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CBSE

CENTRAL BOARD SECONDARY EXAMINATION

2023

CLASS XII

Questions & Solutions

Date: 11 March 2023 | TIME : (10:30 a.m. to 01:30 p.m)

Duration: 3 hr | Max. Marks: 80

SUBJECT: MATHEMATICS

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SET-3

Series EF1GH/2

Code No. **65/2/3**

Roll No. | | | | | | | |

Candidates must write the Q.P. Code on the title page of the answer-book

MATHEMATICS

Time allowed : 3 Hours **Maximum Marks : 80**

Note :

(i) Please check that this question paper contains 23 printed pages.

(ii) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer -book by the candidate.

(iii) Please check that this question paper contains 38 questions.

(iv) Please write down the serial number of the question- in the answer book before attempting it.

(v) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period .

General Instructions :

Read the following instructions very carefully and follow them:

(i) This Question Paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into **FIVE** Sections - Section **A, B, C, D** and **E**.

(iii) In **Section A** - Question Nos. **1** to **18** are Multiple Choice Questions (MCQs) and Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each.

(iv) In **Section B** - Question Nos. **21** to **25** are Very Short Answer (VSA) type questions of 2 marks each.

(v) In **Section C** - Question Nos. **26** to **31** are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In **Section D** - Question Nos. **32** to **35** are Long Answer (LA) type questions carrying 5 marks each.

(vii) In **Section E** - Question Nos. **36** to **38** are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section **C**, **2** questions in Section **D** and **2** questions in Section **E**.

(ix) Use of calculators is **NOT** allowed.

SECTION - A

(Multiple Choice Questions)

Each question carries 1 mark.

1. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a}\times\vec{b}| = 3\sqrt{3}$, then the value of $\vec{a}.\vec{b}$ is 1.

(A) 9 (B) 3 (C) $\frac{1}{9}$ (D) $\frac{1}{3}$

Sol. $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$3\sqrt{3} = |\vec{a}||\vec{b}|\sin\frac{\pi}{3}$

$3\sqrt{3} = |\vec{a}||\vec{b}|\times\frac{\sqrt{3}}{2}$

$|\vec{a}||\vec{b}| = 6$

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= 6\times\cos\frac{\pi}{3}$

$= 6\times\frac{1}{2} = 3$ Ans.

2. The position vectors of three consecutive vertices of a parallelogram ABCD are $(4\hat{i}+2\hat{j}-6\hat{k})$, B $(5\hat{i}-3\hat{j}+\hat{k})$ and C $(12\hat{i}+4\hat{j}+5\hat{k})$. The position vector of D is given by : **1**

(A) $-3\hat{i}-5\hat{j}-10\hat{k}$ (B) $21\hat{i}+3\hat{j}$ (C) $11\hat{i}+9\hat{j}-2\hat{k}$ (D) $-11\hat{i}-9\hat{j}+2\hat{k}$

Sol. Let position vector D $= x\hat{i}+y\hat{j}+z\hat{k}$ of $\overrightarrow{AC}$

m is mid point

$m = \frac{16\hat{i}+6\hat{j}-\hat{k}}{2}$

$m = 8\hat{i}+3\hat{j}-\frac{1}{2}\hat{k}$

m is mid point of $\vec{BD}$

$8\hat{i}+3\hat{j}-\frac{1}{2}\hat{k} = \frac{(x+5)\hat{i}+(y-3)\hat{j}+(z+1)\hat{k}}{2}$

$16\hat{i}+6\hat{j}-\hat{k}$

$x+5 = 16 \Rightarrow x = 11$

$y-3 = 6 \Rightarrow y = 9$

$z+1 = -1 \Rightarrow z = -2$

$\therefore$ D vector $\Rightarrow 11\hat{i}+9\hat{j}-2\hat{k}$

3. If for two events A and B , P (A – B) = $\frac{1}{5}$ and P (A) = $\frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to **1**

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{2}{3}$

Sol. $P(A-B) = \frac{1}{5}$; $P(A) = \frac{3}{5}$

$\therefore P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$\therefore P(A-B) = P(A) - P(A \cap B)$

$\frac{1}{5} = \frac{3}{5} - P(A \cap B)$

$p(A \cap B) = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

$\therefore P\left(\frac{B}{A}\right) = \frac{2/5}{3/5} = \frac{2}{3}$ here , $P(B \cap A) = P(A \cap B)$

4. If $\int_0^{2\pi} \cos^2 x\,dx = k\int_0^{\pi/2} \cos^2 x\,dx$, then the value of k is **1**

(A) 4 (B) 2 (C) 1 (D) 0

Sol. $\int_0^{2a} f(x) = 2\int_0^{a} f(x)dx$ if $f(2a-x) = f(x)$

$= 0$ if $f(2a-x) = -f(x)$

Property $\int_0^a f(x)\,dx = \int_0^a f(a-x)dx$

LHS $\Rightarrow 2\int_0^{\pi} \cos^2 x\,dx$

$\Rightarrow 2 \times 2\int_0^{\pi/2} \cos^2 x\,dx$

$= 4$

$\therefore k = 4$

5. If (a,b),(c,d), and (e,f) are the vertices of ΔABC and Δ denotes the area of Δ ABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to **1**

(A) $2\Delta^2$ (B) $4\Delta^2$ (C) 2Δ (D) 4Δ

Sol vertices of Δ ABC are

(a,b) (c,d) & (e,f)

$$\Delta \text{ ABC} = \frac{1}{2}\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}$$

$$2\Delta = \begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}$$

value $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2 = (2\Delta)^2$

$= 4\Delta^2$

6. The function $f(x) = x|x|$ is **1**

(A) continuous and differentiable at x = 0

(B) continuous but not differentiable at x= 0

(C) differentiable but not continuous at x =0

(D) neither differentiable nor continuous at x= 0

Sol. $f(x) = x|x|$ is

$f(x) = x|x|$ at point x = 0

LHL = RHL = f (0) = 0

and at point x =0

differentiability Hence,

LHD = RHD = 0

$\therefore$ f(x) is continious and differentiable at x = 0.

