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RBSE

**RAJASTHAN BOARD
SECONDARY EXAMINATION
2022**

**CLASS
XII**

Questions & Solutions

Date: 13 April, 2022 | TIME : (9.00 a.m. to 11.45 a.m)

Duration: 2hr, 45 min. | Max. Marks: 80






SUBJECT: MATHEMATICS

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Roll No.

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Candidates must write the Code on the title page of the answer-book

MATHEMATICS

Time allowed : 2 hr, 45 Min.

Maximum Marks : 80

General Instructions :

परीक्षार्थियों के लिए सामान्य निर्देश:

1. Candidate must write first his/her Roll No. on the question paper compulsorily
परीक्षार्थी सर्वप्रथम अपने प्रश्न-पत्र पर नामांक अनिवार्यतः लिखें।
2. All the question are compulsory.
सभी प्रश्न करने अनिवार्य है।
3. Write the answer to all question in the given answer-book only.
सभी प्रश्नों का उत्तर-पुस्तिका में ही लिखें।
4. For questions having more than one part, the answers to those parts are to be written together in continuity.
जिन प्रश्नों में आन्तरिक खण्ड है उन सभी के उत्तर एक साथ ही लिखें।
5. If there is any error/ difference/ contradiction in Hindi & English versions of the questions paper, the questions of Hindi version should be treated valid.
प्रश्न-पत्र के हिन्दी व अंग्रेजी रूपान्तरण में किसी प्रकार की त्रुटि/अन्तर/विरोधाभास होने पर हिन्दी भाषा के प्रश्न को ही सही मानें।
6. Write down the serial number of the question before attempting it.
प्रश्न का उत्तर लिखने से पूर्व प्रश्न का क्रमांक अवश्य लिखें।
7. There are internal choices in Questions Nos. 17 to 23.
प्रश्न क्रमांक 17 से 23 में आन्तरिक विकल्प है।

SECTION A

खण्ड A

1. Choose the correct answer from multiple choice question 1 (i to xii) and write in given answer book.
बहुविकल्पी प्रश्न 1 (i से xii) : निम्न प्रश्नों के उत्तर का सही विकल्प चयन कर उत्तर पुस्तिका में लिखिए
- (i) If $f : R \rightarrow R$ be defined as $f(x) = x^4$, then the function
- (A) f is one-one and onto (B) f is many one onto
(C) f is one-one but not onto (D) f is neither one-one nor onto

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मान लीजिए कि $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^4$ द्वारा परिभाषित हैं, तो फलन :

- (A) f एकैकी आच्छादक है। (B) f बहु एक आच्छादक है।
(C) f एकैकी है किन्तु आच्छादक नहीं है। (D) f न तो एकैकी है और न ही आच्छादक है।

Ans. (D)

(ii) The value of $2\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ is

- (A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) $\frac{5\pi}{6}$

$2\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ का मान है :

- (A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{2}$ (D) $\frac{5\pi}{6}$

Ans. (B)

(iii) If $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ are two matrices, then AB will be

- (A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

यदि $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ तथा $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ दो आव्यूह हैं, तो AB होगा -

- (A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Ans. (A)

(iv) In the equation $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$, the value of x is

- (A) 3 (B) 4 (C) 5 (D) 2

समीकरण $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ में x का मान है :

- (A) 3 (B) 4 (C) 5 (D) 2

Ans. (D)

(v) If $3x + 2y = \sin x$, then the $\frac{dy}{dx}$ is

- (A) $\frac{\cos x + 3}{2}$ (B) $\frac{\cos x - 2}{3}$ (C) $\frac{\cos x - 3}{2}$ (D) $\frac{\cos x + 2}{3}$

यदि $3x + 2y = \sin x$, तो $\frac{dy}{dx}$ है




- (A) $\frac{\cos x + 3}{2}$ (B) $\frac{\cos x - 2}{3}$ (C) $\frac{\cos x - 3}{2}$ (D) $\frac{\cos x + 2}{3}$

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Ans. (C)

Sol. $3x + 2y = \sin x$

$$3 + 2 \frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{\cos x - 3}{2}$$

(vi) If $\begin{vmatrix} 3 & 3 \\ x & 1 \end{vmatrix} = \begin{vmatrix} -3 & x \\ 1 & 1 \end{vmatrix}$, then value of x is

(A) 2

(B) 3

(C) -3

(D) -2

यदि $\begin{vmatrix} 3 & 3 \\ x & 1 \end{vmatrix} = \begin{vmatrix} -3 & x \\ 1 & 1 \end{vmatrix}$, तो x का मान है

(A) 2

(B) 3

(C) -3

(D) -2

Ans. (B)

Sol. $3 - 3x = -3 - x$

$$6 = 2x$$

$$x = 3$$

(vii) The second order derivative of $x^3 \log x$. w.r.t x is

(A) $x(5 + 6 \log x)$

(B) $x^2(5 + 6 \log x)$

(C) $x(6 + 5 \log x)$

(D) $x^2(6 + 5 \log x)$

$x^3 \log x$. का x सापेक्ष द्वितीय कोटि का अवकलन है -

(A) $x(5 + 6 \log x)$

(B) $x^2(5 + 6 \log x)$

(C) $x(6 + 5 \log x)$

(D) $x^2(6 + 5 \log x)$

Ans. (A)

