

CODE

9

PAPER-2

P2-15-9

459619

Time : 3 Hours

Maximum Marks : 240

**READ THE INSTRUCTIONS CAREFULLY****GENERAL :**

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The ORS CODE is printed on its left part as well as the right part. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name and roll number in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet, verify that the booklet contains **32** pages and that all the **60** questions along with the options are legible.

**QUESTION PAPER FORMAT AND MARKING SCHEME :**

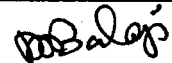
8. The question paper has three parts: Physics, Chemistry and Mathematics. Each part has three sections.
9. Carefully read the instructions given at the beginning of each section.
10. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).  
**Marking scheme:** +4 for correct answer and 0 in all other cases.
11. Section 2 contains 8 multiple choice questions with one or more than one correct option.  
**Marking scheme:** +4 for correct answer, 0 if not attempted and -2 in all other cases.
12. Section 3 contains 2 "paragraph" type questions. Each paragraph describes an experiment, a situation or a problem. Two multiple choice questions will be asked based on this paragraph. One or more than one option can be correct.  
**Marking scheme:** +4 for correct answer, 0 if not attempted and -2 in all other cases.

**OPTICAL RESPONSE SHEET :**

13. The ORS consists of an original (top sheet) and its carbon-less copy (bottom sheet).
14. Darken the appropriate bubbles on the original by applying sufficient pressure. This will leave an impression at the corresponding place on the carbon-less copy.
15. The original is machine-gradable and will be collected by the invigilator at the end of the examination.
16. You will be allowed to take away the carbon-less copy at the end of the examination.
17. Do not tamper with or mutilate the ORS.
18. Write your name, roll number and the name of the examination center and sign with pen in the space provided for this purpose on the original. **Do not write any of these details anywhere else.** Darken the appropriate bubble under each digit of your roll number.

DO NOT BREAK THE SEAL WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

Please see the last page of this booklet for rest of the instructions.



**SPACE FOR ROUGH WORK**

11/11/11

$$dn \cos r_1 + n \cos r_1 \cdot dr_1 = 0 \quad \cos(60 - r_1) \cdot dr_1 = \cos \theta \cdot d\theta$$

$$\frac{dn}{n} + \frac{\sqrt{3}}{2} n dr_1 = 0$$

$$\frac{\sqrt{3}}{2} = \frac{+dn}{\sqrt{3}n} = \frac{1}{2} \frac{d\theta}{\theta}$$

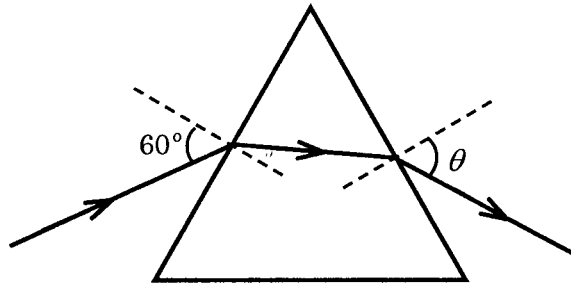
# PART I : PHYSICS

## SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  - +4 If the bubble corresponding to the answer is darkened
  - 0 In all other cases

Q.1 A monochromatic beam of light is incident at  $60^\circ$  on one face of an equilateral prism of refractive index  $n$  and emerges from the opposite face making an angle  $\theta(n)$  with the normal (see the figure). For  $n = \sqrt{3}$  the value of  $\theta$  is  $60^\circ$  and  $\frac{d\theta}{dn} = m$ . The value of  $m$  is

~~1~~  
2



$$r_1 = \sin^{-1} \frac{\sqrt{3}}{2n} \quad \frac{1}{\sqrt{1 - \frac{3}{4n^2}}}$$

$$60 - r_1 = \theta$$

$$\frac{\sqrt{3}}{2} = \sqrt{3} \sin r_1 \quad r_1 = 30^\circ$$

Space for rough work

$$n \sin r_1 = \frac{\sqrt{3}}{2}$$

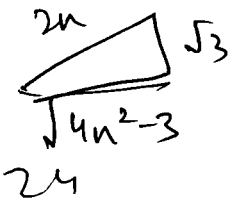
$$n \sin(60 - r_1) = \sin \theta$$

$$\frac{\sqrt{3}}{2} = \frac{3}{2\sqrt{3}}$$

$$\sin r_1 = \frac{\sqrt{3}}{2n}$$

$$\frac{\sqrt{3}}{2} \cos r_1 - \frac{\sin r_1}{2} = \sin \theta$$

$$\frac{3}{4} - \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{4} = \frac{1}{2}$$



$$\frac{\sqrt{3}}{2} \frac{\sqrt{4n^2 - 3}}{2n} - \frac{\sqrt{3}}{\sqrt{4n^2 - 3} \cdot 2n} = \sin \theta$$

$$\frac{1\sqrt{3}}{2\sqrt{3}}$$

$$\frac{\sqrt{3}}{4} = \frac{3}{2\sqrt{3}}$$

$$\frac{3}{8} - \frac{1}{8}$$

$$dn \frac{1}{2} - n \cos(60 - r_1) dr_1 = \cos \theta d\theta$$

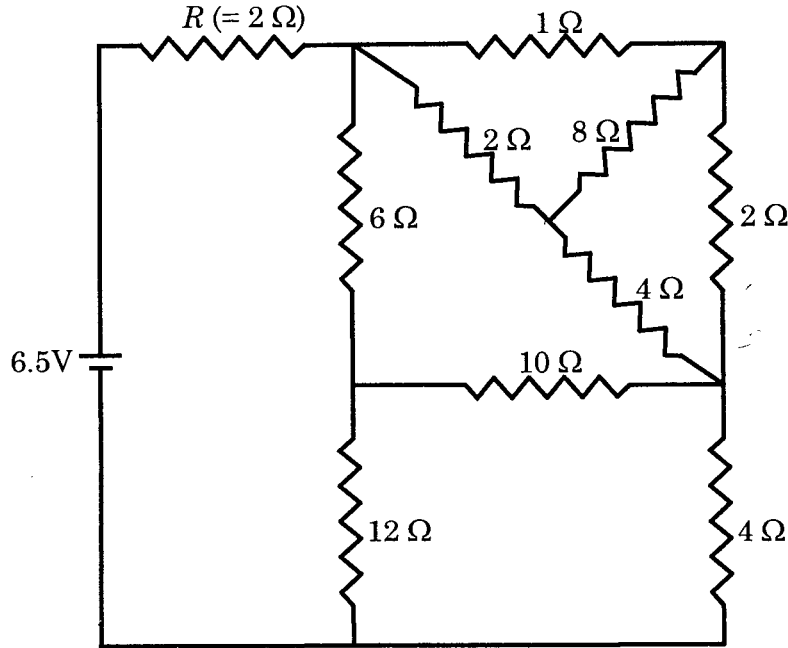
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$$n \sin r_1 = \frac{\sqrt{3}}{2}$$

$$\frac{dn}{n} + \frac{\sqrt{3}}{2} n dr_1 = \frac{d\theta}{\sin \theta}$$

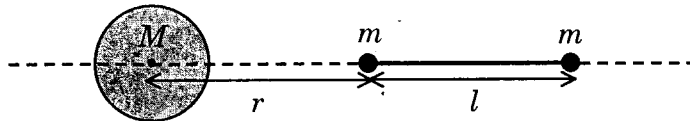
$$\frac{3}{32} \quad \frac{d\theta}{dn} = \frac{1}{2}$$

Q.2 In the following circuit, the current through the resistor  $R (= 2\Omega)$  is  $I$  Amperes. The value of  $I$  is



Q.3 An electron in an excited state of  $\text{Li}^{2+}$  ion has angular momentum  $3h/2\pi$ . The de Broglie wavelength of the electron in this state is  $p\pi a_0$  (where  $a_0$  is the Bohr radius). The value of  $p$  is

Q.4 A large spherical mass  $M$  is fixed at one position and two identical point masses  $m$  are kept on a line passing through the centre of  $M$  (see figure). The point masses are connected by a rigid massless rod of length  $l$  and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to  $M$  is at a distance  $r = 3l$  from  $M$ , the tension in the rod is zero for  $m = k\left(\frac{M}{288}\right)$ . The value of  $k$  is



$$\frac{GMm}{9l^2} - \frac{Gmm}{l^2} = \frac{GM}{16}$$

$$\frac{M}{9} - \frac{M}{16} = 2m.$$

Space for rough work

Handwritten calculations for Q.4:

$$\frac{GMm}{16l^2} + \frac{Gmm}{l^2} = \frac{GM}{16}$$

$$\frac{7M}{32 \times 9} = m.$$

$$I = 6.5$$

$$I = 6.10$$

$$2 + \frac{9}{2}$$

$$\frac{9 \times 8 \times 6}{24 \times 2}$$

$$\frac{GMm}{9l^2} = \frac{GMm}{16}$$

$$\frac{1}{18} = \frac{1}{9}$$

$$\frac{19 \times 16}{24 \times 3}$$

Q.5 The energy of a system as a function of time  $t$  is given as  $E(t) = A^2 \exp(-\alpha t)$ , where  $\alpha = 0.2 \text{ s}^{-1}$ . The measurement of  $A$  has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of  $E(t)$  at  $t = 5 \text{ s}$  is

Q.6 The densities of two solid spheres  $A$  and  $B$  of the same radii  $R$  vary with radial distance  $r$  as  $\rho_A(r) = k \left(\frac{r}{R}\right)$  and  $\rho_B(r) = k \left(\frac{r}{R}\right)^5$ , respectively, where  $k$  is a constant. The moments of inertia of the individual spheres about axes passing through their centres are  $I_A$  and  $I_B$ , respectively. If  $\frac{I_B}{I_A} = \frac{n}{10}$ , the value of  $n$  is

Q.7 Four harmonic waves of equal frequencies and equal intensities  $I_0$  have phase angles  $0, \pi/3, 2\pi/3$  and  $\pi$ . When they are superposed, the intensity of the resulting wave is  $nI_0$ . The value of  $n$  is

