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JEE MAIN 2016

ONLINE EXAMINATION

DATE : 09-04-2016

SUBJECT : MATHEMATICS

TEST PAPER
WITH SOLUTIONS & ANSWER KEY

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JEE (Advanced)

Selections (from 2002-2015)	Selections @ 2015
23182	**4124**
(YCCP: 14864 | DLP+eLP: 8318)	(YCCP: 2570 | DLP+eLP: 1554)

Highest selections in JEE (Adv) 2015 in India from any single institute of Kota

JEE (Main)

Selections (from 2009-2015)	Selections @ 2015
93915	**25542**
(YCCP: 66288 | DLP+eLP: 27627)	(YCCP: 18816 | DLP+eLP: 6726)

Highest selections in JEE (Main) 2015 in India from any single institute of Kota

For Classes: XI, XII & XII+

Target: JEE (Main+Advanced) | **JEE** (Main) | Board/ IJSO/ NTSE

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1. If A and B are any two events such that P(A) =2/5 and P(A $\cap$ B) = 3/20, then the conditional probability, P(A/(A' $\cup$ B')), where A' denotes the complement of A, is equal to :

(1) $\frac{8}{17}$ (2) $\frac{1}{4}$ (3) $\frac{5}{17}$ (4) $\frac{11}{20}$

Ans. **(3)**

Sol. $P(A) = \frac{2}{5}$

$P(A \cap B) = \frac{3}{20}$

$P(A/(A' \cup B')) = ?$

$$P(A/(A' \cup B')) = \frac{P(A \cap (A' \cup B'))}{P(A' \cap B')} = \frac{P((A \cap A') \cup (A \cap B'))}{P(A \cap B)'} = \frac{P(\phi \cup (A \cap B'))}{1 - P(A \cap B)}$$

$$= \frac{P(A \cap B')}{1 - \frac{3}{20}} = \frac{P(A) - p(A \cap B)}{\frac{17}{20}} = \frac{\frac{2}{5} - \frac{3}{20}}{\frac{17}{20}} = \frac{\frac{5}{20}}{\frac{17}{20}} = \frac{5}{17}$$

2. For x $\in$ R, x $\neq$ 0, x $\neq$ 1, let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f(_n(X))$, n = 0, 1, 2,……… Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to :

(1) $\frac{4}{3}$ (2) $\frac{1}{3}$ (3) $\frac{5}{3}$ (4) $\frac{8}{3}$

Ans. **(3)**

Sol. $f_0(x) = \frac{1}{1-x}$

$f_1(x) = f_0(f_0(x)) = \frac{1}{1-f_0(x)}$; $f_0(x) \neq 1$

$= \frac{1}{1-\frac{1}{1-x}}$ $x \neq 0$

$= \frac{1-x}{-x}$

$f_2(x) = f_0(f_1(x)) = \frac{1}{1-f_1(x)}$; $f_1(x) \neq 1$

$= \dfrac{1}{1+\dfrac{1-x}{x}} = x$

similarly $f_3(x) = f_0(x)$

$f_4(x) = f_1(x)$ .....

$f_{100}(3) + f_1\left(\dfrac{2}{3}\right) + f_2\left(\dfrac{3}{2}\right) = f_1(3) + f_1\left(\dfrac{2}{3}\right) + \dfrac{3}{2}$

$= 1 - \dfrac{1}{3} + 1 - \dfrac{3}{2} + \dfrac{3}{2} = \dfrac{5}{3}$

3. The distance of the point (1, –2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes $x - y + 2z = 3$ and $2x - 2y + z + 12 = 0$, is

(1) $\dfrac{1}{\sqrt{2}}$ (2) 2 (3) $\sqrt{2}$ (4) $2\sqrt{2}$

Ans. **(4)**

Sol. Equation of plane $\perp$ to the planes.