Sol. $y = x^3 \log x$

$$y' = x^3 \times \frac{1}{x} + \log x \times 3x^2$$

$$y' = x^2 + 3x^2 \log x$$

$$y'' = 2x + 3 \left[x^2 \times \frac{1}{x} + \log x \times 2x \right]$$

$$= 2x + 3x + 6x \log x$$

$$= 5x + 6x \log x$$

$$= x [5 + 6 \log x] \quad (a)$$

(viii) The integration of the function $\frac{x}{e^{x^2}}$, with respect to x is

(A) $\frac{1}{2e^{x^2}} + C$

(B) $\frac{2}{e^{x^2}} + C$

(C) $-\frac{2}{e^{x^2}} + C$

(D) $-\frac{1}{2e^{x^2}} + C$

फलन $\frac{x}{e^{x^2}}$ का x के सापेक्ष समाकलन है :

(A) $\frac{1}{2e^{x^2}} + C$

(B) $\frac{2}{e^{x^2}} + C$

(C) $-\frac{2}{e^{x^2}} + C$

(D) $-\frac{1}{2e^{x^2}} + C$

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Ans. (D)

Sol.

$$\int \frac{x}{e^{x^2}} dx$$

$$\text{Let } x^2 = t$$

$$2x dx = dt$$

$$x dx = \frac{dt}{2}$$

$$\frac{1}{2} \int \frac{dt}{e^t} = -\frac{1}{2} e^{-t} + c$$

$$= -\frac{1}{2} e^{-x^2}$$

$$= -\frac{1}{2e^{x^2}} + c$$

(ix) The degree of differential equation $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^3 - y \frac{dy}{dx} = 0$ is

(A) 3

(B) 1

(C) 0

(D) 2

अवकल समीकरण $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^3 - y \frac{dy}{dx} = 0$ की घात है

(A) 3

(B) 1

(C) 0

(D) 2

Ans. (B)

(x) The magnitude of the vector $\frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} - \frac{1}{\sqrt{3}} \hat{k}$ is

(A) 3

(B) 1

(C) -1

(D) 2

सदिश $\frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} - \frac{1}{\sqrt{3}} \hat{k}$ का परिमाण है -

(A) 3

(B) 1

(C) -1

(D) 2

Ans. (B)

(xi) The unit vector in the direction of the vector $\vec{a} = -2\hat{i} + 3\hat{j} - \hat{k}$ is

(A) $\frac{2\hat{i}}{\sqrt{14}} - \frac{3\hat{j}}{\sqrt{14}} + \frac{\hat{k}}{\sqrt{14}}$

(B) $\frac{2\hat{i}}{\sqrt{14}} - \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$

(D) $\frac{2\hat{i}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$

(D) $\frac{-2\hat{i}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$

सदिश $\vec{a} = -2\hat{i} + 3\hat{j} - \hat{k}$ के अनुदिश मात्रक (इकाई) सदिश है:

(A) $\frac{2\hat{i}}{\sqrt{14}} - \frac{3\hat{j}}{\sqrt{14}} + \frac{\hat{k}}{\sqrt{14}}$

(B) $\frac{2\hat{i}}{\sqrt{14}} - \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$

(C) $\frac{2\hat{i}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$

(D) $\frac{-2\hat{i}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$

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Ans. (D)

(xii) If a die is thrown twice, then the probability of getting the sum of the numbers equal to 7, will be

- (A) $\frac{5}{36}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{1}{9}$

यदि एक पासे को दो बार उछाला जाता है, तो प्रकट हुई संख्याओं का योग 7 पाए जाने की प्रायिकता होगी

- (A) $\frac{5}{36}$ (B) $\frac{1}{12}$ (C*) $\frac{1}{6}$ (D) $\frac{1}{9}$

Ans. (C)

2. Fill in the blanks (i) to (vi):

रिक्त स्थानों की पूर्ति कीजिए (i) से (vi) :

(i) If $f(x) = 27x^3$ and $g(x) = x^{1/3}$, then $g \circ f(x) = \underline{\hspace{2cm}}$
यदि $f(x) = 27x^3$ तथा $g(x) = x^{1/3}$ हो, तो $g \circ f(x) = \underline{\hspace{2cm}}$ है।

Ans. $3x$

Sol. $g(f(x)) = g[27x^3]$
 $= g[27x^3]$
 $= (27x^3)^{1/3}$
 $= 3x$

(ii) The principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is $\underline{\hspace{2cm}}$

$\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ का मुख्य मान $\underline{\hspace{2cm}}$ है।

Ans. $\frac{3\pi}{4}$

Sol. $\cos^{-1}(-x) = \pi - \cos^{-1}x$
 $= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

(iii) If $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, then $A - B = \underline{\hspace{2cm}}$

यदि $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ और $B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ तो $A - B = \underline{\hspace{2cm}}$ है।

Ans. $\begin{bmatrix} -1 & -1 \\ -5 & 3 \end{bmatrix}$

Sol. $A - B$

$\begin{bmatrix} -1 & -1 \\ -5 & 3 \end{bmatrix}$ Ans.