Q.8 For a radioactive material, its activity  $A$  and rate of change of its activity  $R$  are defined as  $A = -\frac{dN}{dt}$  and  $R = -\frac{dA}{dt}$ , where  $N(t)$  is the number of nuclei at time  $t$ . Two radioactive sources  $P$  (mean life  $\tau$ ) and  $Q$  (mean life  $2\tau$ ) have the same activity at  $t = 0$ . Their rates of change of activities at  $t = 2\tau$  are  $R_P$  and  $R_Q$ , respectively. If  $\frac{R_P}{R_Q} = \frac{n}{e}$ , then the value of  $n$  is

Space for rough work

$N = N_0 e^{-kt}$   
 $\frac{dN}{dt} = -kN_0 e^{-kt}$   
 $\frac{dA}{dt} = -k^2 N_0 e^{-kt}$

$E = A^2 e^{-\alpha t}$   
 $\frac{dE}{dt} = 2A e^{-\alpha t} \cdot dA + \frac{A^2 e^{-\alpha t} \cdot (-\alpha) dt}{A^2 e^{-\alpha t}}$

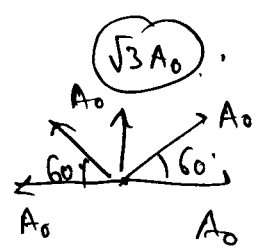
$k_1 = \frac{1}{2\tau} = 0.2$   
 $k_2 = \frac{1}{\tau} = 1.50$

$2 \times 1.25 = 2.5$   
 $2.5 + 1.5 = 4$

$\frac{1}{k} = \tau$   
 $\int_0^R r^2 k \left(\frac{r}{R}\right) \cdot 4\pi r^2 \cdot dr$   
 $\int_0^R r^2 k \left(\frac{r}{R}\right)^5 \cdot 4\pi r^2 \cdot dr$

$\frac{4\pi k}{R} \cdot \frac{R^6}{6} = \frac{4\pi k R^5}{6}$   
 $\frac{10}{4\pi k R^5}$

$\frac{k_1}{k_2} = \frac{e^{-2\tau}}{e^{-\tau}}$



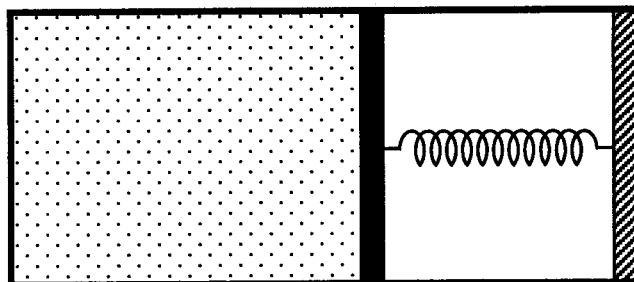
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SECTION 2 (Maximum Marks : 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - 2 In all other cases

Q.9 An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature  $T_1$ , pressure  $P_1$  and volume  $V_1$  and the spring is in its relaxed state. The gas is then heated very slowly to temperature  $T_2$ , pressure  $P_2$  and volume  $V_2$ . During this process the piston moves out by a distance  $x$ . Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are)



- (A) If  $V_2 = 2V_1$  and  $T_2 = 3T_1$ , then the energy stored in the spring is  $\frac{1}{4}P_1V_1$
- (B) If  $V_2 = 2V_1$  and  $T_2 = 3T_1$ , then the change in internal energy is  $3P_1V_1$
- (C) If  $V_2 = 3V_1$  and  $T_2 = 4T_1$ , then the work done by the gas is  $\frac{7}{3}P_1V_1$
- (D) If  $V_2 = 3V_1$  and  $T_2 = 4T_1$ , then the heat supplied to the gas is  $\frac{17}{6}P_1V_1$

Space for rough work

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \cdot \frac{2}{3}$$

$$\frac{2}{3} k x = k x$$

- Q.10 A fission reaction is given by  ${}_{92}^{236}\text{U} \rightarrow {}_{54}^{140}\text{Xe} + {}_{38}^{94}\text{Sr} + x + y$ , where  $x$  and  $y$  are two particles. Considering  ${}_{92}^{236}\text{U}$  to be at rest, the kinetic energies of the products are denoted by  $K_{\text{Xe}}$ ,  $K_{\text{Sr}}$ ,  $K_x$  (2 MeV) and  $K_y$  (2 MeV), respectively. Let the binding energies per nucleon of  ${}_{92}^{236}\text{U}$ ,  ${}_{54}^{140}\text{Xe}$  and  ${}_{38}^{94}\text{Sr}$  be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option(s) is(are)
- (A)  $x = n, y = n, K_{\text{Sr}} = 129 \text{ MeV}, K_{\text{Xe}} = 86 \text{ MeV}$
- (B)  $x = p, y = e^-, K_{\text{Sr}} = 129 \text{ MeV}, K_{\text{Xe}} = 86 \text{ MeV}$
- (C)  $x = p, y = n, K_{\text{Sr}} = 129 \text{ MeV}, K_{\text{Xe}} = 86 \text{ MeV}$
- (D)  $x = n, y = n, K_{\text{Sr}} = 86 \text{ MeV}, K_{\text{Xe}} = 129 \text{ MeV}$

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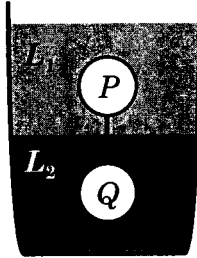
**Space for rough work**

$$\left(\frac{\rho_1 \sigma_1}{\eta_1}\right) \propto v_p$$

$$\rho_1 V g - \sigma_1 V g = 6\pi r v_p$$

Q.11 Two spheres  $P$  and  $Q$  of equal radii have densities  $\rho_1$  and  $\rho_2$ , respectively. The spheres are connected by a massless string and placed in liquids  $L_1$  and  $L_2$  of densities  $\sigma_1$  and  $\sigma_2$  and viscosities  $\eta_1$  and  $\eta_2$ , respectively. They float in equilibrium with the sphere  $P$  in  $L_1$  and sphere  $Q$  in  $L_2$  and the string being taut (see figure). If sphere  $P$  alone in  $L_2$  has terminal velocity  $\vec{V}_P$  and  $Q$  alone in  $L_1$  has terminal velocity  $\vec{V}_Q$ , then

$$\eta_1 v_p = \eta_2 v_q$$



- (A)  $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$     
  (B)  $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$     
  (C)  $\vec{V}_P \cdot \vec{V}_Q > 0$     
  (D)  $\vec{V}_P \cdot \vec{V}_Q < 0$

Q.12 In terms of potential difference  $V$ , electric current  $I$ , permittivity  $\epsilon_0$ , permeability  $\mu_0$  and speed of light  $c$ , the dimensionally correct equation(s) is(are)

- (A)  $\mu_0 I^2 = \epsilon_0 V^2$     
  (B)  $\epsilon_0 I = \mu_0 V$     
  (C)  $I = \epsilon_0 c V$     
  (D)  $\mu_0 c I = \epsilon_0 V$

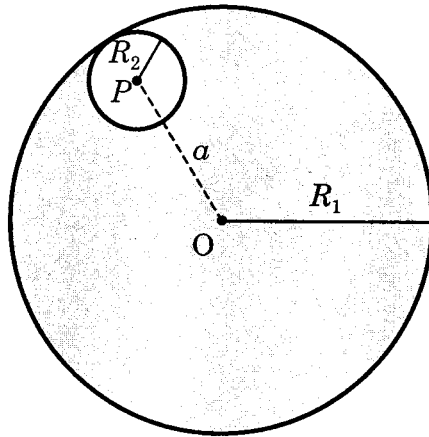
Space for rough work

Handwritten rough work for Q.12:

- $\frac{V}{I} = \frac{kg m^2}{s^2 C} \times \frac{s}{C} = \frac{kg m^2}{s^2 C^2}$
- $\frac{kg m^2}{s^2 C} = V$
- $I = \frac{C}{s}$
- $\frac{Br}{\mu_0} = \epsilon_0 \times \frac{Q}{\epsilon_0^2 r^2}$
- $\epsilon_0 = \frac{s^2 C^2}{m^3 kg}$
- $\frac{kg m}{s^2} \times \frac{m^2}{C^2} = \frac{1}{\epsilon_0}$
- $\frac{1}{\mu_0} \times \frac{kg m^3}{s^2 C^2} = \frac{1}{\mu_0}$
- $\epsilon_0 \times \frac{Br}{\mu_0} = \mu_0 \times \frac{Q}{\epsilon_0 r}$
- $\epsilon_0^2 Br = \mu_0^2 \times Q$
- $\mu_0 = \frac{kg m}{C^2}$



Q.13 Consider a uniform spherical charge distribution of radius  $R_1$  centred at the origin O. In this distribution, a spherical cavity of radius  $R_2$ , centred at P with distance  $OP = a = R_1 - R_2$  (see figure) is made. If the electric field inside the cavity at position  $\vec{r}$  is  $\vec{E}(\vec{r})$ , then the correct statement(s) is(are)



- (A)  $\vec{E}$  is uniform, its magnitude is independent of  $R_2$  but its direction depends on  $\vec{r}$
- (B)  $\vec{E}$  is uniform, its magnitude depends on  $R_2$  and its direction depends on  $\vec{r}$
- (C)  $\vec{E}$  is uniform, its magnitude is independent of  $a$  but its direction depends on  $\vec{a}$
- (D)  $\vec{E}$  is uniform and both its magnitude and direction depend on  $\vec{a}$