$x - y + 2z = 3$ & $2x - 2y + z + 12 = 0$

and passes through (1, 2, 2) is

$$\begin{vmatrix} x-1 & y-2 & z-2 \\ 1 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix} = 0$$

$3(x - 1) + 3(y - 2) = 0$

$x + y = 3$ ..... (1)

distance of plane $x + y - 3 = 0$ from (1, – 2, 4) is

$= \left|\dfrac{1-2-3}{\sqrt{1+1}}\right| = 2\sqrt{2}$

4. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from – 1, then | b | is equal to

(1) $\sqrt{2}$ (2) 2 (3) $\sqrt{3}$ (4) 3

Ans. **(3)**

Sol. $x^2 + bx - 1 = 0$ & $x^2 + x + b = 0$ have common root α.

$\Rightarrow \alpha^2 + b\alpha - 1 = 0$

$\alpha^2 + \alpha + b = 0$

$\Rightarrow \dfrac{\alpha^2}{b^2+1} = \dfrac{\alpha}{-(b+1)} = \dfrac{1}{(1-b)} \Rightarrow (b + 1)^2 = (b^2 + 1)(1 - b)$

$\Rightarrow b^2 + 2b + 1 = b^2 - b^3 + 1 - b \Rightarrow b^3 + 3b = 0$

$\Rightarrow b = 0$ or $b^2 = -3$

when b = 0 then common roots is (–1) hence b = 0 rejected.

so $b^2 = -3 \Rightarrow b = \pm\sqrt{3}\,i \Rightarrow |b| = \sqrt{3}$

5. If $2\int_0^1 \tan^{-1} x dx = \int_0^1 \cot^{-1}(1-x+x^2)\, dx$ then $\int_0^1 \tan^{-1}(1-x+x^2)\, dx$ is equal to :

(1) log2 (2) $\frac{\pi}{2}+\log 2$ (3) log4 (4) $\frac{\pi}{2}-\log 4$

Ans. **(1)**

Sol. $2\int_0^1 \tan^{-1} x dx = \int_0^1 \cot^{-1}(1-x+x^2)dx$ ...(1)

$\int_0^1 \tan^{-1}(1-x+x^2)dx = \int_0^1 \left\{\frac{\pi}{2} - \cot^{-1}(1-x+x^2)dx\right\}$

$= \frac{\pi x}{2}\Big|_0^1 - 2\int_0^1 \tan^{-1} x dx = \frac{\pi}{2} - 2\left(\frac{\pi}{4} - \frac{1}{2}\ell n2\right) = \ell n2$

6. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is

(1) $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$

Ans. **(2)**

Sol. $P P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = P^T P$

New $P^T Q^{2015} P = P^T \underbrace{PAP^T \quad PAP^T \quad ....PAP^T}_{2015 \text{ times}} \quad P$

because $= A^{2015}$

Now $A^2 - 2A + I = 0$

$\Rightarrow \quad A^n = nA - (n-1) I \quad \Rightarrow \quad A^{2015} = 2015 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - (2014)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

7. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then A + B + C equals

(1) $\frac{16}{5}$ (2) $\frac{21}{5}$ (3) $\frac{7}{10}$ (4) $\frac{27}{10}$

Ans. **(1)**

Sol. $I = \int \frac{dx}{\cos^3 x \sin^{\frac{1}{2}} x \cos^{\frac{1}{2}} x} = \frac{1}{2}\int \frac{(\tan^2 x + 1)\sec^2 x}{(\tan x)^{\frac{1}{2}}} dx$

tanx = t

$I = \frac{1}{2}\int t^{\frac{3}{2}} dt + \frac{1}{2}\int t^{-\frac{1}{2}} dt$

$= \frac{t^{\frac{5}{2}}}{5} + t^{1/2} + c = \frac{(\tan x)^{\frac{5}{2}}}{5} + (\tan x)^{1/2}$