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(iv) If $3x+2y = \cos y$, then $\frac{dy}{dx} =$ _____

यदि $3x+2y = \cos y$, तो $\frac{dy}{dx} =$ _____ है।

Ans. $\frac{dy}{dx} = \frac{-3}{2+\sin y}$

Sol. $3 + 2 \frac{dy}{dx} = -\sin y \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-3}{2+\sin y}$$

(v) Then value of $\int_0^1 \frac{dx}{1+x^2}$ is _____

$\int_0^1 \frac{dx}{1+x^2}$ का मान _____ है।

Ans. $\frac{\pi}{4}$

Sol. $\int_0^1 \frac{dx}{1+x^2} = \left\{ \tan^{-1} x \right\}_0^1 = \frac{\pi}{4} - 0$

(vi) The vector joining the points A(1, 2,2,) and B(2,3,1) directed from A to B is बिन्दुओं A(1, 2,2,) और B(2,3,1) को मिलाने वाला एवं A से B की तरफ दिष्ट सदिश _____ है।

Ans. $\vec{AB} = (\hat{i} + \hat{j} - \hat{k})$

3. Give the answer of the following questions (i to xii) in one line.

निम्न प्रश्नों (I से xii) के उत्तर एक पंक्ति में दीजिए।

(i) Show that the functions $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = 2x$ is not onto. $f(x) = 2x$ द्वारा प्रदत्त फलन $f : \mathbb{N} \rightarrow \mathbb{N}$ दर्शाइए कि $f(x)$ आच्छादक नहीं है।

Sol. $f : \mathbb{N} \rightarrow \mathbb{N}$

$$f(x) = 2x$$

$$y = 2x$$

$$x = \frac{y}{2}; y \in \mathbb{N}$$

$$y = 3; x = \frac{3}{2} \notin \mathbb{N}$$

∴ it is not onto function

(ii) Find the value of $3\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$3\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ का मान ज्ञात कीजिए।

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Ans. $\frac{5\pi}{6}$

Sol. $3 \cos^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{\sqrt{3}}{2}$

$\Rightarrow 3 \times \frac{\pi}{6} + \frac{\pi}{3}$

$\frac{5\pi}{6}$

(iii) Write the identity matrix of 3×3 order.
 3×3 कोटि का तत्समक आव्यूह लिखिए।

Ans. $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(iv) For which value of x , the value of determinant $\begin{vmatrix} 3 & 2 \\ 5 & x \end{vmatrix}$ will be zero?

x के किस मान के लिए सारणिक $\begin{vmatrix} 3 & 2 \\ 5 & x \end{vmatrix}$ का मान शून्य होगा?

Ans. $\frac{10}{3}$

Sol. $\begin{vmatrix} 3 & 2 \\ 5 & x \end{vmatrix} = 0$

$3x - 10 = 0$

$x = \frac{10}{3}$ Ans.

(v) Find the minor of the element 6 in the determinant $\begin{vmatrix} 1 & 3 & 2 \\ 8 & 6 & 3 \\ 9 & 5 & 4 \end{vmatrix}$

सारणिक $\begin{vmatrix} 1 & 3 & 2 \\ 8 & 6 & 3 \\ 9 & 5 & 4 \end{vmatrix}$ में अवयव 6 का उपसारणिक ज्ञात कीजिए।

Ans. -14

Sol. $\begin{vmatrix} 1 & 3 & 2 \\ 8 & 6 & 3 \\ 9 & 5 & 4 \end{vmatrix}$

Minor of elements 6 $\Rightarrow \begin{vmatrix} 1 & 2 \\ 9 & 4 \end{vmatrix} = 4 - 18$






$= -14$ Ans.

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(vi) Show that the function $f(x) = x^2$ is continuous at $x = 0$
दर्शाए कि फलन $f(x) = x^2$, $x = 0$ पर सतत है।

Ans. It is Continuous

Sol. $f(x) = x^2$; $x = 0$ continues

$$\text{LHL} \quad \lim_{h \rightarrow 0} f(x)$$

$$\text{RHL} \quad \lim_{h \rightarrow 0} f(a + h)$$

$$\Rightarrow \lim_{h \rightarrow 0} f(a - x)$$

$$\Rightarrow \lim_{h \rightarrow 0} f(0 + h)$$

$$\Rightarrow \lim_{h \rightarrow 0} (0-h)^2$$

$$\Rightarrow \lim_{h \rightarrow 0} (0 + h)^2$$

$$\Rightarrow 0$$

$$\Rightarrow 0$$

and at $x = 0$; $f(x) = 0$

∴ LHL = RHL = $f(0)$

∴ $f(x)$ is continuous at $x = 0$

(vii) Find $\int \sqrt[3]{x^4} dx$.