Space for rough work

$$\frac{S^2 C^2}{m^3 kg \cdot kg m}$$

$$\frac{C^4 S^2}{kg^2 m^4}$$

$$kg \frac{m}{s^2} = \frac{N}{s} > \frac{C}{s} = 2$$

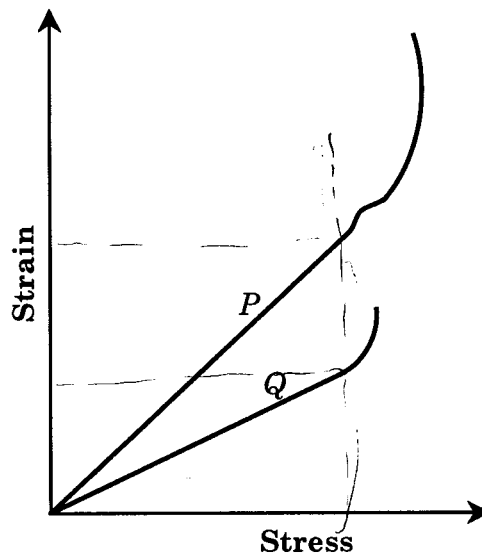
$$\frac{S^2 C^2}{m^3 kg} > kg \frac{m^2}{s^2 C}$$

$$E = \frac{\rho a}{2\epsilon_0} = \rho$$

$$\frac{\mu_0}{\epsilon_0} = \frac{1}{V^2}$$

$$\frac{\epsilon_0}{\mu_0} = \frac{1}{V^2}$$

Q.14 In plotting stress versus strain curves for two materials  $P$  and  $Q$ , a student by mistake puts strain on the  $y$ -axis and stress on the  $x$ -axis as shown in the figure. Then the correct statement(s) is(are)



- (A)  $P$  has more tensile strength than  $Q$
- (B)  $P$  is more ductile than  $Q$
- (C)  $P$  is more brittle than  $Q$
- (D) The Young's modulus of  $P$  is more than that of  $Q$

Space for rough work

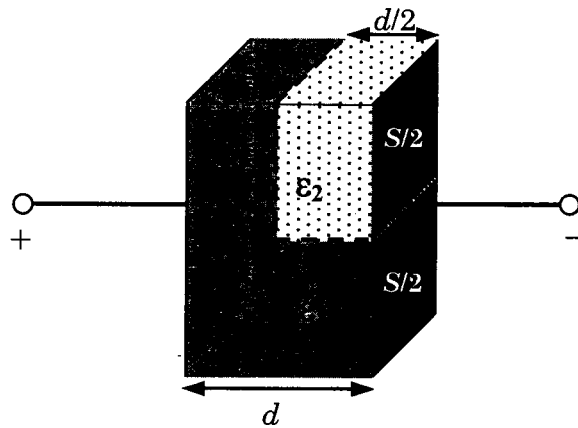
$\frac{F}{A} = Y \frac{\Delta L}{L}$   
 Stress =  $Y$  Strain

Q.15 A spherical body of radius  $R$  consists of a fluid of constant density and is in equilibrium under its own gravity. If  $P(r)$  is the pressure at  $r$  ( $r < R$ ), then the correct option(s) is(are)

- (A)  $P(r=0) = 0$ 
(B)  $\frac{P(r=3R/4)}{P(r=2R/3)} = \frac{63}{80}$   
(C)  $\frac{P(r=3R/5)}{P(r=2R/5)} = \frac{16}{21}$ 
(D)  $\frac{P(r=R/2)}{P(r=R/3)} = \frac{20}{27}$

Q.16 A parallel plate capacitor having plates of area  $S$  and plate separation  $d$ , has capacitance  $C_1$  in air. When two dielectrics of different relative permittivities ( $\epsilon_1 = 2$  and  $\epsilon_2 = 4$ ) are introduced between the two plates as shown in the figure, the capacitance becomes  $C_2$ . The

ratio  $\frac{C_2}{C_1}$  is



- (A) 6/5      (B) 5/3      (C) 7/5       (D) 7/3

Space for rough work

$\frac{A\epsilon_0/2}{\frac{d}{2\epsilon_1}}$

$\frac{A\epsilon_0/2}{\frac{d}{2\epsilon_2}}$

$C_1 + C_2$

$\frac{7}{3}C_1 = 2C_2$

$\frac{7}{3} \times \frac{9}{16} = \frac{9}{4}$

$\frac{7}{4} \times \frac{9}{4} = \frac{63}{16}$

$\frac{1}{4} \times \frac{9}{1}$

$\frac{A\epsilon_0/2}{2d} + \frac{d}{2\epsilon_2}$

$\frac{2A\epsilon_0}{d}$

$\frac{7}{3}C_1 = 2C_2$

$\frac{9}{16} \times \frac{9}{4}$

$\frac{7}{4} \times \frac{9}{4}$

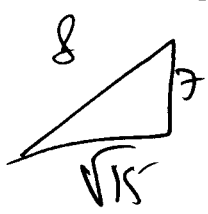
$\frac{1}{4} \times \frac{9}{1}$

SECTION 3 (Maximum Marks : 16)

- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - 2 In all other cases

Space for rough work

$$\frac{5}{64} \frac{49}{15}$$



sin i m .

~~n1 sin theta~~

$$n_1 \cos \theta = n_2 NA$$

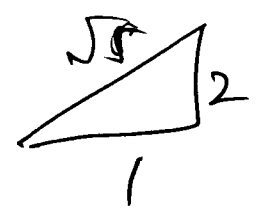
$$NA = n_2 NA$$

~~sin theta = 7/8~~  
 $\sin \theta = \frac{7}{8}$



NA.  
 $\frac{6}{\sqrt{15}} NA = 2$

$$\frac{\sqrt{45}}{2 \cdot 4} \sin \theta = \frac{2}{2}$$



$\frac{6}{\sqrt{15}} NA = \frac{3}{4}$

$$\frac{\sqrt{45} \sin \theta}{2} = \sqrt{3}$$

$$\sin \theta = \frac{2}{\sqrt{5}}$$

$$n = \frac{3\sqrt{15}}{24}$$

$$\frac{4}{3} NA = \frac{\sqrt{45}}{4} \times \frac{1}{\sqrt{3}}$$

$$\frac{\sqrt{15} \sin \theta}{2} = 2$$

$\cos \theta = \frac{1}{\sqrt{5}}$

$$NA = \frac{8}{16}$$

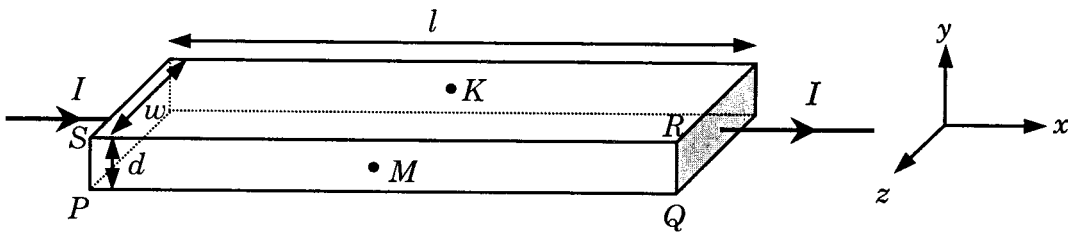
$$n = \frac{9}{16}$$

~~sin theta = 3/4~~  
 $\frac{8}{\sqrt{15}} = \frac{3}{4}$

PARAGRAPH 1

In a thin rectangular metallic strip a constant current  $I$  flows along the positive  $x$ -direction, as shown in the figure. The length, width and thickness of the strip are  $l$ ,  $w$  and  $d$ , respectively.

A uniform magnetic field  $\vec{B}$  is applied on the strip along the positive  $y$ -direction. Due to this, the charge carriers experience a net deflection along the  $z$ -direction. This results in accumulation of charge carriers on the surface  $PQRS$  and appearance of equal and opposite charges on the face opposite to  $PQRS$ . A potential difference along the  $z$ -direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



Q.17 Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are  $w_1$  and  $w_2$  and thicknesses are  $d_1$  and  $d_2$ , respectively. Two points  $K$  and  $M$  are symmetrically located on the opposite faces parallel to the  $x$ - $y$  plane (see figure).  $V_1$  and  $V_2$  are the potential differences between  $K$  and  $M$  in strips 1 and 2, respectively. Then, for a given current  $I$  flowing through them in a given magnetic field strength  $B$ , the correct statement(s) is(are)

- (A) If  $w_1 = w_2$  and  $d_1 = 2d_2$ , then  $V_2 = 2V_1$
- (B) If  $w_1 = w_2$  and  $d_1 = 2d_2$ , then  $V_2 = V_1$
- (C) If  $w_1 = 2w_2$  and  $d_1 = d_2$ , then  $V_2 = 2V_1$
- (D) If  $w_1 = 2w_2$  and  $d_1 = d_2$ , then  $V_2 = V_1$

$$V \propto \frac{B_1}{n_1}$$

Q.18 Consider two different metallic strips (1 and 2) of same dimensions (length  $l$ , width  $w$  and thickness  $d$ ) with carrier densities  $n_1$  and  $n_2$ , respectively. Strip 1 is placed in magnetic field  $B_1$  and strip 2 is placed in magnetic field  $B_2$ , both along positive  $y$ -directions. Then  $V_1$  and  $V_2$  are the potential differences developed between  $K$  and  $M$  in strips 1 and 2, respectively. Assuming that the current  $I$  is the same for both the strips, the correct option(s) is(are)

- (A) If  $B_1 = B_2$  and  $n_1 = 2n_2$ , then  $V_2 = 2V_1$
- (B) If  $B_1 = B_2$  and  $n_1 = 2n_2$ , then  $V_2 = V_1$
- (C) If  $B_1 = 2B_2$  and  $n_1 = n_2$ , then  $V_2 = 0.5V_1$
- (D) If  $B_1 = 2B_2$  and  $n_1 = n_2$ , then  $V_2 = V_1$

$$I = n e A v_d$$

Space for rough work

$$E = \frac{V}{w}$$

$$V \propto \frac{I}{wd}$$

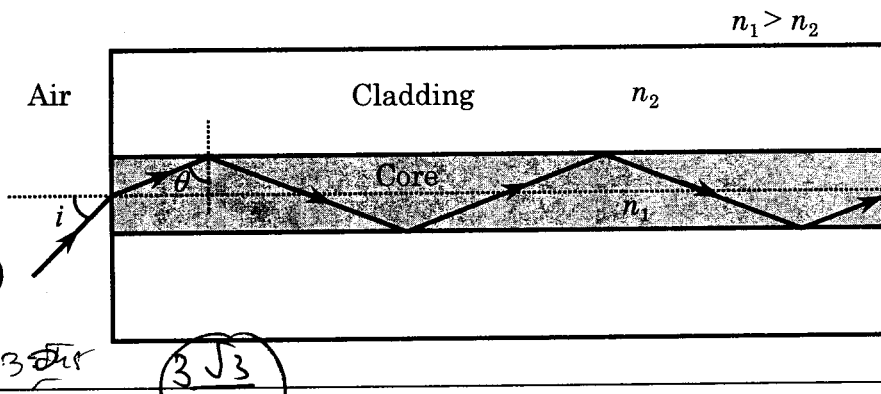
$$E \propto \frac{I}{wd}$$

$$E \propto \frac{I B}{n A}$$

$\sin \theta_c = \frac{2}{\sqrt{15}}$   
 $\cos \theta_c = \frac{2}{\sqrt{15}}$   
 $n_1 \sqrt{15} \sqrt{\frac{4}{15}} \sin \theta_c = \frac{8}{2} \rightarrow S_1 \sqrt{15}$

PARAGRAPH 2

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index  $n_1$  surrounded by a medium of lower refractive index  $n_2$ . The light guidance in the structure takes place due to successive total internal reflections at the interface of the media  $n_1$  and  $n_2$  as shown in the figure. All rays with the angle of incidence  $i$  less than a particular value  $i_m$  are confined in the medium of refractive index  $n_1$ . The numerical aperture (NA) of the structure is defined as  $\sin i_m$ .