$A = \frac{1}{2}, B = \frac{5}{2}, C = \frac{1}{5}$

$A + B + C = \frac{16}{5}$

8. The point (2, 1) is translated parallel to the line L : x – y = 4 by $2\sqrt{3}$ units. If the new point Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is :

(1) $2x + 2y = 1 - \sqrt{6}$ (2) $x = y = 3 - 3\sqrt{6}$

(3) $x + y = 2 - \sqrt{6}$ (4) $x + y = 3 - 2\sqrt{6}$

Ans. **(4)**

Sol. Slopes of x – y = 4

$\Rightarrow \tan\theta = 1 \Rightarrow \left(\sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = \frac{1}{\sqrt{2}}\right)$

or $\left(\sin\theta = -\frac{1}{\sqrt{2}}, \cos\theta = -\frac{1}{\sqrt{2}}\right)$

Q is $\left(2 + 2\sqrt{3}\left(-\frac{1}{\sqrt{2}}\right), 1 + 2\sqrt{3}\left(-\frac{1}{\sqrt{2}}\right)\right)$

$(2 - \sqrt{6},\ 1 - \sqrt{6})$

equation of required line is $x + y = 3 - 2\sqrt{6}$

9. If the function $f(x) = \begin{cases} -x, & x < 1 \\ a + \cos^{-1}(x + b), & 1 \le x \le 2 \end{cases}$ is differentiable at x = 1, then $\frac{a}{b}$ is equal to :

(1) $\frac{-\pi - 2}{2}$ (2) $-1 - \cos^{-1}(2)$ (3) $\frac{\pi + 2}{2}$ (4) $\frac{\pi - 2}{2}$

Ans. **(3)**

Sol. L.H.L. at x = 1 is – 1

R.H.L at x = 1 is $a + \cos^{-1}(1 + b)$

$\Rightarrow \quad -1 = a + \cos^{-1}(1 + b)$

$\cos^{-1}(1 + b) = -1 - a$ ....(i)

now L.H.D. at x = 1 is – 1

R.H.D at x = 1 is $\dfrac{-1}{\sqrt{1-(1+b)^2}}$

$\Rightarrow \quad (1 + b)^2 = 0 \quad \Rightarrow \quad b = -1$

Now $\cos^{-1}(1 - 1) = -1 - a$

$a = -1 - \dfrac{\pi}{2}$

$\dfrac{a}{b} = \dfrac{-(2+\pi)}{2(-1)} = \dfrac{2+\pi}{2}$

10. The value of $\sum_{r=1}^{15} r^2\left(\dfrac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$ is equal to :

(1) 1085 (2) 560 (3) 680 (4) 1240

Ans. (3)

Sol. $\sum_{r=1}^{15} r^2\left(\dfrac{{}^{15}C_r}{{}^{15}C_{r-1}}\right) = \sum_{r=1}^{15} r^2\left(\dfrac{15-r+1}{r}\right) = \sum_{r=1}^{15} r(16-r) = 16\left(\dfrac{15\times16}{2}\right) - \dfrac{15\times16\times31}{6} = \dfrac{15\times16}{6}(17) = 680.$

11. In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}$, $-\hat{i}+3\hat{j}+p\hat{k}$ and $5\hat{i}+q\hat{j}-4\hat{k}$, then the point (p, q) lies on a line

(1) parallel to y-axis

(2) making an acute angle with the positive direction of x-axis

(3) parallel to x-axis

(4) making an obtuse angle with the position direction of x-axis.