$\int \sqrt[3]{x^4} dx$ ज्ञात कीजिए।

Ans. $\frac{3}{7} x^{\frac{7}{3}} + c$

Sol. $\int x^{\frac{4}{3}} dx$

$$\Rightarrow \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + c$$

$$\Rightarrow \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + c$$

$$\Rightarrow \frac{3}{7} x^{\frac{7}{3}} + c$$

(viii) Find the general solution of the differential equation $(1 + x^2) dy = (1 + y^2) dx$.

अवकल समीकरण $(1 + x^2) dy = (1 + y^2) dx$ का व्यापक हल ज्ञात कीजिए।

Ans. $\tan^{-1} y = \tan^{-1} x + c$

Sol. $(1+x^2) dy = (1+y^2) dx$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\Rightarrow \tan^{-1} y = \tan^{-1} x + c$$

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(ix) Find the equation of a curve if the slope to the tangent at the point (x, y) of the curve is $\frac{3x^2}{y^2}$.

ऐसे वक्र का समीकरण ज्ञात कीजिए जिसके किसी बिन्दु (x, y) पर स्पर्श-रेखा की प्रवणता $\frac{3x^2}{y^2}$ है।

Ans. $\frac{y^3}{3} = x^3 + c$

Sol. Slope = $\frac{3x^2}{y^2}$

$$\frac{dy}{dx} = \frac{3x^2}{y^2}$$

$$\int y^2 dy = \int 3x^2 dx$$

$$\frac{y^3}{3} = \frac{3x^3}{3} + c$$

$$\frac{y^3}{3} = x^3 + c$$

(x) Find the unit vector in the direction of the sum of the vector $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k}$.

सदिशों $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ और $\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ के योगफल के अनुदिश मात्रक सदिश ज्ञात कीजिए।

Ans. $\frac{4\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{29}}$

Sol. $\vec{a} + \vec{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

Unit vector

$$\Rightarrow \frac{4\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{16+9+4}}$$

$$\Rightarrow \frac{4\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{29}}$$

(xi) Find the vector components of the vector with initial point (2, 1) and terminal point (-5, 7).

एक सदिश का प्रारम्भिक बिन्दु (2, 1) है और अन्तिम बिन्दु (-5, 7) है। इस सदिश के सदिश घटक ज्ञात कीजिए।

Ans. $-7\hat{i}$ and $6\hat{j}$

Sol. $\vec{AB} = -7\hat{i} + 6\hat{j}$

Vector component are: $-7\hat{i}$ and $6\hat{j}$

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- (xii) If a die is thrown once, then find the probability that the number appeared on the die is a multiple of 2.
यदि एक पासे को एक बार उछाला जाता है, तो पासे पर संख्या 2 का अपवर्त्य आने की प्रायिकता ज्ञात कीजिए।

Ans. $\frac{3}{6} = \frac{1}{2}$

Sol. die is thrown one
= {1, 2, 3, 4, 5, 6}

Probability of multiple of 2 is

$\frac{3}{6} = \frac{1}{2}$ Ans.

SECTION B खण्ड B

4. Considering $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 2x + 3$, prove that f is invertible
 $f(x) = 2x + 3$ द्वारा प्रदत्त फलन $f : \mathbb{R} \rightarrow \mathbb{R}$ पर विचार करते हुए सिद्ध कीजिए कि f व्युत्क्रमणीय है।

Sol. $f : \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = 2x + 3$

For one-one $f(x_1) = f(x_2)$

$2x_1 + 3 = 2x_2 + 3$

$x_1 = x_2 \quad x_1, x_2 \in \mathbb{R}$

∴ it is one-one

For onto Let $f(x) = y$

$y = 2x + 3$

$x = \frac{y-3}{2}; y \in \mathbb{R}$

For every value of y has pre image in domain \mathbb{R} it is onto.

∴ it will be invertible

5. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B [1 \ 3 \ -6]$, then verify that $(AB)' = B'A'$

यदि $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B [1 \ 3 \ -6]$, तो सत्यापित कीजिए कि $(AB)' = B'A'$

Sol. $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ $B = [1, 3, -6]$

$(AB)' = B'A'$

LHS $(AB)' = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} [1 \ 3 \ -6]$

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$$= \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$\text{RHS } B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \quad A' = [-2 \ 4 \ 5]$$

$$B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} [-2 \ 4 \ 5]$$

$$= \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ +12 & -24 & -30 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

6. If $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then find X and Y

यदि $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ तथा $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, तो X तथा Y ज्ञात कीजिए।

Sol. $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ 1

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
2

Adding equation 1 and 2

$$X = \begin{bmatrix} 5 & 8 \\ 1 & 4 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

7. If $\begin{vmatrix} 2 & 3 \\ y & x \end{vmatrix} = 3$, $\begin{vmatrix} x & y \\ 4 & 2 \end{vmatrix} = 5$, then find the values of x and y