$\frac{\sqrt{45}}{24} \times \frac{2}{\sqrt{15}} = \frac{4}{3}$  (C)  
 $\frac{3\sqrt{3}}{8}$

Q.19 For two structures namely  $S_1$  with  $n_1 = \sqrt{45}/4$  and  $n_2 = 3/2$ , and  $S_2$  with  $n_1 = 8/5$  and  $n_2 = 7/5$  and taking the refractive index of water to be  $4/3$  and that of air to be 1, the correct option(s) is(are)

- (A) NA of  $S_1$  immersed in water is the same as that of  $S_2$  immersed in a liquid of refractive index  $\frac{16}{3\sqrt{15}}$
- (B) NA of  $S_1$  immersed in liquid of refractive index  $\frac{6}{\sqrt{15}}$  is the same as that of  $S_2$  immersed in water
- (C) NA of  $S_1$  placed in air is the same as that of  $S_2$  immersed in liquid of refractive index  $\frac{4}{\sqrt{15}}$
- (D) NA of  $S_1$  placed in air is the same as that of  $S_2$  placed in water

$n_1 \frac{3\sqrt{3}}{8} = \frac{8}{5} \sqrt{\frac{4}{15}}$

Q.20 If two structures of same cross-sectional area, but different numerical apertures  $NA_1$  and  $NA_2$  ( $NA_2 < NA_1$ ) are joined longitudinally, the numerical aperture of the combined structure is

- (A)  $\frac{NA_1 NA_2}{NA_1 + NA_2}$
- (B)  $NA_1 + NA_2$
- (C)  $NA_1$
- (D)  $NA_2$

$\sin \theta_c = \frac{\sqrt{15}}{24}$   
 $\sin i_m = \frac{2}{\sqrt{15}}$   
 $n_1 \cos \theta_c = n_2 \sin i_m$

END OF PART I : PHYSICS

$n_1 \cos \theta_c = NA_2$

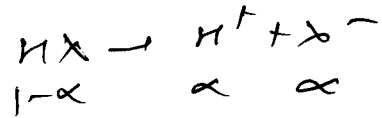
$NA_1 = \frac{2}{\sqrt{15}}$ ,  $NA_2 = \frac{238}{92}$  →  $82 RB + 8 \frac{14}{32}$

$n_2 \cos \theta_c = n_1 \sin i_m$   
 $\frac{\sqrt{15}}{11} \times 2 = \frac{8}{5} \sqrt{\frac{4}{15}}$

$$\alpha^2 = \frac{K_a}{K_a + C}$$

$$\alpha = \sqrt{\frac{K_a}{K_a + C}}$$

# PART II : CHEMISTRY



## SECTION 1 (Maximum Marks: 32)

$$\frac{1}{2} pK_a - \frac{\log C}{2}$$

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:  
 +4 If the bubble corresponding to the answer is darkened  
 0 In all other cases

1

Q.21 The molar conductivity of a solution of a weak acid HX (0.01 M) is 10 times smaller than the molar conductivity of a solution of a weak acid HY (0.10 M). If  $\lambda_X^0 \approx \lambda_Y^0$ , the difference in their  $pK_a$  values,  $pK_a(HX) - pK_a(HY)$ , is (consider degree of ionization of both acids to be  $\ll 1$ )

9

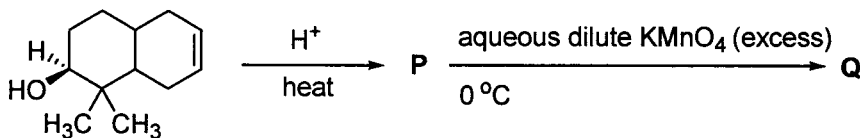
Q.22 A closed vessel with rigid walls contains 1 mol of  $^{238}_{92}\text{U}$  and 1 mol of air at 298 K. Considering complete decay of  $^{238}_{92}\text{U}$  to  $^{206}_{82}\text{Pb}$ , the ratio of the final pressure to the initial pressure of the system at 298 K is

8

Q.23 In dilute aqueous  $\text{H}_2\text{SO}_4$ , the complex diaquodioxalatoferrate(II) is oxidized by  $\text{MnO}_4^-$ . For this reaction, the ratio of the rate of change of  $[\text{H}^+]$  to the rate of change of  $[\text{MnO}_4^-]$  is

4

Q.24 The number of hydroxyl group(s) in Q is



$$\frac{1+9}{1} = 10$$

$$\frac{5}{84} = 0.0595$$

$$\frac{49}{15} = 3.266$$

Space for rough work

$\frac{pK_{a1} + \log C_1}{2} - \frac{pK_{a2} + \log C_2}{2} = 2 - 1 = 1$

$\frac{4 \sin \theta}{3} = \frac{\sqrt{49}}{4} \times \frac{\sqrt{11}}{\sqrt{14}}$

$\frac{3\sqrt{33}}{16}$

$[\text{Fe}(\text{H}_2\text{O})_2(\text{C}_2\text{O}_4)_2]^{2-} + \text{MnO}_4^- \rightarrow \text{Mn}^{2+} + 4\text{H}^+ + 2\text{CO}_2 + 2\text{e}^-$

$5\text{e}^- + 8\text{H}^+ + \text{MnO}_4^- \rightarrow \text{Mn}^{2+} + 4\text{H}_2\text{O}$

$8 \times 5 = 40 \text{e}^-$


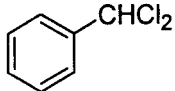
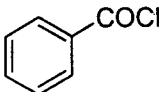
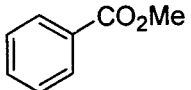
$8 \times 8 = 64$

$49 - 64 = -15$

$\frac{15}{32}$

Q.25 Among the following, the number of reaction(s) that produce(s) benzaldehyde is

4

- I.   $\xrightarrow[\text{Anhydrous AlCl}_3/\text{CuCl}]{\text{CO, HCl}}$
- II.   $\xrightarrow[100^\circ\text{C}]{\text{H}_2\text{O}}$
- III.   $\xrightarrow[\text{Pd-BaSO}_4]{\text{H}_2}$
- IV.   $\xrightarrow[\text{Toluene, } -78^\circ\text{C}]{\text{DIBAL-H}}$   
H<sub>2</sub>O

2

Q.26 In the complex acetylbromidodicarbonylbis(triethylphosphine)iron(II), the number of Fe-C bond(s) is

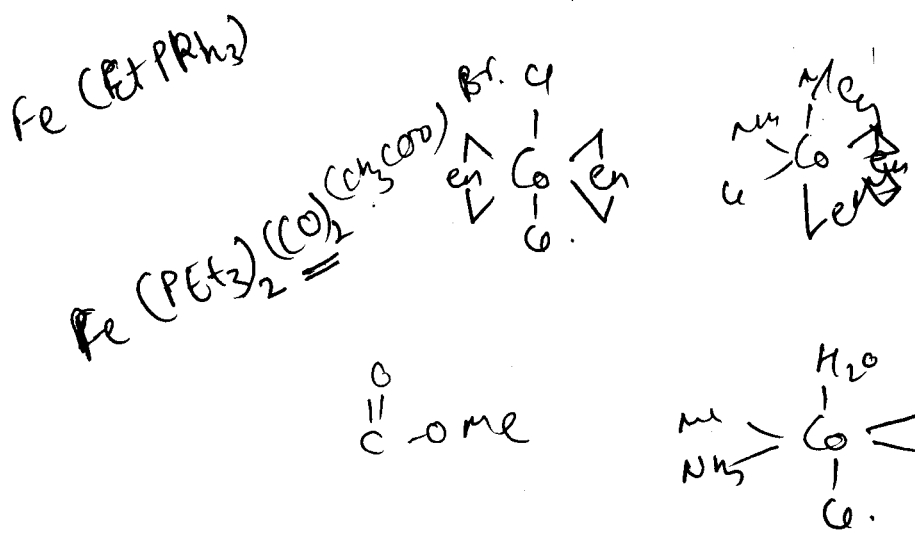
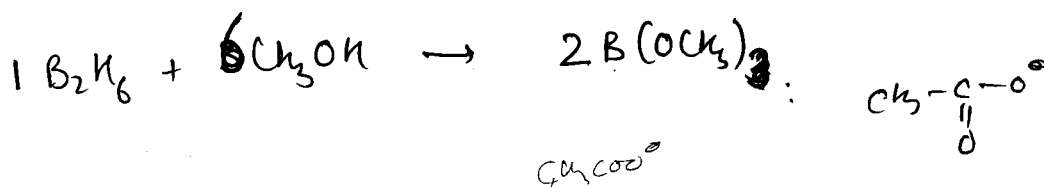
6