7. If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to **1**

(A) $\frac{-y}{x}$ (B) $\frac{y}{x}$ (C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$

Sol. $\tan\left(\frac{x+y}{x-y}\right) = k$

Differentiating both sides : (using chain rule)

$$\Rightarrow \left[\sec^2\left(\frac{x+y}{x-y}\right)\left(\frac{(x-y)\left(1+\frac{dy}{dx}\right)-\left(1-\frac{dy}{dx}\right)(x+y)}{(x-y)^2}\right)\right] = 0$$

so, $(x-y)\left(1+\frac{dy}{dx}\right)-\left(1-\frac{dy}{dx}\right)(x+y) = 0$

$$\frac{dy}{dx}(x-y+x+y)+x-y-x-y = 0$$

$$\frac{dy}{dx} = \frac{2y}{2x} = \frac{y}{x}$$

option (B)

8. The objective function Z = ax + by of an LPP has maximum value 42 at (4,6) and minimum value 19 at (3,2). Which of the following is true ? **1**

(A) a = 9, b = 1 (B) a = 5, b =2 (C) a =3, b =5 (D) a = 5, b =3

Sol. Z = ax + by

max. value = 42 at point (4,6)

Z_{max} = 4a +6b = 42

Z_{min} = 3a + 2b = 19

OptionC , satisfy both equation

$\therefore$ C a = 3, b = 5

9. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3},\frac{4}{3}\right)$.

If Z = 30x + 24y is the objective function, then (maximum value of Z - minimum value of Z) is equal to **1**

(A) 40 (B) 96 (C) 120 (D) 136

Sol. Corner point (0,4) (8 ,0) and $\left(\frac{20}{3},\frac{4}{3}\right)$

Z =30x +24y

For (0,4) : Z = 96

For (8,0): Z = 240

For $\left(\frac{20}{3},\frac{4}{3}\right)$:Z = 240

$\therefore$ $Z_{max} - Z_{min}$ = 240 – 96

= 144

Option not here

10. Number of symmetric matrices of order 3×3 with each entry 1 or –1 is **1**

(A) 512 (B) 64 (C) 8 (D) 4

Sol. No. symmetric matrices of order 3×3 with each entry 1 or $-1 \Rightarrow 2^6 = 64$

11. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ = P+Q, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to **1**

(A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

Sol. $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ = P + Q

Where Q is a skew symmetric matrix

We know $Q = \frac{1}{2}(A - A^T)$, where $A = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$

$$Q = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$$

option (B)

12. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is **1**

(A) R (B) {0} (C) {4} (D) R–{4}

Sol. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & a & 1 \end{bmatrix}$

non singular matrix

$|A| \neq 0$

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{vmatrix} \neq 0$$

$(3-a)-2(2-3)+1(2a-9) \neq 0$

$3-a+2+2a-9 \neq 0$

$a-4 \neq 0$

$a \neq 4$

$\therefore$ option (D) R–{4}

13. If IA I = IkA I, where A is a square matrix of order 2, then sum of all possible values of k is **1**

(A) 1 (B) –1 (C) 2 (D) 0

Sol. $|A| = |KA|$

$|A| = k^2 |A|$

$K = \pm 1$

$\therefore$ Sun of all value of k $\Rightarrow + 1 - 1 = 0$

option (D)

14. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to **1**

(A) a + b (B) $\frac{ax^2}{2} + bx$ (C) $\frac{ax^2}{2} + bx + c$ (D) b

Sol. If $\frac{d}{dx} f(x) = ax + b$

$f(0) = 0$

$= f(x) = \int (ax + b)dx$

$= f(x) = \frac{ax^2}{2} + bx + c$

at point x = 0

$f(0) = c$

$c = 0$

$f(x) = \frac{ax^2}{2} + bx$

option (B)

15. Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is **1**

(A) 2 (B)1 (C) not defind (D) 0

Sol. $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$

Degree not defined

Because it is not polynomial in derivative

Option(c)

16. The integrating factor of the differential equation **1**

$(1- y^2)\ \frac{dx}{dy} + y\,x = ay,\ (-1 < y < 1)$ is

(A) $\frac{1}{y^2-1}$ (B) $\frac{1}{\sqrt{y^2-1}}$ (C) $\frac{1}{1-y^2}$ (D) $\frac{1}{\sqrt{1-y^2}}$

Sol. $\frac{dx}{dy}+\frac{xy}{1-y^2}=\frac{ay}{1-y^2}$

$\frac{dx}{dy}+px=Q$

I.F.= $e^{\int p\,dy}$

$e^{\int \frac{y}{1-y^2}dy}$

$e^{\int \frac{y}{1-y^2}dy} \Rightarrow e^{-\frac{1}{2}\int \frac{2y}{y^2-1}dy}$

$\Rightarrow e^{-\frac{1}{2}\log(y^2-1)}$

$\frac{1}{\sqrt{y^2-1}}$

17. Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, -1) and (4, 4, -7), is **1**

(A) $2\hat{i}+3\hat{j}-6\hat{k}$ (B) $-2\hat{i}-3\hat{j}+6\hat{k}$

(C) $\frac{-2\hat{i}}{7}-\frac{3\hat{j}}{7}+\frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7}+\frac{3\hat{j}}{7}-\frac{6\hat{k}}{7}$

Sol. $\overrightarrow{PQ}=2\hat{i}-3\hat{j}-6\hat{k}$

$\therefore$ Unit vector along $\overrightarrow{PQ}$

$\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}=\frac{2\hat{i}-3\hat{j}-6\hat{k}}{\sqrt{4+9+36}}$

$\Rightarrow \overrightarrow{PQ}=\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}$

option (D)

18. Equation of a line passing through point (1, 2, 3) and equally inclined to the coordinate axis, is **1**

(A) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (B) $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

(C) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ (D) $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$

Sol. Equation of line passing through point (1, 2, 3)
Equally inclined to co-ordinate

$$\frac{x-1}{1/\sqrt{3}}=\frac{y-2}{1/\sqrt{3}}=\frac{z-3.}{1/\sqrt{3}}$$

$$\Rightarrow \frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$$

option(D)

ASSERTION-REASON BASED QUESTIONS

In the following questions 19 & 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:
(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

19. **Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2.
Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ **1**

Sol. Maximum value of
$(\cos^{-1}x)^2$ is π^2
$\cos^{-1}x$ range $[0, \pi]$
$\Rightarrow$ Not $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$
option (C) correct
(A) is true but (R) is false.