यदि $\begin{vmatrix} 2 & 3 \\ y & x \end{vmatrix} = 3$, $\begin{vmatrix} x & y \\ 4 & 2 \end{vmatrix} = 5$, तो x तथा y के मान ज्ञात कीजिए।

Sol. $2x - 3y = 3$ 1

$$2x - 4y = 5$$
2

Subtracting equation 2 from equation 1

$$y = -2$$

$$\therefore 2x + 6 = 3$$






$$x = \frac{-3}{2}$$

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8. Prove that the points A(a, b + c), B(b, c + a) and C(c, a + b) are collinear

सिद्ध कीजिए कि बिन्दु A(a, b + c), B(b, c + a) तथा C(c, a + b) सरेख हैं।

Sol. Points collinear if

$$\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 + C_2$$

$$\begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix}$$

Taking common (a + b + c) from c₁

$$(a+b+c) \begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix}$$

$$\therefore [C_1 = C_3]$$

$$(a+b+c) \times 0 = 0 \quad \text{RHS.}$$

9. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right); 0 < x < 1$, then find $\frac{dy}{dx}$

यदि $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right); 0 < x < 1$, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

Sol. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan \theta$

$$y = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$y = 2\theta$$

$$y = 2 \tan^{-1} x$$

$$\frac{dy}{dx} = 2 \times \frac{1}{1+x^2}$$

$$= \frac{2}{1+x^2} \quad \text{ans.}$$

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10. If $y = 3\cos x - 2\sin x$, then prove that $\frac{d^2y}{dx^2} + y = 0$.

यदि $y = 3\cos x - 2\sin x$ है, तो सिद्ध कीजिए कि $\frac{d^2y}{dx^2} + y = 0$.

Sol. $y = 3\cos x - 2\sin x$

$$\frac{dy}{dx} = -3\sin x - 2\cos x$$

Again differentiate

$$\frac{d^2y}{dx^2} = -3\cos x - 2\sin x$$

$$\therefore \frac{d^2y}{dx^2} + 3\cos x - 2\sin x = 0$$

$$\frac{d^2y}{dx^2} + y = 0 \quad \text{H.P.}$$

11. Find the antiderivative $F(x)$ of f defined by $f(x) = 5x^4 - 5$, where $F(0) = 2$.

$f(x) = 5x^4 - 5$ द्वारा परिभाषित फलन f का प्रतिअवकलज $F(x)$ ज्ञात कीजिए जहाँ $F(0) = 2$ है।

Sol. $f(x) = 5x^4 - 5$

Antiderivate of $f(x)$

$$\Rightarrow \int f(x) dx$$

$$F(x) = \int (5x^4 - 5) dx$$

$$F(x) = \frac{5x^5}{5} - 5x + c$$

$$\text{Put } x = 0 \Rightarrow F(0) = c$$

$$2 = c$$

$$\therefore F(x) = x^5 - 5x + 2$$

12. Find $\int \frac{\cos^2 x}{1 + \sin x} dx$.

$$\int \frac{\cos^2 x}{1 + \sin x} dx \quad \text{ज्ञात कीजिए।}$$

Sol. $I = \int \frac{\cos^2 x}{1 + \sin x} dx$

$$I = \int \frac{1 - \sin^2 x}{1 + \sin x} dx$$

$$\int \frac{(1 + \sin x)(1 - \sin x)}{(1 + \sin x)} dx$$

$$\int (1 - \sin x) dx$$

$$I = x + \cos x + c$$

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13. Find the integration factor of the differential equation $x \frac{dy}{dx} - y = 2x^2$.

अवकल समीकरण $x \frac{dy}{dx} - y = 2x^2$ का समाकलन गुणक ज्ञात कीजिए।

Sol. $x \frac{dy}{dx} - y = 2x^2$

$$\frac{dy}{dx} - \frac{y}{x} = 2x$$

$$P = -\frac{1}{x}; Q = 2x$$

$$I.F. = e^{\int pdx}$$

$$= e^{\int -\frac{1}{x} dx}$$

$$= e^{-\log x}$$

$$= \frac{1}{x}$$

14. Find $|\vec{a} - \vec{b}|$; if two vector \vec{a} and \vec{b} are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$.

यदि दो सदिश \vec{a} और \vec{b} इस प्रकार है कि $|\vec{a}| = 2$, $|\vec{b}| = 3$ और $\vec{a} \cdot \vec{b} = 4$ तो $|\vec{a} - \vec{b}|$ ज्ञात कीजिए।

Sol. $|\vec{a}| = 2$, $|\vec{b}| = 3$,

$$\vec{a} \cdot \vec{b} = 4$$

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

$$= 4 + 9 - 2 \times 4$$

$$= 13 - 8$$

$$|\vec{a} - \vec{b}|^2 = 5$$

$$\therefore |\vec{a} - \vec{b}| = \sqrt{5}$$

15. If a die is thrown three times, then find the probability of getting an odd number on the die in each throw.

यदि एक पासे को तीन बार उछाला जाता है, तो प्रत्येक बार पासे पर विषम संख्या प्राप्त होने की प्रायिकता ज्ञात कीजिए।

Sol. Probability of getting an odd number in a single throw of a die = $\frac{3}{6} = \frac{1}{2}$

$$\therefore \text{probability of getting an odd number on each throw} \Rightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

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16. If $P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ and $P(A \cap B) = \frac{4}{11}$, then find $P(A/B)$.