Q.27 Among the complex ions,  $[\text{Co}(\text{NH}_2\text{-CH}_2\text{-CH}_2\text{-NH}_2)_2\text{Cl}_2]^+$ ,  $[\text{CrCl}_2(\text{C}_2\text{O}_4)_2]^{3-}$ ,  $[\text{Fe}(\text{H}_2\text{O})_4(\text{OH})_2]^+$ ,  $[\text{Fe}(\text{NH}_3)_2(\text{CN})_4]^-$ ,  $[\text{Co}(\text{NH}_2\text{-CH}_2\text{-CH}_2\text{-NH}_2)_2(\text{NH}_3)\text{Cl}]^{2+}$  and  $[\text{Co}(\text{NH}_3)_4(\text{H}_2\text{O})\text{Cl}]^{2+}$ , the number of complex ion(s) that show(s) *cis-trans* isomerism is

6

Q.28 Three moles of B<sub>2</sub>H<sub>6</sub> are completely reacted with methanol. The number of moles of boron containing product formed is

Space for rough work





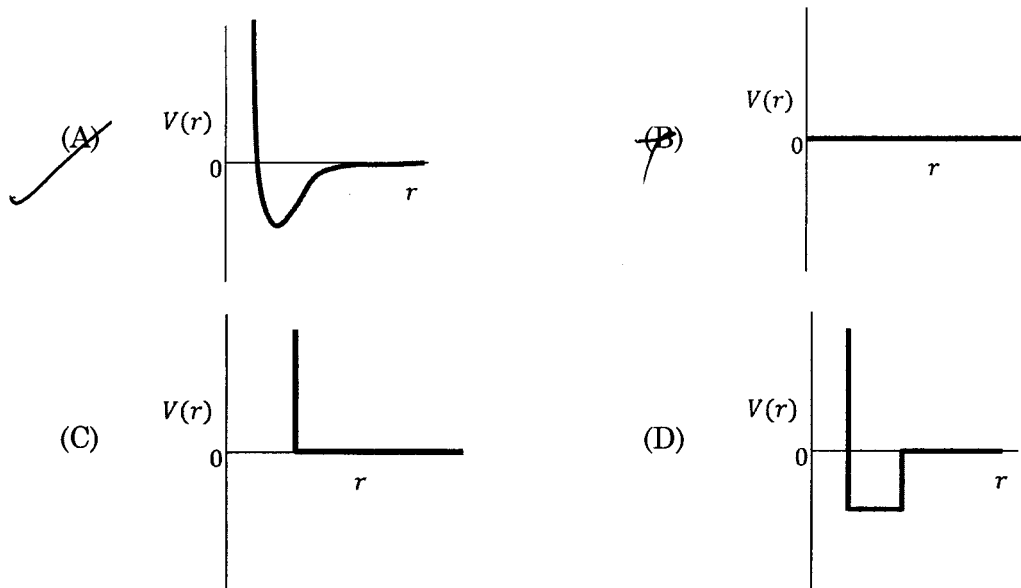
SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - 2 In all other cases

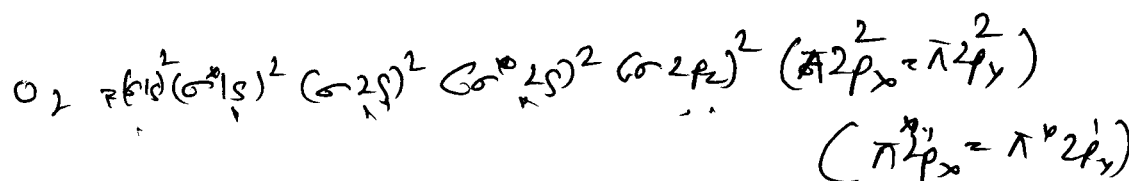
Q.29 When  $O_2$  is adsorbed on a metallic surface, electron transfer occurs from the metal to  $O_2$ . The **TRUE** statement(s) regarding this adsorption is(are)

- (A)  $O_2$  is physisorbed       (B) heat is released  
 (C) occupancy of  $\pi_{2p}^*$  of  $O_2$  is increased       (D) bond length of  $O_2$  is increased

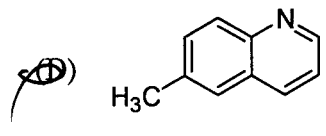
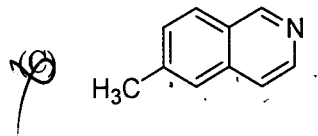
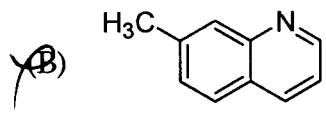
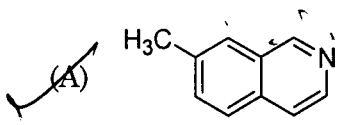
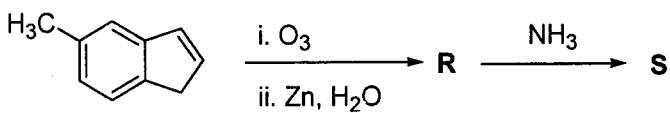
Q.30 One mole of a monoatomic real gas satisfies the equation  $p(V - b) = RT$  where  $b$  is a constant. The relationship of interatomic potential  $V(r)$  and interatomic distance  $r$  for the gas is given by



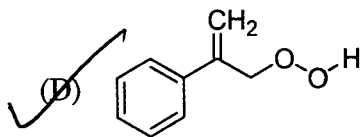
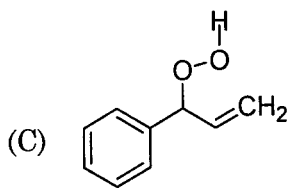
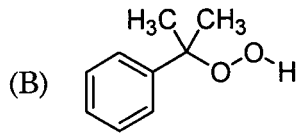
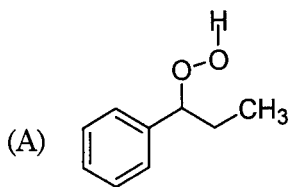
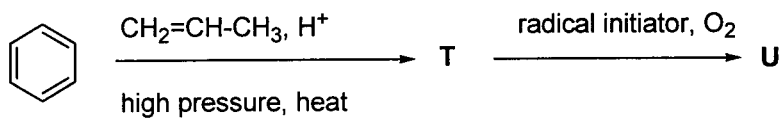
Space for rough work



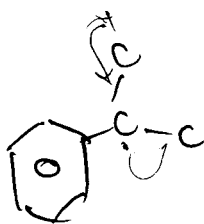
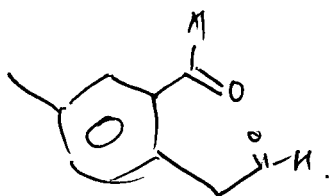
Q.31 In the following reactions, the product S is



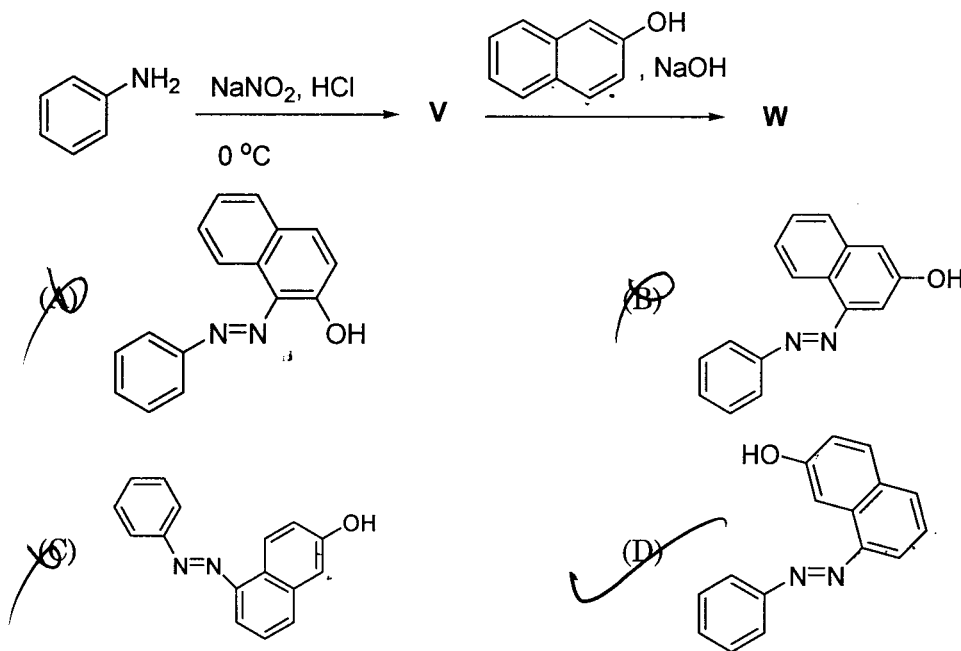
Q.32 The major product U in the following reactions is



Space for rough work



Q.33 In the following reactions, the major product W is



Q.34 The correct statement(s) regarding, (i) HClO, (ii) HClO<sub>2</sub>, (iii) HClO<sub>3</sub> and (iv) HClO<sub>4</sub>, is(are)

- (A) The number of Cl=O bonds in (ii) and (iii) together is two  
 (B) The number of lone pairs of electrons on Cl in (ii) and (iii) together is three  
 (C) The hybridization of Cl in (iv) is  $sp^3$   
 (D) Amongst (i) to (iv), the strongest acid is (i)

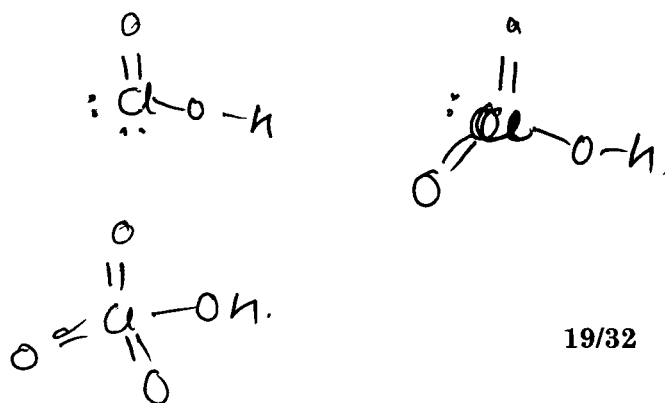
Q.35 The pair(s) of ions where BOTH the ions are precipitated upon passing H<sub>2</sub>S gas in presence of dilute HCl, is(are)