20. **Assertion (A):** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.
Reason (R) : The sum of squares of the direction cosines of a line is 1. **1**

Sol $\sqrt{l^2+m^2+n^2}=1$
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma$
$\therefore \sin^2\alpha + \sin^2\beta + \sin^2\gamma$
Asseration is true
Reason (R) is also true
$l^2+m^2+n^2 = 1$
$\therefore$ option (A) both (A) and (R) are the and (R) is true correct explation of A.
Option (A) is correct

SECTION - B

This section comprises of Very Short Answer Type (VSA) questions, each of 2 marks.

21. If points A, B and C have position vectors $2\hat{i}$, $\hat{j}$ and $2\hat{k}$ respectively, then show that Δ ABC is an isosceles triangle. **2**

Sol. $\overrightarrow{OA} = 2\hat{i}$

$\overrightarrow{OB} = \hat{j}$

$\overrightarrow{OC} = 2\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \hat{j} - 2\hat{i} \Rightarrow \quad |\overrightarrow{AB}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

$\overrightarrow{BC} = \quad \overrightarrow{OC} - \overrightarrow{OB}$

$= 2\hat{k} - \hat{j} \qquad \Rightarrow |\overrightarrow{BC}| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$|\overrightarrow{AB}| = |\overrightarrow{BC}|$

$\Rightarrow \Delta ABC$ is isosceles triangle.

22. (a) Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$. **2**

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.

Sol. $\sin^{-1}(\sin\frac{3\pi}{4}) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$

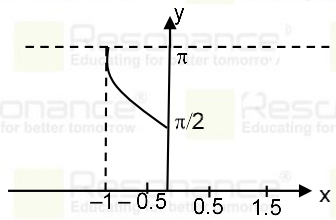
$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{4}\right)\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$

$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \cos^{-1}(-1) + \tan^{-1}(1)$

$= \frac{\pi}{4} + \pi + \frac{\pi}{4} = \frac{3\pi}{2}$

OR

(b) Graph of $\cos^{-1} x$



Range of $\cos^{-1}$ in $[-1,0]$ is $[\pi/2, \pi]$.

23. If equal sides of an isosceles triangle with fixed base 10 cm are increasing at the rate of 4 cm/sec, how fast is the area of triangle increasing at an instant when all sides become equal? **2**

Sol. Let two equal to sides of ΔABC be

$AB = AC = x$ cm

$S = \frac{a+b+c}{2} = \frac{2x+10}{2} = (x + 5)$ cm

Area of $\Delta \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{(x+5)(x+5-10)\times 5\times 5}$

$A = 5\sqrt{x^2-25}$ cm²

$\frac{dA}{dt} = 5\times1(x^2\text{-}25)^{-1/2} \times \frac{dx}{dt}$ [Using chain rule]

Now $\frac{dx}{dt} = 4$ cm/s and $x = 10$

$\therefore$ (i) gives

$\frac{dA}{dt} = 5 \times (10^2\text{–}25)^{1/2}\ 10 \times 4$ cm²/sec

$= 5\sqrt{75}\times10\times4$ cm²/sec

$= 25 \times10 \times 4\ \sqrt{3}$ cm²/sec

$= 1000\sqrt{3}$ cm²/sec

1732.05 cm² /sec

$(\sqrt{3} \cong 1.73205)$

24. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin. **2**

Sol. Equation of given line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$$

General point on the above line is

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2} = \lambda$$

Point P : $(\lambda, 2\lambda+1, 2\lambda\text{–}1)$

Distance of point P from the origin (0,0,0) is $\sqrt{11}$

OP = $\sqrt{11}$ Units {given}

From distance formula

$$\sqrt{(\lambda-0)^2+(2\lambda+1-0)^2+(2\lambda-1-0)^2} = \sqrt{11}$$

$\therefore$ squaring on both side

$\lambda^2 + (2\lambda+1)^2+(2\lambda\text{–}1)^2= 11$

$\lambda^2 + 4\lambda^2 +4\lambda+1+4\lambda^2\text{–}4\lambda+1=11$

$9\lambda^2 + 2 = 11$

$9\lambda^2 = 9$

$\lambda^2 = 1$

$\lambda = \pm1$

$\therefore$ The co-ordinates of points are (–1, –1, –3) and (1, 3, 1)

25. (a) If $y = \sqrt{ax+b}$ then prove that $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2 = 0$ **2**

OR

(b) $f(x) = \begin{cases} ax+b; & 0 < x \le 1 \\ 2x^2 - x; & 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b

Sol. (a) $y = \sqrt{ax+b}$

squaring both sides

$y^2 = ax + b$

Differentation both sides

$2y\frac{dy}{dx} = a$

$\Rightarrow y\frac{dy}{dx} = \frac{a}{2}$

Differentation both sides

$\Rightarrow y\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dy}{dx} = 0 \quad \left[\because \frac{d(uv)}{dx} = \frac{udv}{dx} + \frac{vdu}{dx}\right]$

$\Rightarrow y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2 = 0$

OR

(b) $f(x) = \begin{Bmatrix} ax+b: & 0 < x \le 1 \\ 2x^2 - x: & 1 < x < 2 \end{Bmatrix}$ is differentiable is (0,2)

$\Rightarrow$ f(x) differential at x = 1

$\Rightarrow$ f(x) is continuous at x = 1

(Every differentiable function is continuous)

$\therefore$ LHL at x = 1 = RHL at x= 1

$\lim\limits_{h\to 0} a(1-h)+b = \lim\limits_{h\to 0} 2(1+h)^2 - (1+h)$

$\Rightarrow a + b = 1 \quad$ (i)

Again LHD = RHD at x = 1

$a = 2 \times 2x - 1$

$a = 2 \times 2 - 1 = 3$ and $b = -2$

SECTION - C

The section coprises Short Answer (SA) type question of 3 marks each.