Ans. (2)

Sol. $\overrightarrow{AB} = -4i + 2j + (p + 1)\hat{k}$

$\overrightarrow{AC} = 2\hat{i} + (q - 1)\hat{j} - 3\hat{k}$

$\overrightarrow{AB}.\overrightarrow{AC} = 0 \quad \Rightarrow \quad -8 + 2(9 - 1) - 3(p + 1) = 0 \quad \Rightarrow \quad -3p + 2q - 13 = 0$

$\Rightarrow$ (p, q) lies on line

$3x - 2y + 13 = 0$

slope = $\dfrac{3}{2}$

12. If $\lim\limits_{x \to \infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x} = e^3$, then 'a' is equal to :

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) 2 (4) $\frac{1}{2}$

Ans. **(2)**

Sol. $L = \lim\limits_{x \to \infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}$ must be of the from 1^{∞}

$L = e^{\lim\limits_{x\to\infty}\left(\frac{a}{x}-\frac{4}{x^2}\right)2x}$

$\Rightarrow$ $L = e^{\lim\limits_{x\to\infty}\frac{2(ax-4)}{x}}$

$= e^{2a} = e^3$

$a = \frac{3}{2}$

13. The number of $x \in [0, 2\pi]$ for which $\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is

(1) 6 (2) 4 (3) 8 (4) 2

Ans. **(3)**

Sol. $2\sin^4x + 18\cos^2x = 1 + 2\cos^4x + 18\sin^2x + 2\sqrt{2\cos^4 x + 18\sin^2 x}$

$2(\sin^2x - \cos^2x) + 18(\cos^2x - \sin^2x) = 1 + 2\sqrt{2\cos^4 x + 18\sin^2 x}$

$\Rightarrow$ $16(\cos^2x - \sin^2x) = 1 + 2\sqrt{2\cos^4 x + 18\sin^2 x}$

$\Rightarrow$ $16\cos 2x - 1 = 2\sqrt{2\left(\frac{1+\cos 2x}{2}\right)^2 + 9(1-\cos 2x)}$

$\Rightarrow$ $256\cos^2 2x + 1 - 32\cos 2x = 4\left(\frac{1+2\cos 2x+\cos^2 2x}{2} + 9(1-\cos 2x)\right)$

$\Rightarrow$ $256\cos^2 2x + 1 - 32\cos 2x = 2(19 - 16\cos 2x + \cos^2 2x)$

$\Rightarrow$ $254\cos^2 2x = 37$

$\Rightarrow$ $\cos^2 2x = \frac{37}{254}$ $\Rightarrow$ $\cos 2x = \pm\sqrt{\frac{37}{254}} \in [-1, 1]$

clearly 8 solutions

14. If m and M are the minimum and the maximum values of $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x$, $x \in R$, then M – m is equal to

(1) $\frac{7}{4}$ (2) $\frac{15}{4}$ (3) $\frac{9}{4}$ (4) $\frac{1}{4}$

Ans. **(3)**

Sol. $4 + \frac{1}{2}\sin^2 2x - \frac{1}{2}(2\cos^2 x)^2$

$= 4 + \frac{1}{2}\sin^2 2x - \frac{1}{2}(1 + \cos 2x)^2 = -\cos^2 2x - \cos 2x + 4 = -[\cos^2 2x + \cos 2x - 4] = \frac{17}{4} - \left(\cos 2x + \frac{1}{2}\right)^2$

M = maximum value = $\frac{17}{4}$

m = minimum value = 2

$M - m = \frac{17}{4} - 2 = \frac{9}{4}$.

15. If a variable line drawn through the intersection of the lines $\frac{x}{3} + \frac{y}{4} = 1$ and $\frac{x}{4} + \frac{y}{3} = 1$, meets the coordinate axes at A and B, (A ≠ B), then the locus of the midpoint of AB is

(1) $7xy = 6(x + y)$ (2) $6xy = 7(x + y)$

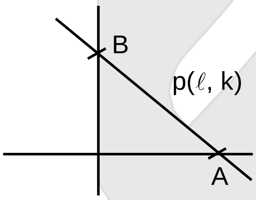
(3) $4(x + y)^2 - 28(x + y) + 49 = 0$ (4) $14(x + y)^2 - 97(x + y) + 168 = 0$