- ~~(A)~~ Ba<sup>2+</sup>, Zn<sup>2+</sup>    ~~(B)~~ Bi<sup>3+</sup>, Fe<sup>3+</sup>     (C) Cu<sup>2+</sup>, Pb<sup>2+</sup>     (D) Hg<sup>2+</sup>, Bi<sup>3+</sup>

Q.36 Under hydrolytic conditions, the compounds used for preparation of linear polymer and for chain termination, respectively, are

- (A) CH<sub>3</sub>SiCl<sub>3</sub> and Si(CH<sub>3</sub>)<sub>4</sub>     (B) (CH<sub>3</sub>)<sub>2</sub>SiCl<sub>2</sub> and (CH<sub>3</sub>)<sub>3</sub>SiCl  
 (C) (CH<sub>3</sub>)<sub>2</sub>SiCl<sub>2</sub> and CH<sub>3</sub>SiCl<sub>3</sub>     (D) SiCl<sub>4</sub> and (CH<sub>3</sub>)<sub>3</sub>SiCl

Space for rough work



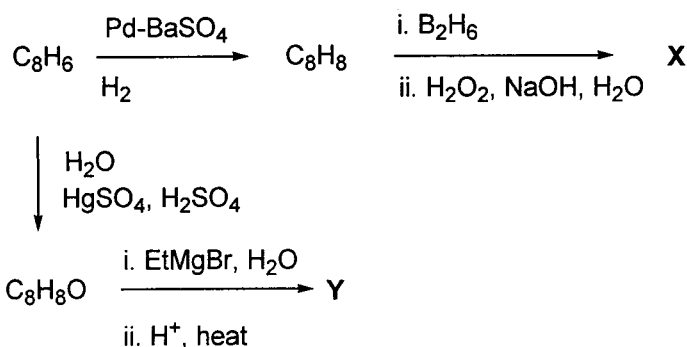
SECTION 3 (Maximum Marks: 16)

- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - 2 In all other cases

PARAGRAPH 1

18-6 ~ 6,  
2

In the following reactions



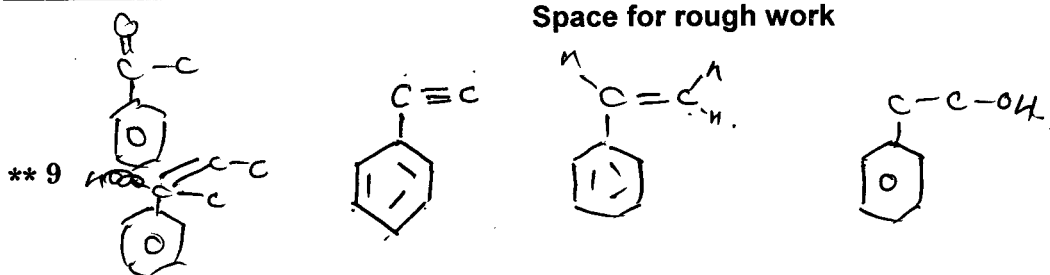
Q.37 Compound X is



Q.38 The major compound Y is



Space for rough work



PARAGRAPH 2

When 100 mL of 1.0 M HCl was mixed with 100 mL of 1.0 M NaOH in an insulated beaker at constant pressure, a temperature increase of 5.7 °C was measured for the beaker and its contents (**Expt. 1**). Because the enthalpy of neutralization of a strong acid with a strong base is a constant ( $-57.0 \text{ kJ mol}^{-1}$ ), this experiment could be used to measure the calorimeter constant.

In a second experiment (**Expt. 2**), 100 mL of 2.0 M acetic acid ( $K_a = 2.0 \times 10^{-5}$ ) was mixed with 100 mL of 1.0 M NaOH (under identical conditions to **Expt. 1**) where a temperature rise of 5.6 °C was measured.

(Consider heat capacity of all solutions as  $4.2 \text{ J g}^{-1} \text{ K}^{-1}$  and density of all solutions as  $1.0 \text{ g mL}^{-1}$ )

Q.39 Enthalpy of dissociation (in  $\text{kJ mol}^{-1}$ ) of acetic acid obtained from the **Expt. 2** is

- (A) 1.0                      (B) 10.0                      (C) 24.5                      (D) 51.4

Q.40 The pH of the solution after **Expt. 2** is

- (A) 2.8                       (B) 4.7                      (C) 5.0                      (D) 7.0

END OF PART II : CHEMISTRY

100

100

100

$$5 - \log 2$$

$$0.3$$

$$10^{-(5 - \log 2)}$$

$$10^{(-5 + \log 2)}$$

$$10^{\log(2 \times 10^{-5})}$$

$$2 \times 10^{-5}$$

$$10^{-2}$$

$$5 \times 10^{-3}$$

1.14

$$9 - 9x + 3 \tan^{-1}(1-x)$$

$$12 + \frac{9(1-x^2)}{1+(1-x^2)}$$

$$9 + \frac{3}{1+x^2}$$

$$e^{9 + \frac{3x}{4}}$$

## PART III : MATHEMATICS

$$9 + 9x^2 + \dots$$

### SECTION 1 (Maximum Marks: 32)

$$1+x^2$$

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  - +4 If the bubble corresponding to the answer is darkened
  - 0 In all other cases

Q.41 If

$$\alpha = \int_0^1 (e^{9x+3 \tan^{-1} x}) \left( \frac{12+9x^2}{1+x^2} \right) dx$$

where  $\tan^{-1} x$  takes only principal values, then the value of  $\left( \log_e |1+\alpha| - \frac{3\pi}{4} \right)$  is

9

Q.42 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous odd function, which vanishes exactly at one point and

$f(1) = \frac{1}{2}$ . Suppose that  $F(x) = \int_{-1}^x f(t) dt$  for all  $x \in [-1, 2]$  and  $G(x) = \int_{-1}^x |f(f(t))| dt$  for all

$x \in [-1, 2]$ . If  $\lim_{x \rightarrow 1} \frac{F(x)}{G(x)} = \frac{1}{14}$ , then the value of  $f\left(\frac{1}{2}\right)$  is

7

Q.43 Suppose that  $\vec{p}$ ,  $\vec{q}$  and  $\vec{r}$  are three non-coplanar vectors in  $\mathbb{R}^3$ . Let the components of a

vector  $\vec{s}$  along  $\vec{p}$ ,  $\vec{q}$  and  $\vec{r}$  be 4, 3 and 5, respectively. If the components of this vector  $\vec{s}$  along  $(-\vec{p} + \vec{q} + \vec{r})$ ,  $(\vec{p} - \vec{q} + \vec{r})$  and  $(-\vec{p} - \vec{q} + \vec{r})$  are  $x$ ,  $y$  and  $z$ , respectively, then the value of  $2x + y + z$  is

6

Space for rough work

$$\vec{s} = 4\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} = 4$$

$$\frac{\vec{s} \cdot \vec{r}}{|\vec{r}|} = 5$$

$$\frac{\vec{s} \cdot \vec{q}}{|\vec{q}|} = 3$$

$$-\hat{i} + \hat{j} + \hat{k}$$

$$-4 + 3 + 5$$

$$\frac{5r - 4p - 3q}{|-\vec{p} - \vec{q} + \vec{r}|} + 2 \left( \frac{3q + 5r - 4p}{|\vec{p} + \vec{q} + \vec{r}|} \right) + \frac{4p - 3q + 5r}{|\vec{p} - \vec{q} + \vec{r}|}$$

$$\frac{-\vec{s} \cdot \vec{p} + \vec{s} \cdot \vec{q} + \vec{s} \cdot \vec{r}}{|\vec{p} + \vec{q} + \vec{r}|}$$

$$= x \cdot (-4 - 3 + 5)$$

$$4 - 3 + 5$$

$$\frac{8}{\sqrt{3}} + \frac{6}{\sqrt{3}} - \frac{2}{\sqrt{3}}$$

$$\frac{12}{\sqrt{3}}$$

Q.44 For any integer  $k$ , let  $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i \sin\left(\frac{k\pi}{7}\right)$ , where  $i = \sqrt{-1}$ . The value of the expression

4

$$\frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^3 |\alpha_{4k-1} - \alpha_{4k-2}|}$$
 is

9

Q.45 Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is  $6 : 11$  and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

1

Q.46 The coefficient of  $x^9$  in the expansion of  $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$  is

**Space for rough work**

$$(1+x+x^2+x^3)(1+x^3)(1+x^4) \dots$$

$$(1+x^3+x+x^4+x^2+x^5+x^6)(1+x^4)(1+x^5)$$

$$(1+x+x^2+x^3 + \dots + x$$

$$\frac{7(2a+3d)}{11(2a+10d)} = \frac{6}{11}$$

$$\frac{f'(x)}{|f'(x)|} = \frac{1}{14}$$

$\vec{p} + \vec{q} = \vec{r}$

$$7a + 21d = 6a + 30d$$

$$a = 9d$$

$$\frac{15}{135} = \frac{1}{9}$$

$$130 \leq a + 6d \leq 140$$

$$130 < 15d < 140$$

$$d = 9$$

d = 9

$f_1 \equiv (2, 0)$   $f_2 \equiv (-2, 0)$

$e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9} \Rightarrow e = \frac{2}{3}$

**MATHEMATICS**

Q.47 Suppose that the foci of the ellipse  $\frac{x^2}{9} + \frac{y^2}{5} = 1$  are  $(f_1, 0)$  and  $(f_2, 0)$  where  $f_1 > 0$  and  $f_2 < 0$ .