26. Solve the following Linear programming problem graphically: **3**

Maximize : $Z = 3x + 3.5y$

Subject to constraints: $x + 2y \geq 240$.

$3x + 1.5y \geq 270$,

$1.5x + 2y \leq 310$,

$x \geq 0, y \geq 0$.

Sol. Subjects to constraints in the problem

$x + 2y \geq 240$ $3x + 1.5 \geq 270$

$1.5x + 2y \leq 310$ $x, y \geq 0$

(i) Group of $x + 2y \geq 240$

Line $x + 2y = 240$

x	240	0
y	0	120

A (240, 0)

B(0,120)

Put x = 0 and y = 0 which is not true $0 + 0 \geq 240$ false

(ii) Graph $3x + 1.5y \geq 270$

Line

$3x + 1.5y = 270$

x	90	0
y	0	180

C (90, 0)

D (0, 180)

Put x = 0 and y = 0

$0 + 0 \geq 270$ which is not true lines on CD or about it

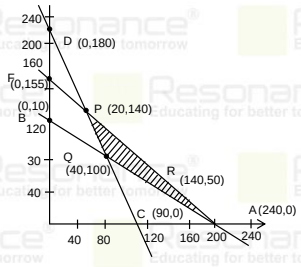
(iii) Graph $1.5x + 2y \leq 310$ line $1.5x + 2y = 310$

Passes through Point E $\left(206\frac{2}{3}, 0\right)$ and F(0,155)

Put x = 0 & y = 0

0 + 0 < 310

which is true is true so region is below line.



(iv) $x \geq 0$ lies on y axis and right side of t.

(v) The points of region j ≥ 0 lies on x-axis and about it.

Intersection of AB & EF is Q (40, 100) intersection of AB& EF R R(140, 50)

Intersection of CD and EF P (20, 140)

feasible region is APQR

at pt p (20, 140)

$Z = 3 \times 20 + 3.5 \times 140$

$= 60 + 490 = 550$

at pt. Q (40, 100)

$Z = 3 \times 40 + 3.5 \times 100$

$120 + 350 = 470$

at pt R (140, 50)

$Z = 3 \times 140 + 3.5 \times 50$

$Z = 420 + 175 = 595$

Z is maximize at x = 140 and y = 50 and max.value is 595

27. (a) Find $\int \frac{x+2}{\sqrt{x^2-4x-5}}\,dx$. **3**

OR

(b) Evaluate $\int_{-a}^{a} f(x)\,dx$, where $f(x)= \frac{9^x}{1+9^x}$ **3**

Sol. **(a)** $\int \frac{x+2}{x^2-4x-5}dx = \int \frac{x+2}{x^2-5x+x-5}dx$

$\int \frac{x+2}{(x-5)(x+1)}dx$ $x^2-4x-5 = t$

$(2x-4)dx = dt$

$2(x-2)dx = dt$

$\int \frac{x-2+4}{\sqrt{x^2-4x-5}}dx$

$\int \frac{(x-2)dx}{\sqrt{x^2-4x-5}}dx + 4\int \frac{1}{x^2-4x-5+4-4}$

$\int \frac{dt/2}{\sqrt{t}} + 4\int \frac{dx}{(x-2)^2-3^2}$

$\frac{1}{2}\int \frac{dt}{\sqrt{t}} + 4\int \frac{dx}{(x-2)^2 3^2}$

$\frac{1}{2}\frac{t^{-1/2+1}}{-1/2+1} + 4\log|(x-2)\sqrt{x^2-4x-5}| + c$

$\sqrt{x^2-4x-5} + 4\log|(x-2)\sqrt{x^2-4x-5}| + c$

$\sqrt{x^2-4x-5} + 4\log|(x-2)\sqrt{x^2-4x-5}| + c$

(b) $\int_{-a}^{a} \frac{9^x}{1+9^x} dx$ .....(i)

$I = \int_a^b f(n)dx = \int_a^b (a+b-x)dx$

$I = \int_{-a}^{a} \frac{1}{9^x+1} dx$ ..... (ii)

Equation (i) + equation (ii)

$2I = \int_{-a}^{a} \frac{1+9^x}{1+9^x} dx$

$2I = \int_{-a}^{a} 1dx$

$2I = [x]_{-a}^{a}$

$2I = [a-(-a)]$

$2I = 2a$

$I = a$

28. Find $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$ **3**

Sol. $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$

$\cot^{-1} x = t$

$\frac{-1}{1+x^2}dx = dt$

$\frac{1dx}{1+x^2} = -dt$

$x = \cot(t)$

$= -\int e^t\left(1-\cot(t)+\cot^2 t\right)dt$

$= -\int e^t\left(\operatorname{cosec}^2 t - \cot t\right)dt$

$= \int e^t\left(\cot t - \operatorname{cosec}^2 t\right)dt$

$= \int \left\{e^x\left(f(x)+f'(x)\right)\right\}dx$

$= e^x f(x) + c$

$\Rightarrow \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

29. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})}dx$. **3**

Sol. $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{\left(e^x+e^{-x}\right)\left(e^x-e^{-x}\right)}dx$

$\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{e^{2x}}{e^{4x}-1}dx$

$\frac{1}{2}\int_2^3 \frac{dt}{(t+1)(t-1)}$

$\frac{1}{4}\left[\int \frac{(t+1)-(t-1)}{(t+1)(t-1)}dt\right]$ let $e^{2x} = 1$

$2e^{2x}\,dx = dt$

$e^{2x}\,dx = \frac{dt}{2}$

$\frac{1}{4}\int \frac{1}{(t-1)}dt - \frac{1}{4}\int \frac{1}{(t+1)}dt$

$\frac{1}{4}\left[\log\frac{(t-1)}{(t+1)}\right]_2^3$

$\frac{1}{4}\left[\log\frac{(3-1)}{(3+1)} - \log\left(\frac{2-1}{2+1}\right)\right]$

