of the areas of two squares is 544 m^2. If the diffeence of their perimeter is 32 m, find the sides of the two squares.

Sol. Let a, b are the sides of two square. Area's will be a^2 & b^2, Perimeter will be 4a, 4b

Given $a^2 + b^2 = 544$ …(i)

and $4a - 4b = 32$

$a - b = 8$ …(ii)

Put a from (ii) in equation (i)

$(8 + b)^2 + b^2 = 544$

$64 + b^2 + 16b + b^2 = 544$

$2b^2 + 16b - 480 = 0$

$b^2 + 8b - 240 = 0$

$b^2 + 20b - 12b - 240 = 0$

$b(b + 20) - 12(b + 20) = 0$

$(b + 20)(b - 12) = 0$

$b = 12$ or $b \neq -20$ as it is side

from equation (ii)

$a - 12 = 8$

$a = 20$

Sides will be 20m & 12m

OR

A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Speed of boat in still water (x) = 18 km/hr

Speed of stream = y km/hr

$$\frac{24}{18-y} - \frac{24}{18+y} = 1$$

$$\frac{1}{18-y} - \frac{1}{18+y} = \frac{1}{24}$$

$$\frac{18+y-18+y}{324-y^2} = \frac{1}{24}$$

$$\frac{2y}{24-y^2} = \frac{1}{24}$$

$48y = 324 - y^2$

$y^2 + 48y - 324 = 0$

$y^2 + 54y - 6y - 324 = 0$

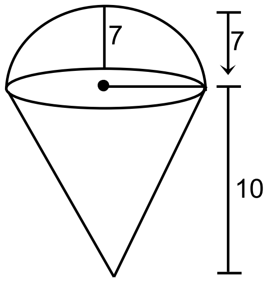
$y(y + 54) - 6(y + 54) = 0$

$(y + 54)(y - 6) = 0$

$y = 6$ km/hr $\quad$ $y = -54$ km/hr (Not possible)

36. A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of the base is 7 cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy. $\left(\text{Use } \pi = \dfrac{22}{7} \text{ and } \sqrt{149} = 12.2\right)$

Sol. r = 7 cm, h = 10 cm



Volume to Toy = Volume of hemisphere + Volume of cone.

$= \dfrac{2}{3}\pi r^3 + \dfrac{1}{3}\pi r^2 h \;= \dfrac{2}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 7 + \dfrac{1}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 10$

$= \dfrac{2156}{3} + \dfrac{1540}{3} = \dfrac{3696}{3} = 1232 \text{ cm}^3$

= slant height $\ell = \sqrt{10^2 + 7^2} = \sqrt{149}$

Area of colour sheet required = C.S.A of cone + C.S.A of hemisphere

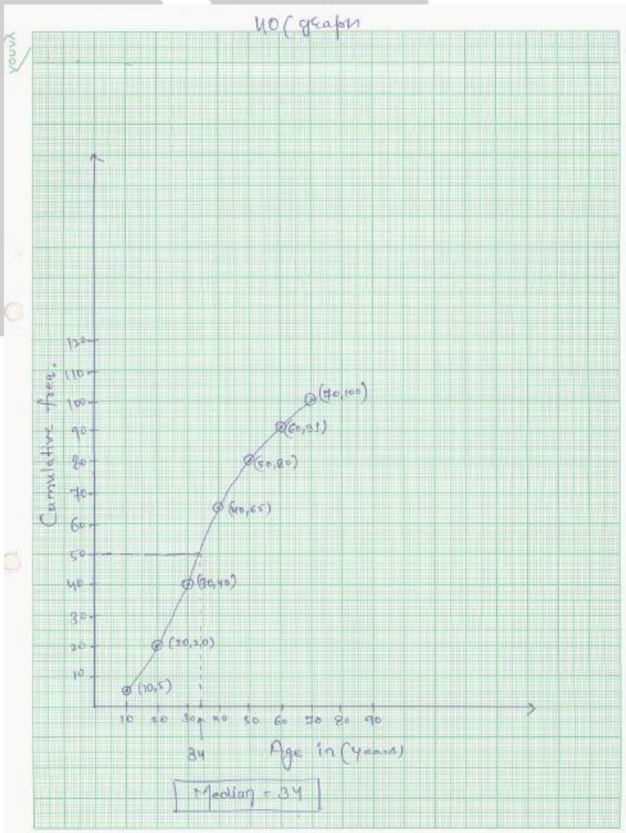
$= \pi r\ell + 2\pi r^2 = \dfrac{22}{7} \times 7 \times \sqrt{149} + 2 \times \dfrac{22}{7} \times 7 \times 7 = 22 \times 12.2 + 308 = 576.4 \text{ cm}^2$

37. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (in years)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of persons	5	15	20	25	15	11	9

Sol.

Age (In Years)	Less than C.F.
Less than 10	5
Less than 20	20
Less than 30	40
Less than 40	65
Less than 50	80
Less than 60	91
Less than 70	100



OR

The distribution given below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Sol.