4

Let  $P_1$  and  $P_2$  be two parabolas with a common vertex at  $(0, 0)$  and with foci at  $(f_1, 0)$  and  $(2f_2, 0)$ , respectively. Let  $T_1$  be a tangent to  $P_1$  which passes through  $(2f_2, 0)$  and  $T_2$  be a tangent to  $P_2$  which passes through  $(f_1, 0)$ . If  $m_1$  is the slope of  $T_1$  and  $m_2$  is the slope of  $T_2$ , then the value of  $\left(\frac{1}{m_1^2} + m_2^2\right)$  is

then the value of  $\left(\frac{1}{m_1^2} + m_2^2\right)$  is

Q.48 Let  $m$  and  $n$  be two positive integers greater than 1. If

2

$\lim_{\alpha \rightarrow 0} \left( \frac{e^{\cos(\alpha^n)} - e}{\alpha^m} \right) = -\left(\frac{e}{2}\right)$

$1 + \cos \alpha^n + \frac{\cos^2 \alpha^n}{2!} + \frac{\cos^3 \alpha^n}{3!} - e \approx -\frac{e}{2}$   
 $\propto \alpha^m$

then the value of  $\frac{m}{n}$  is

Space for rough work

$e^{\cos \alpha^n} = 1 + \cos \alpha^n + \frac{\cos^2 \alpha^n}{2!} + \dots$

$e^{\cos \alpha^n} - e = \cos \alpha^n - 1 + \frac{\cos^2 \alpha^n - 1}{2} + \dots$

$\approx -\frac{1}{2} \alpha^{2n} + \frac{1}{24} \alpha^{4n} - \dots$

$\lim_{\alpha \rightarrow 0} \frac{-\frac{1}{2} \alpha^{2n} + \dots}{\alpha^m} = -\frac{e}{2}$

$\Rightarrow \frac{2n}{m} = 1 \Rightarrow \frac{m}{n} = 2$

For Q.47, the ellipse is  $\frac{x^2}{9} + \frac{y^2}{5} = 1$ . Foci are  $(\pm 2, 0)$ . Parabolas have vertices at  $(0, 0)$  and foci at  $(\pm 4, 0)$ . Tangent lines  $T_1$  and  $T_2$  are shown. The calculation shows  $m_1^2 = \frac{1}{2}$  and  $m_2^2 = 2$ , so  $\frac{1}{m_1^2} + m_2^2 = 2 + 2 = 4$ .



SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - 2 In all other cases

Q.49 Let  $f(x) = 7 \tan^8 x + 7 \tan^6 x - 3 \tan^4 x - 3 \tan^2 x$  for all  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ . Then the correct expression(s) is(are)

(A)  $\int_0^{\pi/4} x f(x) dx = \frac{1}{12}$

(B)  $\int_0^{\pi/4} f(x) dx = 0$

(C)  $\int_0^{\pi/4} x f(x) dx = \frac{1}{6}$

(D)  $\int_0^{\pi/4} f(x) dx = 1$

Q.50 Let  $f'(x) = \frac{192x^3}{2 + \sin^4 \pi x}$  for all  $x \in \mathbb{R}$  with  $f\left(\frac{1}{2}\right) = 0$ . If  $m \leq \int_{1/2}^1 f(x) dx \leq M$ , then the possible values of  $m$  and  $M$  are

(A)  $m = 13, M = 24$

(B)  $m = \frac{1}{4}, M = \frac{1}{2}$

(C)  $m = -11, M = 0$

(D)  $m = 1, M = 12$

Space for rough work

$$[\tan^7(x) - \tan^3(x)] \left\{ x - \frac{1}{3} \right\} \left\{ 7 \tan^6 x (\tan^2 x + 1) - 3 \tan^2 x (\tan^2 x + 1) \right\}$$

$$4 \sin^3 \pi x \cdot \cos \pi x \cdot \pi$$

$$\frac{7 \tan^7 x}{7} - \frac{3 \tan^3 x}{3}$$

$$\int 7x \cdot \tan^6 x \sec^2 x \cdot dx$$

$$**9 \quad \frac{7}{7} \left[ x \cdot \frac{\tan^7 x}{7} - \frac{\tan^7 x}{7} \right] - \frac{3}{3} \left[ x \cdot \frac{\tan^3 x}{3} - \frac{\tan^3 x}{3} \right]^{25/32}$$

Q.51 Let  $S$  be the set of all non-zero real numbers  $\alpha$  such that the quadratic equation  $\alpha x^2 - x + \alpha = 0$  has two distinct real roots  $x_1$  and  $x_2$  satisfying the inequality  $|x_1 - x_2| < 1$ . Which of the following intervals is(are) a subset(s) of  $S$ ?

- (A)  $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$    (B)  $\left(-\frac{1}{\sqrt{5}}, 0\right)$    (C)  $\left(0, \frac{1}{\sqrt{5}}\right)$    (D)  $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Q.52 If  $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$  and  $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$ , where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are)

- (A)  $\cos \beta > 0$    (B)  $\sin \beta < 0$    (C)  $\cos(\alpha + \beta) > 0$    (D)  $\cos \alpha < 0$

Space for rough work

$D = 1 - 4\alpha^2 > 0$

$1 > 4\alpha^2$   
 $\alpha^2 < \frac{1}{4}$

The rough work contains several diagrams and calculations:

- Q.51 Analysis:**
  - Graphs of parabolas  $y = \alpha x^2 - x + \alpha$  for different values of  $\alpha$ . One graph shows a parabola opening upwards with two real roots, and another shows a parabola opening downwards.
  - Notes:  $\alpha^2 < \frac{1}{4}$ ,  $1 > 4\alpha^2$ .
  - Calculated values for  $\alpha$ :  $\frac{\pi}{6}$ ,  $\frac{\pi}{3}$ ,  $\frac{2\pi}{3}$ ,  $\frac{5\pi}{6}$ .
  - Final conclusion for Q.51:  $\frac{\pi}{6}$  and  $\frac{5\pi}{6}$  are circled as the correct options.
- Q.52 Analysis:**
  - Graphs of  $y = \sin^{-1}(x)$  and  $y = \cos^{-1}(x)$ .
  - Notes:  $\frac{\pi}{3} > \sin^{-1}\left(\frac{6}{11}\right)$ .
  - Final conclusion for Q.52: (D)  $\cos \alpha < 0$  is circled as the correct option.

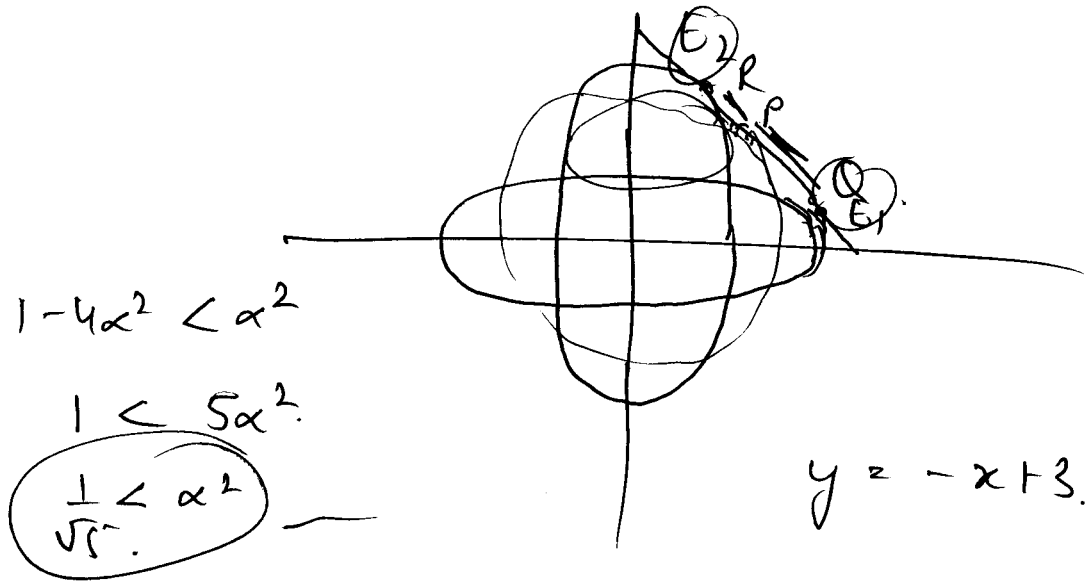
Q.53 Let  $E_1$  and  $E_2$  be two ellipses whose centers are at the origin. The major axes of  $E_1$  and  $E_2$  lie along the  $x$ -axis and the  $y$ -axis, respectively. Let  $S$  be the circle  $x^2 + (y-1)^2 = 2$ . The straight line  $x + y = 3$  touches the curves  $S$ ,  $E_1$  and  $E_2$  at  $P$ ,  $Q$  and  $R$ , respectively. Suppose that  $PQ = PR = \frac{2\sqrt{2}}{3}$ . If  $e_1$  and  $e_2$  are the eccentricities of  $E_1$  and  $E_2$ , respectively, then the correct expression(s) is(are)

- (A)  $e_1^2 + e_2^2 = \frac{43}{40}$     (B)  $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$     (C)  $|e_1^2 - e_2^2| = \frac{5}{8}$     (D)  $e_1 e_2 = \frac{\sqrt{3}}{4}$

Q.54 Consider the hyperbola  $H: x^2 - y^2 = 1$  and a circle  $S$  with center  $N(x_2, 0)$ . Suppose that  $H$  and  $S$  touch each other at a point  $P(x_1, y_1)$  with  $x_1 > 1$  and  $y_1 > 0$ . The common tangent to  $H$  and  $S$  at  $P$  intersects the  $x$ -axis at point  $M$ . If  $(l, m)$  is the centroid of the triangle  $\Delta PMN$ , then the correct expression(s) is(are)

- (A)  $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$  for  $x_1 > 1$     (B)  $\frac{dm}{dx_1} = \frac{x_1}{3(\sqrt{x_1^2 - 1})}$  for  $x_1 > 1$   
 (C)  $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$  for  $x_1 > 1$     (D)  $\frac{dm}{dy_1} = \frac{1}{3}$  for  $y_1 > 0$

Space for rough work



$$\left( \frac{\pm am}{\sqrt{a^2 m^2 + b^2}}, \frac{\mp b}{\sqrt{a^2 m^2 + b^2}} \right)$$

$$\left( \frac{\mp a}{\sqrt{a^2 m^2 + b^2}}, \frac{\mp b}{\sqrt{a^2 m^2 + b^2}} \right)$$

$$e^x (\sin^6(2t) + \cos^4(2t))$$

MATHEMATICS

Q.55 The option(s) with the values of  $a$  and  $L$  that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t (\sin^6 at + \cos^4 at) dt}{\int_0^{\pi} e^t (\sin^6 at + \cos^4 at) dt} = L?$$