$\frac{1}{4}\left[\log\left(\frac{1}{2}\right) - \log\left(\frac{1}{3}\right)\right]$

$\frac{1}{4}\left[\log\frac{3}{2}\right]$ **Ans.**

30. (a) Find the general solution of the differential equation: **3**

$(xy - x^2)\, dy = y^2\, dx$

OR

(b) Find the general solution of the differential equation: **3**

$(x^2+1)\,\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$

Sol. (a) $(xy - x^2)dy = y^2\, dx$

$$v + x\frac{dv}{dx} = \frac{v^2x^2}{\left(x(vx) - x^2\right)} = \frac{v^2x^2}{x^2(v-1)}$$

$$v + x\frac{dv}{dx} = \frac{v^2}{v-1}$$

$$x\frac{dv}{dx} = \frac{v^2}{v-1} - v$$

$$x\frac{dv}{dx} = \frac{v^2 - v^2 + v}{v-1}$$

$$x\frac{dv}{dx} = \frac{v}{v-1}$$

$$\int \frac{v-1}{v} dv \int \frac{dx}{x}$$

$$\int \left(1 - \frac{1}{v}\right) dv = \log x + c$$

$$v - \log v = \log x + c$$

$$\frac{y}{x} - \log_e \frac{y}{x} = \log_e^x + c$$

$$\frac{y}{x} = \log_e^x = \log_e \frac{y}{x} + c$$

$$y = x \log cy$$

(b) $(x^2+1)\,\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$

divided by $(x^2 + 1)$

$$\frac{dy}{dx} + \frac{2xy}{x^2+1} = \frac{\sqrt{x^2+4}}{x^2+1}$$

this differential equation a type of

$$\frac{dy}{dx} + py = Q$$

Here, $P = \frac{2x}{x^2+1}$, $Q = \frac{\sqrt{x^2+4}}{x^2+1}$

$e^{\int P dx} \Rightarrow e^{\int \frac{2x}{x^2+1} dx}$

$\Rightarrow e^{\log_e^t}$

Here let $x^2 + 1 = t$

$2x\,dx = dt$

$\Rightarrow$ IF $\Rightarrow (x^2 + 1)$

$y\ e^{\int pdx} = \int Q(x) \cdot (IF) + C$

$y \cdot (IF) = \int Q(x) \cdot (IF) + C$

$y(x^2+1) = \int \sqrt{x^2 + 4} + C$

$y(x^2+1) = \int \sqrt{x^2 + 2^2} dx + C$

$y(x^2+1) = \frac{x}{2}\sqrt{x^2 + 4} + 2\log\left|x + \sqrt{4 + x^2}\right| + C$

31. (a) Two numbers are selected from first six even natural numbers at random without replacement. If X denotes the greater of two numbers selected, find the probability distribution of X. **3**

OR

(b) A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B be the event, "3 comes on the die". Find whether A and B are independent events or not. **3**

Sol. **(a)**

S = {2, 4,6, 8, 10, 12} 2 number are selected

X = greater number in (2 numbers selected)

X = {4, 6, 8, 10, 12}

$X \neq 2$

So, $P(X = 4) = \frac{1}{6_{C_2}} = \frac{1}{15}$

$P(X = 6) = \frac{2}{6_{C_2}} = \frac{2}{15}$

$P(X = 8) = \frac{3}{6_{C_2}} = \frac{3}{15} = \frac{1}{5}$

$P(X = 10) = \frac{4}{6_{C_2}} = \frac{4}{15}$

$P(X = 12) = \frac{5}{6_{C_2}} = \frac{5}{15} = \frac{1}{3}$

(b) A fair coin & aw unbiased die are tossed

sample space = $\begin{Bmatrix}(H,1) & (H,2) & (H,3) & (H,4) & (H,5) & (H,6)\\ (T,1) & (T,2) & (T,3) & (T,4) & (T,5) & (T,6)\end{Bmatrix}$

event A : Head appears on the coin

$\{(H,1)\ (H,2)\ (H,3)\ (H,4)\ (H,5)\ (H,6)\}$

event B: 3 comes on the die

$\{(H,3)\ \ (T,1)\}$

Now, P(A) = $\frac{6}{12}=\frac{1}{2}$ P(B) = $\frac{2}{12}=\frac{1}{6}$

Now, $(A\cap B)=\{H,3\}$

P $(A\cap B)$ = $\frac{1}{12}$

If 2 events are independent than P $(A\cap B)$ P(A).P(B)

$\Rightarrow \frac{1}{12}=\frac{1}{2}\times\frac{1}{6} \Rightarrow \frac{1}{12}=\frac{1}{12}$ Hence proved

SECTION – D

This section comprises Long Answer type(LA) question of 5 marks each.

32. A function f:[-4, 4] $\rightarrow$ [0, 4] is given by f(x)= $\sqrt{16-x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which f(a)= $\sqrt{7}$ **5**

Sol. Given, Domain: [-4, 4]

Range [0, 4]

function: f(x)= $\sqrt{16-x^2}$

To prove f(x) is onto function, we show that range of f(x) is same as its codomain.

let, f(x)=y

so, y = $\sqrt{16-x^2}$

$y^2 = 16 - x^2$ ($\because$ squaring both sides)

$x^2 = 16 - y^2$

$x = \pm\sqrt{16-y^2}$

Since, $x\in[-4,4]$ for all $y\in[0,4]$, so range of f is given as [0, 4]

Also codomain of f=[0, 4]

Hence f is onto function

To prove one-one

$f(x_1)=f(x_2)$ $y\in[0,4] \Rightarrow \sqrt{16-x_1^2}=\sqrt{16-x_2^2}$

$\Rightarrow 16-x_1^2 = 16-x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