यदि $P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ और $P(A \cap B) = \frac{4}{11}$ हो, तो $P(A/B)$ ज्ञात कीजिए।

Sol. $\therefore P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{4/11}{6/11} \Rightarrow \frac{4}{6} = \frac{2}{3}$$

SECTION C

खण्ड C

17. Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \frac{3}{4}$

सिद्ध कीजिए कि $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \frac{3}{4}$

OR/अथवा

If $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$, then find the value of x.

यदि $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$, तो x का मान ज्ञात कीजिए।

Sol. $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \frac{3}{4}$

LHS

$$\Rightarrow \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11}$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{1}{2} + \frac{2}{11}}{1 - \frac{1}{2} \times \frac{2}{11}} \right]$$

$$\Rightarrow \tan^{-1} \left[\frac{11+4}{10} \right]$$

$$\Rightarrow \tan^{-1} \left[\frac{15}{10} \right]$$

$$\Rightarrow \tan^{-1} \left[\frac{15}{22} \times \frac{11}{10} \right]$$

$$\Rightarrow \tan^{-1} \left(\frac{3}{4} \right) = \text{RHS}$$

OR/अथवा

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$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x-2}\right) \times \frac{x+1}{x+2}}\right] = \frac{\pi}{4}$$

$$\Rightarrow \frac{(x-1)(x+2) + (x+1)(x-2)}{x^2 - 4 - (x^2 - 1)} = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{x^2 + 2x - x - 2 + x^2 - 2x + x - 2}{x^2 - 4 - x^2 + 1} = 1$$

$$\Rightarrow \frac{2x^2 - 4}{-3} = 1$$

$$\Rightarrow 2x^2 - 4 = -3$$

$$\Rightarrow 2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \frac{1}{\sqrt{2}}$$

18. If x and y are connected parametrically by the equation $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$, then without

eliminating the parameters, find $\frac{dy}{dx}$.

यदि x तथा y दिए समीकरण $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$ द्वारा एक दूसरे से प्राचलिक रूप में संबंधित हों, तो

प्राचलों का विलोप किए बिना $\frac{dy}{dx}$ ज्ञात कीजिए।

OR/अथवा

Find $\frac{dy}{dx}$, if $y \sqrt{1-x^2} = \sin^{-1} x$.

यदि $y \sqrt{1-x^2} = \sin^{-1} x$, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

Sol. Given $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$

$$\text{Then, } \frac{dx}{dt} = a \left[\frac{d}{dt}(\cos t) + \frac{d}{dt} \left(\log \tan \frac{t}{2} \right) \right]$$

$$= a \left[-\sin t + \frac{1}{\tan \frac{t}{2}} \frac{d}{dt} \left(\tan \frac{t}{2} \right) \right] = a \left[-\sin t + \cot \frac{t}{2} \sec^2 \frac{t}{2} \frac{d}{dt} \left(\frac{t}{2} \right) \right]$$

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$$= a \left[-\sin t + \frac{\cot \frac{t}{2}}{\sin \frac{t}{2}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2} \right] = a \left[-\sin t + \frac{t}{2 \sin \frac{t}{2} \cos \frac{t}{2}} \right]$$

$$= a \left[-\sin t + \frac{t}{\sin t} \right] = a \left(\frac{-\sin^2 t + 1}{\sin t} \right)$$

$$= a \left(\frac{\cos^2 t}{\sin t} \right) = a \left(\frac{\cos^2 t}{\sin t} \right)$$

$$y = a \sin t$$

$$\frac{dy}{dt} = a \cot t$$

$$\frac{dy}{dx} = \frac{a \cot t}{a \left(\frac{\cos^2 t}{\sin t} \right)}$$

$$\frac{dy}{dx} = \tan t$$

OR/अथवा

$$y \sqrt{1-x^2} = \sin^{-1} x$$

differentiate w.r.t x...