No.of Wickets	No.of bowlers (f_i)	x_i	$f_i\,x_i$
20 – 60	7	40	280
60 – 100	5	80	400
100 – 140	16	120	1920
140 – 180	12	160	1920
180 – 220	2	200	400
220 – 260	3	240	720
	$\sum f_i = 45$	$\sum x_i = 840$	$\sum f_i x_i = 5640$

Mean = $(\overline{X}) = \dfrac{\sum f_i\,x_i}{\sum f_i} \Rightarrow \dfrac{5640}{45} = 125.33$

No.of wi	f_i	c.f.
20 – 60	7	7
60 – 100	5	12
100 – 140	16	28
140 – 180	12	40
180 – 220	2	42
220 – 260	3	45

Median $\Rightarrow$ $m = \ell + \left(\dfrac{\frac{N}{2} - c.f}{2}\right) \times h$

$h = 40,\ \dfrac{N}{2} = \dfrac{\sum f_i}{2} = \dfrac{45}{2} = 22.5$

c.f. = 12

f = 16

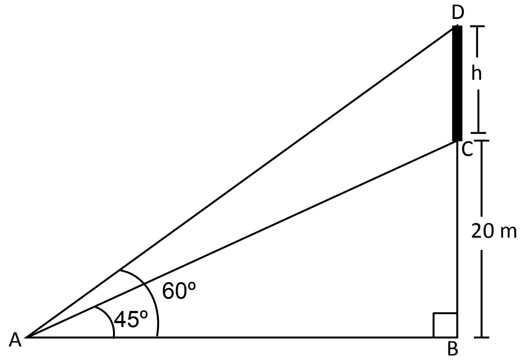
ℓ = 100

$m = 100 + \left(\dfrac{22.5 - 12}{16}\right)\times 40 \Rightarrow 100 + \left(\dfrac{10.5}{16}\right)\times 40 \Rightarrow 100 + 26.25$

$m \Rightarrow 126.25$

38. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. (Use $\sqrt{3}$ =1.73)

Sol. A.T.Q.



Let the height of the tower be h m.

in ΔABC

ten 45º = $\frac{BC}{AB}$

$1 = \frac{20}{AB}$

AB = 20m ---------(1)

in ΔABD

tan 60º = $\frac{BD}{AB}$

$\sqrt{3} = \frac{h+20}{AB}$

$AB = \frac{h+20}{\sqrt{3}}$ -----------(2)

By eq^n (1) & (2)

$\frac{h+20}{\sqrt{3}} = 20$

$h + 20 = 20\sqrt{3}$

$h = 20\sqrt{3} - 20$

$h = 20(\sqrt{3} - 1)$

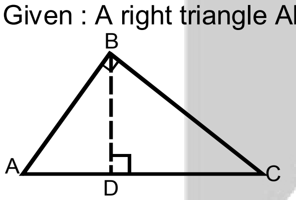
$h = 20(1.73 - 1)$

$h = 20 \times 0.73$

h = 14.6 m.

39. Prove that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides.

Sol. Given : A right triangle ABC, right angled at B.



To prove : $AC^2 = AB^2 + BC^2$

Construction : BD $\perp$ AC

Proof : Δ ADB & Δ ABC

$\angle DAB = \angle CAB$ [Common]

$\angle BDA = \angle CBA$ [90º each]

So, $\Delta ADB \sim \Delta ABC$ [By AA similarity]

$\frac{AD}{AB} = \frac{AB}{AC}$ [Sides are proportional]

or, $AD . AC = AB^2$ ... (i)

Similarly $\Delta BDC \sim \Delta ABC$

So, $\frac{CD}{BC} = \frac{BC}{AC}$

or $CD . AC = BC^2$ ... (ii)

Adding (i) and (ii),

$AD . AC + CD . AC = AB^2 + BC^2$

or, $AC (AD + CD) = AB^2 + BC^2$

or, $AC. AC = AB^2 + BC^2$

or, $AC^2 = AB^2 + BC^2$ Hence Proved.

40. Obtain other zeroes of the polynomial p(x) = $2x^4 - x^3 - 11x^2 + 5x + 5$ if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Sol. P(x) = 2x4 – x3 – 11x2 + 5x + 5

As $x = \sqrt{5}$ is zero, $\left(x - \sqrt{5}\right)$ will be factor, of P(x)

As $x = -\sqrt{5}$ is zero $\left(x + \sqrt{5}\right)$ will be factor , of P(x)

Therefore x2 – 5 will be factor of P(x)

$$
\begin{array}{l}
\qquad\quad 2x^2 - x - 1 \\
x^2 - x\overline{\big)2x^4 - x^3 - 11x^2 + 5x + 5} \\
\qquad 2x^4 \qquad - 10x^2 \\
\qquad \underline{- \qquad\quad +} \\
\qquad -x^3 - x^2 + 5x + 5 \\
\qquad -x^3 \qquad\quad + 5x \\
\qquad \underline{+ \qquad\qquad -} \\
\qquad\qquad -x^2 + 5 \\
\qquad\qquad -x^2 + 5 \\
\qquad\qquad \underline{+ \quad -} \\
\qquad\qquad\quad 0
\end{array}
$$

P(x) = $(x^2 - 5)(2x^2 - x - 1) + 0$

= $(x^2 - 5)(2x^2 - 2x + x - 1)$

= $(x^2 - 5)$ [2x(x –) + 1 (x –)]

= $(x^2 - 5)(x - 1)(2x + 1)$

Therefore other zeros will be x = 1, x = $-\frac{1}{2}$

OR

What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$?

Sol.

$$
\begin{array}{l}
x^2 - 4x + 8 \,\big)\, 2x^3 - 3x^2 + 6x + 7 \,\big(\, 2x + 5 \\
\qquad\qquad\quad 2x^3 - 8x^2 + 16x \\
\qquad\qquad\quad \underline{- \qquad + \qquad -} \\
\qquad\qquad\qquad\quad 5x^2 - 10x + 7 \\
\qquad\qquad\qquad\quad 5x^2 - 20x + 40 \\
\qquad\qquad\qquad\quad \underline{- \qquad + \qquad -} \\
\qquad\qquad\qquad\qquad\quad 10x - 33
\end{array}
$$

So $-10x + 33$ should be added to polynomial so that resulting polynomial will be divisible by $x^2 - 4x + 8$.