$\Rightarrow f(x_1) = f(x_2) \quad \Rightarrow x_1 = x_2$

so, f is not one-one

Now, given, $f(a) = \sqrt{7}$ …(i)

also, $f(a) = \sqrt{16 - a^2}$ …(ii)

from (i) + (ii)

$\sqrt{16 - a^2} = \sqrt{7}$

$\Rightarrow 16 - a^2 = 7$

$\Rightarrow a^2 = 16 - 7$

$\Rightarrow a^2 = 9$

$\Rightarrow a = \pm\sqrt{9}$

$\Rightarrow a = \pm 3$

so, a=3, –3

33. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations:

$X - 2y = 3$

$2x - y - z = 2$

$-2y + z = 3$ **5**

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$ **5**

Sol. **(a)** $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

$$A \times B = \begin{bmatrix} -3\times1 - 2\times(-2) - 4\times0 & -3\times2 - 2(-1) - 4(-1) & -3\times0 - 2\times(-2) - 4(1) \\ 2(1) + 1(-2) + 2(0) & 2(2) + 1(-1) + 2(-1) & 2(0) + 1(-2) + 2(1) \\ 2(1) + 1(-2) + 3(0) & 2(2) + 1(-1) + 3(-1) & 2(0) + 1(-2) + 3(1) \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A \times B = I$$

So $\Rightarrow \quad B = A^{-1} \quad \& \quad A = B^{-1}$

Now writing the system of equations in matrix form as;

$$\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 2\\ 3\end{bmatrix}$$

Here, we can see that $\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}=B^T$

So, $\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\left(B^T\right)^{-1}\begin{bmatrix}3\\ 2\\ 3\end{bmatrix}\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=A^T\begin{bmatrix}3\\ 2\\ 3\end{bmatrix}\qquad\left[\because\left(B^T\right)^{-1}=A^T\right]$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}\begin{bmatrix}3\\ 2\\ 3\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-3(3)+2(2)+2(3)\\ -2(3)+1(2)+1(3)\\ -4(3)+2(2)+3(3)\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ -1\\ 1\end{bmatrix}$$

So, x=1, y= –1, z=1.

(b) $f(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$ $\qquad f(-\beta)=\begin{bmatrix}\cos(-\beta) & -\sin(-\beta) & 0\\ \sin(-\beta) & \cos(-\beta) & 0\\ 0 & 0 & 1\end{bmatrix}$

As, $\cos(-x)=\cos x$ & $\sin(-x)=-\sin x$ so,

$$f(-\beta)=\begin{bmatrix}\cos\beta & \sin\beta & 0\\ -\sin\beta & \cos\beta & 0\\ 0 & 0 & 1\end{bmatrix}$$

$f(\alpha)\times f(-\beta)=$

$$\begin{bmatrix}\cos\alpha.\cos\beta-\sin\alpha(-\sin\alpha)+0+0 & \cos\alpha.\sin\beta-\sin\alpha.\cos\beta+0 & \cos\alpha\times0-\sin\alpha\times0+0\times1\\ \sin\alpha.\cos\beta+\cos\alpha(-\sin\beta)+0\times0 & \sin\alpha.\sin\beta+\cos\alpha.\cos\beta+0 & \sin\alpha\times0+\cos\alpha\times0+0\times1\\ 0.\cos\beta+0\times(-\sin\beta)+1\times0 & 0.\sin\beta+0.\cos\beta+1\times0 & 0\times0+0\times0+1\times1\end{bmatrix}$$

$$f(\alpha)\times f(-\beta)=\begin{bmatrix}\cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0\\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0\\ 0 & 0 & 1\end{bmatrix}\qquad\left[\because\cos\alpha.\cos\beta+\sin\alpha.\sin\beta=\cos(\alpha-\beta)\right]$$

$$\left[\because\sin\alpha.\cos\beta-\cos\alpha.\sin\beta=\sin(\alpha-\beta)\right]$$

$f(\alpha)\times f(-\beta)=f(\alpha-\beta)$: proved

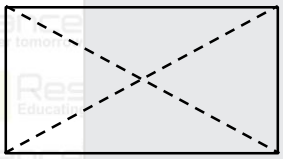
34. (a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, -6), Q (5, -3, 1), R(12, 4, 5) and S(11, 9, -2). Use these equations to find the point of intersection of diagonals. **5**

OR

(b) A line ℓ passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. Find the vector equation of the line ℓ. Hence, obtain its distance from origin. **5**

Sol. (a) The eqⁿ of diagonal PR will be

P(4,2–6) Q(5,–3,1)



S(11,9,–2) R(12,4,5)

$$\frac{(x-4)}{12-4}=\frac{(y-2)}{4-2}=\frac{(z-(-6))}{5-(-6)}$$

$$\Rightarrow \frac{x-4}{8}=\frac{y-2}{2}=\frac{z+6}{11} \quad .....(i)$$

The equation of diagonal QS will be

$$\frac{x-5}{11-5}=\frac{y-(-3)}{9-(-3)}=\frac{z-1}{-2-1}$$

$$\Rightarrow \frac{x-5}{6}=\frac{y+3}{12}=\frac{z-1}{-3} \quad .....(ii)$$

From equation (i)

$$\frac{x-4}{8}=\frac{y-2}{2}=\frac{z+6}{11}\ \lambda \Rightarrow x = 8\lambda + y,\ y = 2\lambda + 2,\ z = 11\lambda - 6$$

Any point the above line will be

$(x,y,z) = (8\lambda + y, 2\lambda +2, 11\lambda – 6)$..... (iii)

For point of intersection, put this point in

$$\frac{8\lambda+y-5}{6}=\frac{2\lambda+2+3}{12}=\frac{11-6-1}{-3}$$

$$\Rightarrow \frac{8\lambda-1}{6}=\frac{2\lambda+5}{12}=\frac{11\lambda-7}{-3}$$