$$y \cdot \frac{1}{2\sqrt{1-x^2}} (-2x) + \sqrt{1-x^2} \cdot \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{-xy}{\sqrt{1-x^2}} + \sqrt{1-x^2} \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$-xy + (1-x^2) \frac{dy}{dx} = 1$$

$$(1-x^2) \frac{dy}{dx} = 1 + xy$$

$$\frac{dy}{dx} = \frac{1+xy}{1-x^2}$$

19. Integrate the function $\frac{1}{x^2 - 6x + 13}$ with respect to x.

फलन $\frac{1}{x^2 - 6x + 13}$ का x के सापेक्ष समाकलन कीजिए।

OR अथवा

$$\text{Find } \int \frac{x+1}{x^2+4x+5} dx$$

$$\int \frac{x+1}{x^2+4x+5} dx \text{ ज्ञात कीजिए।}$$

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Sol. $x^2 - 6x + 13$

$$\Rightarrow (x-3)^2 + 4$$

$$I = \int \frac{1}{(x-3)^2 + (2)^2} dx$$

$$I = \frac{1}{2} \tan^{-1} \left(\frac{x-3}{2} \right) + C$$

OR अथवा

$$I = \int \frac{x+1}{x^2 + 4x + 5} dx$$

Let $x + 1 = A \frac{d}{dx} (x^2 + 4x + 5) + B$

$$= A (2x + 4) + B$$

$$x + 1 = 2xA + 4A + B$$

compare

$$2A = 1 \quad A = \frac{1}{2}$$

$$4A + B = 1$$

$$B = -1$$

$$\therefore x + 1 = \frac{1}{2} (2x + 4) - 1$$

$$I = \int \frac{\frac{1}{2} (2x + 4) - 1}{x^2 + 4x + 5} dx$$

$$I = \frac{1}{2} \int \frac{2x + 4}{x^2 + 4x + 5} dx$$

$$\int \frac{1}{x^2 + 4x + 5} dx$$

$$I = \frac{1}{2} I_1 - I_2 \quad \dots\dots(1)$$

$$\therefore I_1 = \int \frac{2x + 4}{x^2 + 4x + 5} dx$$

$$I_1 = \log |x^2 + 4x + 5|$$

$$\text{and } I_2 = \int \frac{1}{x^2 + 4x + 5} dx$$

$$\text{Let } x^2 + 4x + 5 \Rightarrow (x + 2)^2 + 1$$

$$I_2 = \int \frac{1}{(x + 2)^2 + (1)^2} dx$$

$$I_2 = \tan^{-1} \left(\frac{x + 2}{1} \right)$$

Putting value of I_1 and I_2 in equation (1)






$$\therefore I = \frac{1}{2} \log |x^2 + 4x + 5| - \tan^{-1}(x + 2) + C$$

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20. If two sides of a triangle are represented by the vectors $\hat{i} + 2\hat{j} + 2\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, then prove that the area of the triangle is $\frac{5}{2}\sqrt{5}$ square units.

यदि एक त्रिभुज की दो भुजाएँ सदिश $\hat{i} + 2\hat{j} + 2\hat{k}$ तथा $3\hat{i} - 2\hat{j} + \hat{k}$ से निरूपित हो, तो सिद्ध कीजिए कि त्रिभुज का क्षेत्रफल $\frac{5}{2}\sqrt{5}$ वर्ग इकाई है।

अथवा OR

If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then find the value of $|\vec{a} \times \vec{b}|$

यदि $|\vec{a}| = 10, |\vec{b}| = 2$ तथा $\vec{a} \cdot \vec{b} = 12$, तो $|\vec{a} \times \vec{b}|$ का मान ज्ञात कीजिए।

Sol. Area of $\Delta ABC = \frac{1}{2} |\vec{a} \times \vec{b}|$

$$= \frac{1}{2} |(\hat{i} + 2\hat{j} + 2\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|$$

$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [(2+4)\hat{i} - (1-6)\hat{j} + (-2-6)\hat{k}]$$

$$= \frac{1}{2} |6\hat{i} + 5\hat{j} - 8\hat{k}|$$

$$= \frac{1}{2} \sqrt{36 + 25 + 64}$$

$$= \frac{1}{2} \sqrt{125}$$

$$= \frac{1}{2} 5\sqrt{5} \text{ Square Unit.}$$

OR

$$|\vec{a}| = 10 : \quad |\vec{b}| = 2 \quad \vec{a} \cdot \vec{b} = 12$$

$$|\vec{a} \times \vec{b}| = ?$$

$$\vec{a} \cdot \vec{b} = 12$$

$$|\vec{a}| |\vec{b}| \cos\theta = 12$$

$$10 \times 2 \cos\theta = 12$$

$$\cos\theta = \frac{12}{10 \times 2} = \frac{3}{5}$$

$$\therefore \sin\theta = \frac{4}{5}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin\theta$$

$$|\vec{a} \times \vec{b}| = 10 \times 2 \times \frac{4}{5}$$

$$|\vec{a} \times \vec{b}| = 16 \text{ Ans.}$$

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SECTION D

खण्ड D

21. Find the value of $\int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx$

$\int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx$ का मान ज्ञात कीजिए।

OR (अथवा)

Find the value of $\int_{-\pi/4}^{\pi/4} \sin^2 x dx$

$\int_{-\pi/4}^{\pi/4} \sin^2 x dx$ का मान ज्ञात कीजिए।

Sol. $\int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx$

Let $x^5 + 1 = t$ $x = -1; t = 0$
 $5x^4 dx = dt$ $x = 1; t = 2$

$\therefore \int_0^2 \sqrt{t} dt$

$\Rightarrow \left[\frac{t^{3/2}}{3/2} \right]_0^2$

$= \frac{2}{3} [(2)^{3/2} - 0]$

$= \frac{2}{3} (2)^{3/2}$ Ans.