Ans. **(1)**

Sol. $4x + 3y = 12$ ....(1)

$3x + 4y = 12$ ....(2)

equation of lines passing through the intersection of the lines



$4x + 3y - 12 + \lambda(3x + 4y - 12) = 0$

$A = C\left(\frac{12(1+\lambda)}{4+3\lambda},\ 0\right)$

$B = \left(0,\ \frac{12(1+\lambda)}{3+4\lambda}\right)$

$\ell n = \dfrac{6(1+\lambda)}{4+3\lambda}$ .....(3)

$k = \dfrac{6(1+\lambda)}{3+4\lambda}$ ... (4)

from (3) & (4)

$\lambda = \dfrac{3k-4h}{3h-4k}$ put in (1)

7hk = 6(h + k)

hence locus is 7xy = 6(x + y)

16. If f(x) is a differentiable function in the interval $(0, \infty)$ such that f(1) = 1 and $\lim\limits_{t\to x} \dfrac{t^2f(x)-x^2f(t)}{t-x} = 1$, for each x > 0, then $f\left(\dfrac{3}{2}\right)$ is equal to :

(1) $\dfrac{13}{6}$ (2) $\dfrac{23}{18}$ (3) $\dfrac{25}{9}$ (4) $\dfrac{31}{18}$

Ans. **(4)**

Sol. Differentiate w.r.t. t

$\lim\limits_{t\to x} \dfrac{2t\ \ f(x)-x^2f'(t)}{1} = 1$

$\Rightarrow$ $2x\, f(x) - x^2f'(x) = 1$

$f'(x) = \dfrac{2x\ \ f(x)-1}{x^2}$

$\dfrac{dy}{dx} = \dfrac{2y}{x} - \dfrac{1}{x^2}$

I.F. $= e^{-\int \frac{2}{x}dx}$

$= e^{-2\ell nx} = \dfrac{1}{x^2}$

$y\left(\dfrac{1}{x^2}\right) = \int -\dfrac{1}{x^4}dx$

$\dfrac{y}{x^2} = \dfrac{1}{3x^3} + c$

at x = 1, y = 1

$\Rightarrow$ $c = \dfrac{2}{3}$

$f(x) = \dfrac{1}{3x} + \dfrac{2x^2}{3}$

$f\left(\dfrac{3}{2}\right) = \dfrac{31}{18}$

17. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3$, $y = 8t^3 - 1$, $t \in R$, meets the curve again at a point Q, then the coordinates of Q are :

(1) $(t^2 + 3, -t^3 - 1)$ (2) $(t^2 + 3, t^3 - 1)$ (3) $(16t^2 + 3, -64t^3 - 1)$ (4) $(4t^2 + 3, -8t^3 - 1)$

Ans. **(1)**

Sol. $P(x = 4t^2 + 3, y = 8t^2 - 1)$

let $Q(4t_1^2 + 3, 8y_1^3 - 1)$

at P, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{24t^2}{8t} = 3t$

$\therefore$ tangent at P is $y - 8t^3 + 1 = 3t(x - 4t^2 - 3)$

Q will satisfy it

$\therefore$ $8t_1^3 - 8t^3 = 3t(4t_1^2 - 4t^2)$

$8(t_1 - t)(t_1^2 + t_1t + t^2) = 3t.\ 4(t_1 - t)(t_1 + t)$

$2(t_1^2 + t_1t + t^2) = 3t(t_1 + t)$

$2t_1^2 + 2t_1t + 2t^2 = 3t\,t_1 + 3t^2$

$2t_1^2 - t_1t - t^2 = 0$

$(t_1 - t)(2t_1 + t) = 0$

$t_1 = -\frac{t}{2}$

$\therefore$ $Q(t^2 + 3, -t^3 - 1)$ **Ans. (1)**

18. If the tangent at a point on the ellipse $\frac{x^2}{27} + \frac{y^2}{3} = 1$ meets the coordinate axes at A and B, and O is the origin, then the minimum area (in sq. units) of the triangle OAB is :