From first two factors,

$12(8\lambda–1) = 6(2\lambda+5)$

$\Rightarrow 96\lambda –12 = 12\lambda +30 \Rightarrow \lambda = \frac{1}{2}$

(b) Let L_1; $\frac{n}{1}=\frac{y}{2}=\frac{2}{3}$

L_2 : $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

Given that line 'l' is $\perp$ an to L_1 and L_2

equation of line 'l' passing through (–1,3,–2) + $\left(a\hat{1}+b\hat{j}+c\hat{k}\right)$

Now, if line 'l' is $\perp$ an to L_1 , then we get the dot product of their vector of zero.

a + 26 +3c.....(1)

similarly, if line 'l' js $\perp$ an to L_2

–3a +2b +5c = 0.....(2)

Now solving (1) & (2) by cross multiplication

$\frac{a}{10-6}=\frac{b}{-9-5}=\frac{b}{2+6}=t(say)$

a = 4t, b = –14t , c = 8t

So, if we take the direction ratios of line L_1 we get

(4, –14,8)

So, we get equation of line 'l'

l : $\left(-\hat{i}+3\hat{j}-2\hat{k}\right)+\lambda\left(4\hat{i}-14\hat{j}+8\hat{k}\right)$

cartesian equation:-

$\frac{x+1}{4}=\frac{y-3}{-14}=\frac{z+2}{8}=\lambda(say)$

$\therefore$general point

(4λ–1), –14λ+3, 8λ –2)

directions 's OP =(4λ–1, –14λ + 3, 8λ –2)

Now, OP $\perp$ AB

$\Rightarrow A_1A_2 + B_1+B_2 +C_1C_2 = 0$ OA

$\Rightarrow$ 4(4λ –1) + (–14) (–14λ +3)8 (8λ –2) = 0

= 16λ –4 + 196λ □42 +64λ –16 = 0

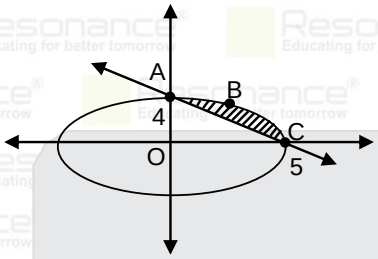
276λ –62 = 0

$\lambda = \frac{62}{276}$

$P = \frac{-7}{69}$

35. Find the area of the smaller region bounded by the curves $\frac{x^2}{25}+\frac{y^2}{16}=1$ and $\frac{x}{5}+\frac{y}{4}=1$, using integration. **5**

Sol. The area of smaller region bounded by ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and $\frac{x}{5}+\frac{y}{4}=1$ is represented by ABCA.



$\therefore$ Area (ABCA) = Area (ABCO) – Area (ACO)

$$= \int_0^5 4\sqrt{1-\frac{x^2}{25}}\,dx - \int_0^5 4\left(1-\frac{x}{5}\right)dx$$

$$= \frac{4}{5}\int_0^5 \sqrt{25-x^2}\,dx - \frac{4}{5}\int_0^5 (5-x)dx$$

$$= \frac{4}{5}\left[\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5 - \frac{4}{5}\left[5x-\frac{x^2}{2}\right]_0^5$$

$$= \frac{4}{5}\left[\frac{5}{2}\sqrt{25-25}+\frac{25}{2}\sin^{-1}\left(\frac{5}{5}\right)-0-\frac{25}{2}\sin^{-1}(0)\right]-\frac{4}{5}\left[5(5)-\frac{(5)^2}{2}-(0-0)\right]$$

$$= \frac{4}{5}\left[0+\frac{25}{2}\left(\frac{\pi}{2}\right)\right]-\frac{4}{5}\left[25-\frac{25}{2}\right]$$

$$= \frac{4}{5}\left[\frac{25\pi}{4}\right]-\frac{4}{5}\left[\frac{25}{2}\right]$$

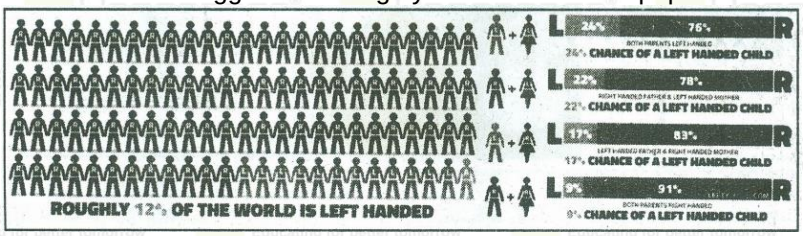
$$= \frac{4}{5}\times\frac{25}{2}\left[\frac{\pi}{2}-1\right]$$

$$= 5(\pi-2) \text{ sq. units}$$

SECTION – E

This section comprises 3 source based case-based/passage based/integrated units of assessment questions of 4 marks each.

6. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A: When both father and mother are left handed:
Chances of left handed child is 24%.

B: When father is right handed and mother is left handed:
Chances of left handed child is 22%.

C: When father is left handed and mother is right handed:
Chances of left handed child is 17%.

D: When both father and mother are right handed:
Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions:

(i) Find P(L/C) **1**

(ii) Find P($\bar{L}$ /A) **1**

(iii) (a) Find P(A/L) **2**

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. **2**

Sol. (i) $P(L/C) = \frac{17}{100}$

(ii) $P\left(\frac{\bar{L}}{A}\right) = 1 - P\left(\frac{L}{A}\right) = 1 - \frac{24}{100} = \frac{76}{100}$

(iii) (a) $P\left(\frac{A}{L}\right) = \frac{P(A)\times P\left(\frac{L}{A}\right)}{P(A)\times P\left(\frac{L}{A}\right) + P(B)\times P\left(\frac{L}{B}\right) + P(C)\times P\left(\frac{L}{C}\right) + P(D)\times P\left(\frac{L}{P}\right)}$

$= \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100} + \frac{1}{4}\times\frac{22}{100} + \frac{1}{4}\times\frac{17}{100} + \frac{1}{4}\times\frac{9}{100}}$