$f(x) - 3g(x)$

- (A)  $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$                       (B)  $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$   
 (C)  $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$                       (D)  $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

Q.56 Let  $f, g: [-1, 2] \rightarrow \mathbb{R}$  be continuous functions which are twice differentiable on the interval  $(-1, 2)$ . Let the values of  $f$  and  $g$  at the points  $-1, 0$  and  $2$  be as given in the following table:

	$x = -1$	$x = 0$	$x = 2$
$f(x)$	3	6	0
$g(x)$	0	1	-1

In each of the intervals  $(-1, 0)$  and  $(0, 2)$  the function  $(f - 3g)''$  never vanishes. Then the correct statement(s) is(are)

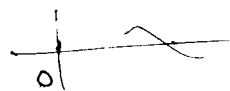
- (A)  $f'(x) - 3g'(x) = 0$  has exactly three solutions in  $(-1, 0) \cup (0, 2)$   
 $\checkmark$  (B)  $f'(x) - 3g'(x) = 0$  has exactly one solution in  $(-1, 0)$   
 $\checkmark$  (C)  $f'(x) - 3g'(x) = 0$  has exactly one solution in  $(0, 2)$   
 (D)  $f'(x) - 3g'(x) = 0$  has exactly two solutions in  $(-1, 0)$  and exactly two solutions in  $(0, 2)$

Space for rough work

$$2n_3 + 2 + 6n_3 = \frac{5}{3}(n_3 + n_4 + 1)$$

$$24n_3 + 6 = 5n_3 + 5n_4 + 5 - 1$$

$$19n_3 + 1 = 5n_4$$



$$\frac{n_1}{n_1 + n_2} \times \frac{n_3 + 1}{n_3 + n_4 + 1} + \frac{n_2}{n_1 + n_2} \times \frac{n_3}{n_3 + n_4 + 1} = \frac{1}{3}$$

$$10n_3 + 1 = 5n_3 + 5n_4 + 5 \quad n_1 n_3 + n_1 + n_2 n_3 = \frac{(n_1 + n_2)(n_3 + n_4 + 1)}{3}$$

$$4n_3 + 4 + 6n_3 = \frac{5}{3}(n_3 + n_4 + 1)$$

SECTION 3 (Maximum Marks : 16)

- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :
  - +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  - 0 If none of the bubbles is darkened
  - 2 In all other cases

PARAGRAPH 1

Let  $n_1$  and  $n_2$  be the number of red and black balls, respectively, in box I. Let  $n_3$  and  $n_4$  be the number of red and black balls, respectively, in box II.

Q.57 One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is  $\frac{1}{3}$ , then the correct option(s) with the possible values of  $n_1, n_2, n_3$  and  $n_4$

- is(are)
- |                                     |                                       |                          |                          |                          |                                     |  |                          |                          |                          |
|-------------------------------------|---------------------------------------|--------------------------|--------------------------|--------------------------|-------------------------------------|--|--------------------------|--------------------------|--------------------------|
|                                     | R                                     | B                        | R                        | B                        |                                     | R                                      | B                        | R                        | B                        |
| <input checked="" type="checkbox"/> | (A)                                   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input checked="" type="checkbox"/> | (B)                                    | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
|                                     | $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ |                          |                          |                          |                                     | $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$ |                          |                          |                          |
| <input checked="" type="checkbox"/> | (C)                                   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> | <input checked="" type="checkbox"/> | (D)                                    | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
|                                     | $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ |                          |                          |                          |                                     | $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$ |                          |                          |                          |

Q.58 A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is  $\frac{1}{5}$ , then the correct option(s) with the possible values of  $n_1$  and  $n_2$  is(are)

- |                                     |     |                           |                                     |     |                         |
|-------------------------------------|-----|---------------------------|-------------------------------------|-----|-------------------------|
| <input checked="" type="checkbox"/> | (A) | $n_1 = 4$ and $n_2 = 6$   | <input checked="" type="checkbox"/> | (B) | $n_1 = 2$ and $n_2 = 3$ |
| <input checked="" type="checkbox"/> | (C) | $n_1 = 10$ and $n_2 = 20$ | <input checked="" type="checkbox"/> | (D) | $n_1 = 3$ and $n_2 = 6$ |

Space for rough work

Handwritten calculations for Q.57 and Q.58:

Q.57:  $\frac{1}{3} = \frac{n_3}{n_3 + n_4}$  (Probability of red ball from box II)

Q.58:  $\frac{1}{5} = \frac{n_1}{n_1 + n_2}$  (Probability of red ball from box I after transfer)

Handwritten work for Q.57:

$$\frac{1}{3} = \frac{n_3}{n_3 + n_4} \Rightarrow n_3 + n_4 = 3n_3 \Rightarrow n_4 = 2n_3$$

Handwritten work for Q.58:

$$\frac{1}{5} = \frac{n_1}{n_1 + n_2} \Rightarrow n_1 + n_2 = 5n_1 \Rightarrow n_2 = 4n_1$$

Handwritten work for Q.57 options:

- (A)  $n_1=3, n_2=3, n_3=5, n_4=15$ :  $\frac{5}{5+15} = \frac{1}{4} \neq \frac{1}{3}$
- (B)  $n_1=3, n_2=6, n_3=10, n_4=50$ :  $\frac{10}{10+50} = \frac{1}{6} \neq \frac{1}{3}$
- (C)  $n_1=8, n_2=6, n_3=5, n_4=20$ :  $\frac{5}{5+20} = \frac{1}{5} \neq \frac{1}{3}$
- (D)  $n_1=6, n_2=12, n_3=5, n_4=20$ :  $\frac{5}{5+20} = \frac{1}{5} \neq \frac{1}{3}$

Handwritten work for Q.58 options:

- (A)  $n_1=4, n_2=6$ :  $\frac{4}{4+6} = \frac{2}{5} \neq \frac{1}{5}$
- (B)  $n_1=2, n_2=3$ :  $\frac{2}{2+3} = \frac{2}{5} \neq \frac{1}{5}$
- (C)  $n_1=10, n_2=20$ :  $\frac{10}{10+20} = \frac{1}{3} \neq \frac{1}{5}$
- (D)  $n_1=3, n_2=6$ :  $\frac{3}{3+6} = \frac{1}{3} \neq \frac{1}{5}$

PARAGRAPH 2

Let  $F: \mathbb{R} \rightarrow \mathbb{R}$  be a thrice differentiable function. Suppose that  $F(1) = 0$ ,  $F(3) = -4$  and  $F'(x) < 0$  for all  $x \in (1/2, 3)$ . Let  $f(x) = xF(x)$  for all  $x \in \mathbb{R}$ .

Q.59 The correct statement(s) is(are)

(A)  $f'(1) < 0$

(B)  $f(2) < 0$

(C)  $f'(x) \neq 0$  for any  $x \in (1, 3)$

(D)  $f'(x) = 0$  for some  $x \in (1, 3)$

Q.60 If  $\int_1^3 x^2 F'(x) dx = -12$  and  $\int_1^3 x^3 F''(x) dx = 40$ , then the correct expression(s) is(are)

(A)  $9f'(3) + f'(1) - 32 = 0$

(B)  $\int_1^3 f(x) dx = 12$

(C)  $9f'(3) - f'(1) + 32 = 0$

(D)  $\int_1^3 f(x) dx = -12$

END OF THE QUESTION PAPER

$f(3) = -12$   
 $f(1) = 0$

$-36 = 2 \int_1^3 f(x) dx = -12$   
 $24$

$f'(x) = \frac{f'(x) - F(x)}{x}$

$\int x f'(x) - x F(x)$

$\int x f'(x) - f(x) = -12$

$\int x f'(x) + f(x) - f(x)$

$f'(x) = \frac{f(x)}{x} + x f'(x)$   
 $f'(1) = \frac{f(1)}{1} + F'(1)$

$f(2) = 2F(2)$

$f''(x) =$

$x f(x)$

$3f(3) - f(1) - 2 \int_1^3 f(x) dx = -12$   
 $30/32$

SPACE FOR ROUGH WORK

$\alpha$

$$f''(x) = f'(x) + F'(x) + F''(x),$$

$$= 2F'(x) + F''(x).$$

$$F''(x) = f''(x) - 2F'(x)$$

$$- 5x^2 \cdot f(x)$$

$$\int x^3 \cdot f''(x) - 2x^3 \underline{F'(x)}$$

$$\int x^3 f''(x) - 2x^2 f'(x) + 2x^2 f(x),$$

$$\int_1^3 x^3 f''(x) - 2x^2 f'(x) + 2x f(x) = 40.$$


$$\int \underline{x^3 f''(x) + 3x^2 f'(x)} - \underline{5x^2 f'(x) + 2x f(x)}$$

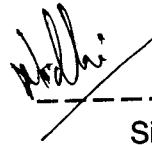
$$\int x f(x)$$

$$- 5x^2 f'(x) - 10x f(x) + \underline{\underline{12x f(x)}}$$

$$27 \underline{f'(3)} - 1 \underline{f'(1)} - 45 f(3) + 5 f(1)$$

**DARKENING THE BUBBLES ON THE ORS :**

19. Use a **BLACK BALL POINT PEN** to darken the bubbles in the upper sheet.
20. Darken the bubble **COMPLETELY**.
21. Darken the bubbles **ONLY** if you are sure of the answer.
22. The correct way of darkening a bubble is as shown here: 
23. There is **NO** way to erase or "un-darken" a darkened bubble.
24. The marking scheme given at the beginning of each section gives details of how **darkened** and **not darkened** bubbles are evaluated.

NAME OF THE CANDIDATE ..... MOHIT KHERWA .....ROLL NO ..... 1051119 .....I HAVE READ ALL THE INSTRUCTIONS  
AND SHALL ABIDE BY THEMI have verified the identity, name and roll  
number of the candidate, and that question  
paper and ORS codes are the same-----  
Signature of the Candidate-----  
Signature of the Invigilator**Space for rough work**