(1) 9 (2) $\frac{9}{2}$ (3) $9\sqrt{3}$ (4) $3\sqrt{3}$

Ans. **(1)**

Sol. Let $P(3\sqrt{3}\cos\theta, \sqrt{3}\sin\theta)$

$\therefore$ tangent is $\frac{x}{3\sqrt{3}}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = 1$

$\Rightarrow A(3\sqrt{3}\sec\theta, 0)$ $B(0, \sqrt{3}\operatorname{cosec}\theta)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}$ OA. OB

$= \frac{1}{2}(3\sqrt{3}\sec\theta.\ \sqrt{3}\operatorname{cosec}\theta$

$= \frac{9}{2\sin\theta\cos\theta} = \frac{9}{\sin 2\theta}$

$\therefore$ minimum area of $\Delta OAB = \frac{9}{1} = 9$

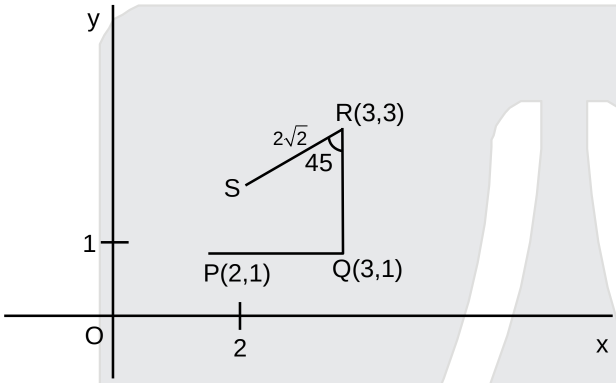
19. The point represented by 2+i in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by :

(1) 2 + 2i (2) – 2 – 2i (3) 1 + i (4) – 1 – i

Ans. **(3)**

Sol. Let P(2 + i)

By rotation theorem



$$\frac{z-(3+3i)}{3+i-(3+3i)} = \frac{2\sqrt{2}}{2} e^{(-\pi/4)\,i}$$

$$\frac{z-3-3i}{-2i} = 1-i$$

$z - 3 - 3i = - 2i - 2$

$z = 1 + i$

20. A circle passes through (–2, 4). Which one of the following equations can represent a diameter of this circle?

(1) 4x + 5y – 6 = 0 (2) 5x + 2y + 4 = 0 (3) 2x – 3y + 10 = 0 (4) 3x + 4y – 3 = 0

Ans. **(3)**

Sol. Required circle is

$(x - 0)^2 + (y - 2)^2 + \lambda(x) = 0$

it passes (– 2, 4)

$\therefore$ 4 + 4 – 2λ = 0

λ = 4

$\therefore$ circle is $x^2 + y^2 - 4y + 4x + 4 = 0$

centre (–2, 2) which satisfy

2x – 3y + 10 = 0 Ans. 3

21. The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is :

(1) 4 (2) 1 (3) 2 (4) 3

Ans. **(3)**

Sol. $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$

$\Rightarrow \cos^3x + \sin^3x + \sin^3x - 3\sin^2x\cos x = 0$

$\Rightarrow (\cos x + \sin x + \sin x)(\cos^2x + \sin^2x + \sin^2x - \cos x\sin x - \cos x\sin x - \sin^2x) = 0$

$\Rightarrow \cos x = -2\sin x$ or $\cos x = \sin x$

$\tan x = -\frac{1}{2}$ $\tan = 1 \Rightarrow x = \pi/4$

$x = -\tan^{-1}\frac{1}{2}$ $\therefore$ two solutions

22. The shortest distance between the lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ lies in the interval :

(1) (2, 3] (2) [0, 1) (3) (3, 4] (4) [1, 2)