$= \frac{24}{24+22+17+9}$

$= \frac{24}{72} = \frac{1}{3}$

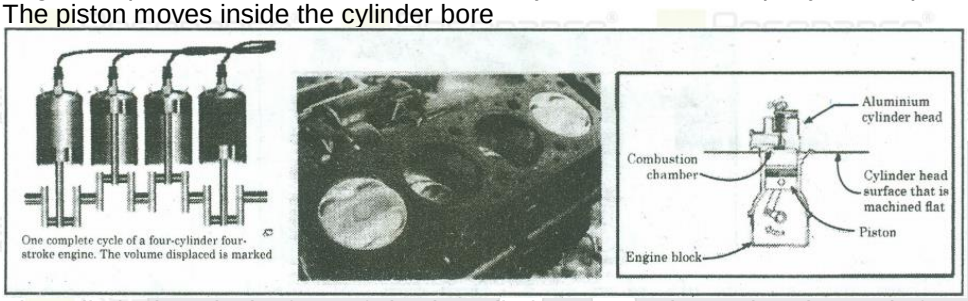
OR

(b) $= P\left(\frac{L}{B}\right) + P\left(\frac{L}{C}\right)$

$= \frac{22}{100} + \frac{17}{100}$

$= \frac{39}{100}$

37. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore



The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions:

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of Radius r. **1**

(ii) Find $\frac{dV}{dr}$. **1**

(iii) (a) Find the radius of cylinder when its volume is maximum. **2**

OR

(b) For maximum volume, h> r. State true or false and justify. **2**

Sol. $2\pi rh + \pi r^2 = 75\pi cm^2$

$2rh + r^2 = 75$

(i) Volume $V = \pi r^2 h$

$V = \pi r^2\left(\frac{75 - r^2}{2r}\right) = \frac{1}{2}\pi r\left(75 - r^2\right)$

(ii) $\frac{dV}{dr} = \frac{d}{dr}\left(\frac{75}{2}\pi r - \frac{1}{2}\pi r^3\right)$

$= \frac{75\pi}{2} - \frac{3}{2}\pi r^2 = \frac{75\pi - 3\pi r^2}{2}$

(ii) for maximum

$\frac{dv}{dr} = 0$

$75\pi = 3\pi r^2$

$r = 5$

$\frac{d^2v}{dr^2} < 0$

At r = 5

Volume is r = 5

38. The use of electric vehicles will curb air pollution in the long run.

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, respectively. Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. **2**

(ii) Prove that the function V(t) is an increasing function. **2**

Sol. (I) $v(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

at t = 0

v (t) < 0

$\therefore$ also < 0

$\therefore$ function cannot be used to estimate no of vehicles in year 2000.

(II) $v(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2$

$v'(t) = \frac{3}{5}t^2 - 5t + 25$

$v'(t) > 0$

D < 0

$\therefore$ v (t) is increasing function.

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BOARD DIVISION (PSPD)

CUET (UG) 2023

Common University Entrance Test

About CUET (UG)

Common University Entrance Test (CUET) is the program that provides equal opportunity to all students from different Boards & different region.

- CUET, known as Common Universities Entrance Test (CUET), is a Computer Based All - India Test for admission to various Undergraduate Programmes in 44 Central Universities and other State Private + Deemed Universities of India.
- CUET (UG) is organized by National Testing Agency (NTA).
- Official Website: <www.samarth.cuet.ac.in> OR <www.cuet.nta.ac.in>

Points to Remember: CUET (UG) 2023

- Candidates can choose any Language/Domain Specific Subjects/General Test or a combination as per the requirements of the course in the specific University.
- The choice of Tests/Subjects depend on the course/s chosen by the candidate and the University/ies where admission is sought.
- A Candidate can take a maximum of **10 tests**.

S.No.	SECTION	NO. OF QUESTIONS	QUESTIONS TO ATTEMPT	DURATION
1.	SECTION-I (A+B)	50	40	45 Minutes
2.	SECTION-II	50/45	40/35	45 Minutes*
3.	SECTION-III	60	50	45 Minutes*

*Not yet announced by NTA.

- **Section IA – 13 Languages (As a medium and "Language")**
  Assamese | Bengali | English | Gujarati | Hindi | Kannada | Malayalam | Marathi | Odia | Punjabi | Tamil | Telugu | Urdu
- **Section IB – 20 Languages**
  Arabic | Bodo | Chinese | Dogri | French | German | Persian | Russian | Sindhi | Tibetan | Italian | Japanese | Kashmiri | Konkani | Maithili | Manipuri | Nepali | Santhali | Spanish | Sanskrit
- **Section II – 27 Domain-Specific Subjects**
  There are 27 Domains specific Subjects being offered under this Section. Candidate may choose a maximum of Six (06) Domains as desired by the applicable University/Universities.
- **Section III – General Test**
  General Knowledge, Current Affairs, General Mental Ability, Numerical Ability, Quantitative Reasoning (Simple application of basic mathematical concepts arithmetic/algebra geometry/mensuration/stat taught till Grade 8).

- Candidates, from any Stream (Arts / Commerce / Science), who are appearing in Class12th Examination in 2022-23 OR who have Passed the class 12th or equivalent examination, irrespective of their age can appear in the CUET (UG)–2023.
- Students of Science stream can explore some unique courses of B. Tech/ M. Tech / Bio-Tech courses through CUET exam at some renowned universities of India like DU / BHU etc.
- Candidates have to fulfil the age criteria if it is specified by a Particular University to which the candidate wishes to apply.

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  - Commerce & Law: Accounts, Business Studies, Business Economics, Law
  - Huminities: History, Political Science, Economics, Geography/Geology, Sociology*
  - Other Subjects: Physical Education, Computer Application*, Teaching Aptitude*, Psychology*, Sanskrit*
- **General Test** — General Test

* Availability of these subjects depends on number of students enrolled.

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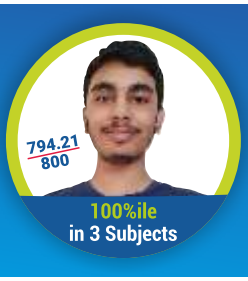

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