OR (अथवा)

$I = \int_{-\pi/4}^{\pi/4} \sin^2 x dx$

$I = 2 \int_0^{\pi/4} \left(\frac{1 - \cos 2x}{2} \right) dx$

$= \frac{2}{2} \left[(x)_0^{\pi/4} - \left(\frac{\sin 2x}{2} \right)_0^{\pi/4} \right] = \left[\left(\frac{\pi}{4} - 0 \right) - \frac{1}{2} (\sin \frac{\pi}{2} - \sin 0) \right]$






$= \frac{\pi}{4} - \frac{1}{2} \times 1 = \frac{\pi}{4} - \frac{1}{2}$ Ans.

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22. Find the general solution of the differential equation $x \frac{dy}{dx} + 2y = x^2, (x \neq 0)$,

अवकल समीकरण $x \frac{dy}{dx} + 2y = x^2, (x \neq 0)$ का व्यापक हल ज्ञात कीजिए।

OR
अथवा

Solve the differential equation $2xy \cdot dy = (x^2 + y^2)dx$
अवकल समीकरण $2xy \cdot dy = (x^2 + y^2)dx$ को हल कीजिए।

Sol. $x \frac{dy}{dx} + 2y = x^2$

$$\frac{dy}{dx} + \frac{2y}{x} = x$$

$$\therefore p = \frac{2}{x}; Q = x$$

$$\text{I.F.} = e^{\int p dx}$$

$$= e^{\int \frac{2}{x} dx}$$

$$= e^{2 \log x}$$

$$= x^2$$

\therefore **Solution:** $y \times \text{I.F.} = \int Q \cdot \text{I.F.} dx + c$

$$y \times x^2 = \int x \times x^2 dx + c$$

$$yx^2 = \frac{x^4}{4} + c \quad \text{Ans.}$$

OR

$$2xy dy = (x^2 + y^2) dx$$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

This is Homogenous differential Equation.

$$y = vx$$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{2x vx}$$

$$x \frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$

$$x \frac{dv}{dx} = \frac{1 - v^2}{2v}$$

$$\int \frac{2v}{1 - v^2} dv = \int \frac{dx}{x}$$

$$-\log |1 - v^2| = \log x + c$$

$$-\log \left| 1 - \frac{y^2}{x^2} \right| = \log x + c \quad \text{Ans.}$$

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23. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.
- (i) Find the probability that he reads neither Hindi nor English newspapers
(ii) If he reads Hindi newspaper, find the probability that he reads English newspaper also.

एक छात्रवास में 60% विद्यार्थी हिन्दी का अखबार, 40% अंग्रेजी का अखबार और 20% दोनों हिन्दी तथा अंग्रेजी का अखबार पढ़ते हैं। एक छात्र को यदृच्छया चुना जाता है।

- (i) प्रायिकता ज्ञात कीजिए कि वह न तो हिन्दी और न ही अंग्रेजी का अखबार पढ़ता है
(ii) यदि वह हिन्दी का अखबार पढ़ता है तो अंग्रेजी का अखबार भी पढ़ने वाला होने की प्रायिकता ज्ञात कीजिए।

OR अथवा

If three cards are drawn successively without replacement from a pack of 52 well shuffled cards, then find the probability that first two cards are aces and the third card drawn is a king

यदि 52 पत्तों की अच्छी तरह फेटी गई गड्डी में से एक के बाद एक तीन पत्ते बिना प्रतिस्थापित किए निकाले जाते हैं, तो पहले दो पत्तों का इक्का होने और तीसरे पत्ते का बादशाह होने की प्रायिकता ज्ञात कीजिए।

Sol. Let H denote the student who read Hindi newspaper and E denote the students who read English newspaper

$$P(H) = 60\% = \frac{60}{100} = \frac{3}{5}$$

$$P(E) = 40\% = \frac{40}{100} = \frac{2}{5}$$

$$P(H \cap E) = 20\% = \frac{20}{100} = \frac{1}{5}$$

$$(i) (H' \cap E') = P(H \cup E)' = 1 - P(H \cup E)$$

$$= 1 - [P(H) + P(E) - P(H \cap E)]$$

$$= 1 - \left[\frac{3}{5} + \frac{2}{5} - \frac{1}{5} \right] = 1 - \frac{4}{5} = \frac{1}{5} \text{ Ans}$$

$$(ii) P(E/H) = \frac{P(E \cap H)}{P(H)} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$$

OR

Let E_1 : first card is drawing a Ace

E_2 : Second card is drawing a Ace

E_3 : Third card is drawing a king

∴ By multiplication law of probability

$$\Rightarrow P(E_1) \times P(E_2) \times P(E_3)$$

$$\Rightarrow \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50}$$

$$\Rightarrow \frac{2}{5525} \text{ Ans.}$$

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