Ans. **(1)**

Sol. $\frac{x}{z} = \frac{y}{z}\ \frac{z}{1}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

shortest distance

$= (\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)$

here $\vec{b}_1 - \vec{b}_2$ = (2i + 2j + k) × (–i + 8j + 4k)

= – 9j + 18k

$(\vec{b}_1 \times \vec{b}_2) = \frac{-j+2k}{\sqrt{5}}$

$\vec{a}_2 - \vec{a}_1$ = – 2i + 4j + 5k

$\therefore$S.D. (–2i + 4j + 6k). $\frac{(-j+2k)}{\sqrt{5}} = \frac{6}{\sqrt{5}}$ which lies in (2,3]

23. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is :

(1) $\frac{11!}{(2!)^3}$ (2) 59 (3) 110 (4) 56

Ans. **(1)**

Sol. There are 1M, 3E, 1D, II, IT, 2R, 2A, 2N

R– –E – – – – – – – – – – – –

rest of 11 letters can be arranged in $\frac{11!}{(2\ \ !)^3}$

24. Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e\text{-}^2 - 18e + 5 = 0$. If S(5, 0) is a focus and 5x = 9 is the corresponding directrix of hyperbola, then $a^2 - b^2$ is equal to

(1) – 7 (2) – 5 (3) 5 (4) 7

Ans. **(1)**

Sol. $9e^2 - 18e + 5 = 0$

$\Rightarrow e = \frac{5}{3}$

$\therefore 1 + \frac{b^2}{a^2} = e^2 = \frac{25}{9}$ .... (i)

Also distance between foci and directrix is

$= \left(ae - \frac{a}{e}\right) = 5 - \frac{9}{5}$

$\Rightarrow a\left(\frac{5}{3} - \frac{3}{5}\right) = \frac{16}{5} \Rightarrow a = 3$

from (i)

$1 + \frac{b^2}{9} = e^2 = \frac{25}{9} \Rightarrow b^2 = 16$

$\therefore a^2 - b^2 = 9 - 16 = -7$

25. Consider the following two statements :

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals :

(1) (F, T) (2) (T, F) (3) (F, F) (4) (T, T)

Ans. **(1)**

Sol. Statement P is False
Statement Q is True.
$V_1 \equiv F$
$V_2 \equiv T$
Ans. 1

26. The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is :

(1) $\frac{\sqrt{15}}{2}$ (2) $\frac{\sqrt{19}}{2}$ (3) $\sqrt{\frac{15}{2}}$ (4) $\sqrt{\frac{19}{2}}$

Ans. **(1)**

Sol. Let point at minimum distance from O is
$(h, h^2 - 4)$
$\therefore OP^2 = h^2 + (h^2 - 4)^2$

$$\frac{d(OP^2)}{dh} = 2h + 2(h^2 - 4)2h = 0$$

$$\Rightarrow h = \pm\sqrt{\frac{7}{2}}, 0$$

$$\left(\frac{d^2(OP^2)}{dh^2}\right)_{h=\pm\sqrt{\frac{7}{2}}} > 0$$

$\therefore$ OP is min at $h = \pm\sqrt{\frac{7}{2}}$

$$OP_{min} = \sqrt{\frac{7}{2} + \left(\frac{7}{2} - 4\right)^2} = \frac{\sqrt{15}}{2}$$

27. Let x, y, z be positive real numbers such that $x + y + z = 12$ and $x^3y^4z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to

(1) 270 (2) 258 (3) 216 (4) 342

Ans. **(3)**

Sol. $x + y + z = 12$
$x^3y^4z^5 = (0.1)(600)^3$

$$\frac{3\left(\frac{x}{3}\right) + 4\left(\frac{y}{4}\right) + 5\left(\frac{z}{5}\right)}{12} \geq \left\{\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)^4\left(\frac{z}{5}\right)^5\right\}^{1/12}$$

$$1 \geq \frac{x^3y^4z^5}{(60)^3(4\times25)}$$

$x^3y^4z^5 \leq (0.1)(600)^3$
But $x^3y^4z^5 = (10.1)(600)^3$
Clearly AM = GM

Hence $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ $\Rightarrow x = 3, y = 4, z = 5$

$\Rightarrow x^3 + y^3 + z^3 = 27 + 64 + 125 = 216$

28. If the mean deviation of the numbers 1, 1 + d, ….., 1 + 100d from their mean is 255, then a value of d is :

(1) 10 (2) 20.2 (3) 5.05 (4) 10.1

Ans. **(4)**

Sol. Mean is $\dfrac{101+\dfrac{100\times101}{2}}{101}$ = 1 + 50 d

sum of deviation about mean is

50 d + 49d + ... d + 0 + d + ... + 50 d

= 50. 51 d

Mean deviation = $\dfrac{50\times51d}{101}$ = 255

d = $\dfrac{255\times101}{2550}$ = 10.1

29. For x ∈ R, x = – 1, if $(1 + x)^{2016} + x\,(1 + x)^{2015} + x^2\,(1 + x)^{2014}$ + …….. + $x^{2016} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to :

(1) $\dfrac{2016!}{16!}$ (2) $\dfrac{2017!}{2000!}$ (3) $\dfrac{2017!}{17!\ 2000!}$ (4) $\dfrac{2016!}{17!\ 1999!}$

Ans. **(3)**

Sol. $\sum_{i=0}^{2016} c_i.x^i = (1 + x)^{2016} + x(1 +x)^{2015} + x^2\,(1 + x)^{2014} + ... + x^{2016}$

$$= \frac{(1+x)^{2016}\left(1-\left(\dfrac{x}{1+x}\right)^{2017}\right)}{1-\dfrac{x}{1+x}}$$

$$= \frac{\dfrac{(1+x)^{2016}}{1}-\dfrac{x^{2017}}{(1+x)}}{\dfrac{x+1-x}{1+x}} = \frac{(1+x)^{2017}-x^{2017}}{1}$$

$\therefore\ a_{17} = {}^{2017}C_{17} = \dfrac{2007!}{17!\ \ 2000!}$

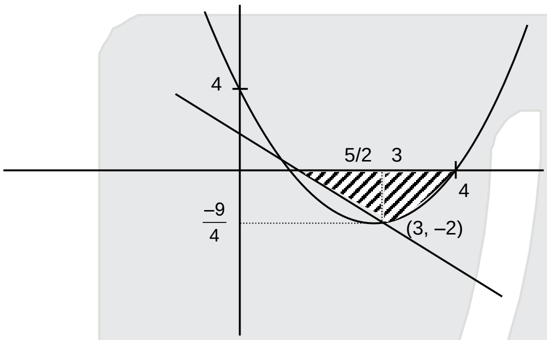
30. The area (in sq. units) of the region described by $A = \{(x, y) \mid y \geq x^2 - 5x + 4,\ x + y \geq 1,\ y \leq 0\}$ is :

(1) $\frac{7}{2}$ (2) $\frac{13}{6}$ (3) $\frac{17}{6}$ (4*) $\frac{19}{6}$

Ans. **(4)**

Sol. $A = \{(x,y) \mid y \geq x^2 - 5x + 4,\ x + y \geq 1,\ y \leq 0\}$

Here $y \geq x^2 - 5x + 4$, $x + y \geq 1$, $y \leq 0$



Required area $= \frac{1}{2} .2\ 2 + \int_3^4 (5x - x^2 - 4)dx$

$= 2 + \left[\frac{5x^2}{2} - \frac{x^3}{3} - 4x\right]_3^4$

$= 2 + \frac{5}{2}(16 - 9) - \frac{1}{3}(64 - 27) - 4\ (4 - 3)$

$- 2 + \frac{35}{2} - \frac{37}{3} - 4 = \frac{19}{6}$