SET-3

Series DPQRS

Code No. 430/4/3

Roll No. | | | | | | | | |

Candidates must write the Q.P. Code on the title page of the answer-book.

- Please check that this question paper contains **10** printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- **Please write down the Serial Number of the questions in the answer-book beforeattempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS (BASIC) THEORY
HINTS & SOLUTIONS

Time allowed: 2 hours **Maximum Marks: 40**

General Instructions:

(i) This Question paper contains **14** questions. **All** Questions are compulsory.
(ii) This Question paper is is divided into 3 Sections- **A, B** and **C**
(iii) **Section-A** comprises of **6** questions (Q Nos. **1 to 6**) of **2** marks each. Internal choice has been provided in two questions.
(iv) **Section-B** comprises of **4** questions (Q Nos. **7 to 10**) of **3** marks each. Internal choice has been provided in **one** question.
(v) **Section C** comprises of **4** questions (Q Nos. **11 to 14**) of **4** marks each. An Internal choice has been provided in one question. It also contains two case study based questions.
(vi) There is no overall choice in the question paper. However, internal, choice has been provided in some questions. Attempt any one choice in such qustions.
(vii) Use of calculator is not permitted

SECTION A

Question Numbers 1 to 6 carry 2 marks each.

Question numbers 1 to 6 carry 2 marks each

1. **(a)** Find the 15th term from the end (towards first term) of the A.P. 3, 8, 13...... 253. **2**

OR

(b) Write the next two terms of the A.P. : $\sqrt{5}, \sqrt{20}, \sqrt{45}, ...$

Sol. **(a)** 15th term from the end = ?

Given AP : 3, 8, 13..... 253

Reverse the AP

253, ... 13, 8, 3

Now first term a = 253

common difference d = 3 – 8 = – 5

nth term $a_n = a + (n – 1) d$

(n = 15) $a_{15} = a + (15 – 1) d$

$= 253 + 14 \times (–5)$

$= 253 – 70$

$= 183$

So 15th term from end is 183

(b) Given AP : $\sqrt{5}, \sqrt{20}, \sqrt{45}, ...$

$\sqrt{5}, 2\sqrt{5}, 3\sqrt{5}, ...$

First term $a = \sqrt{5}$

Common diff $d = a_2 – a_1$

$= 2\sqrt{5} - \sqrt{5} = \sqrt{5}$

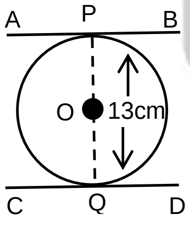
Now next two terms

Fourth term $a_4 = a_3 + d = 3\sqrt{5} + \sqrt{5} = 4\sqrt{5}$

Fifth term $a_5 = a_4 + d = 4\sqrt{5} + \sqrt{5} = 5\sqrt{5}$

2. The distance between two tangents parallel to each other of a circle is 13 cm. Find the radius of the circle. **2**

Sol.



Given that the distance between two tangents parallel to each other to a circle is 13 cm

∵ Two parallel tangents to a circle are found when the line joining the point of contacts are diameter.

∴ d = 13 cm

so radius 'r' = $\frac{d}{2} = \frac{13}{2}$ = 6.5 cm

3. Find the sum of the lower limit of the modal class and upper limit of the median class for the following distribution. **2**

Class	50-55	55-60	60-65	65-70	70-75	75-80
Frequency	10	15	8	13	9	5

Sol.

Class	F	C.F.
50-55	10	10
55-60	15	25
60-65	8	33
65-70	13	46
70-75	9	55
75-80	5	60
	N = 60	

At here maximum frequency is 15
so modal class is 55 – 60
Lower limit of modal class is 55

$\frac{N}{2} = \frac{60}{2} = 30$

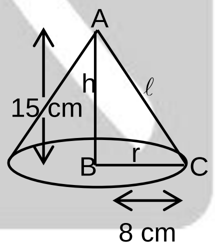
At here CF just greater than 30, is 33
So median class is 60 – 65
Upper limit of median class is 65
So required sum = 55 + 65 = 120

4. **(a)** Find the curved surface area of a right circular cone whose height is 15 cm and base radius is 8 cm $\left[\text{Use } \pi = \frac{22}{7}\right]$ **2**

OR

(b) The surface area of a sphere is 616 sq cm. Find its radius $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. **(a)** Given:
Height of cone = 15 cm
base radius = 8 cm
we know that
curved surface area of cone = $\pi r \ell$
In fig.
In ΔABC



$\ell^2 = h^2 + r^2$ { $\because$ ΔABC is right angle triangle}

$\Rightarrow \ell^2 = (15)^2 + 8^2$

$\Rightarrow \ell^2 = 225 + 64$

$\Rightarrow \ell^2 = 289$

$\Rightarrow \ell = 17$ cm

Now curved surface area = $\pi r \ell = \frac{22}{7} \times 8 \times 17 = \frac{2992}{7} = 427.428 \text{ cm}^2$

(b) Given :
surface area of sphere = 616 sq cm

$\Rightarrow \quad 4\pi r^2 = 616$

$4 \times \frac{22}{7} \times r^2 = 616$

$r^2 = \frac{616 \times 7}{4 \times 22}$

$\Rightarrow \quad r^2 = 7 \times 7 \Rightarrow r = 7$ cm

So, radius of sphere is 7 cm

5. Solve for x : $2x^2+\frac{7}{2}x+\frac{3}{4}=0$ **2**

Sol. $2x^2+\frac{7}{2}x+\frac{3}{4}=0$

$\Rightarrow$ $8x^2 + 14x + 3 = 0$

$\Rightarrow$ $8x^2 + 2x + 12x + 3 = 0$

$\Rightarrow$ $2x(4x + 1) + 3(4x + 1) = 0$

$\Rightarrow$ $(4x + 1)(2x + 3) = 0$

$\Rightarrow$ $4x + 1 = 0$ & $2x + 3 = 0$

$\Rightarrow$ $x=-\frac{1}{4}$ & $x=-\frac{3}{2}$

6. The frequency distribution table of agriculture holding in a village is given below:

Area of Land (in hectares)	1-3	3-5	5-7	7-9	9-11	11-13
Number of families	20	45	80	55	40	12

Find the modal agriculture holding per family. **2**

Sol. Mode = ?

Area of land (in hect)	**No. of families (f)**
1–3	20
3–5	45
5–7	80
7–9	55
9–11	40
11–13	12

At here, maximum frequency is 80, whose corresponding class is 5–7

So modal class is 5–7

Now $\ell = 5$, $f_1 = 80$, $f_0 = 45$, $f_2 = 55$, $h = 2$

Now modal agriculture holding per family

$$= \ell+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$$

$$= 5+\left(\frac{80-45}{2\times80-45-55}\right)\times 2$$

$$= 5+\frac{35}{60}\times 2$$

= 5 + 1.16 = 6.16

SECTION B

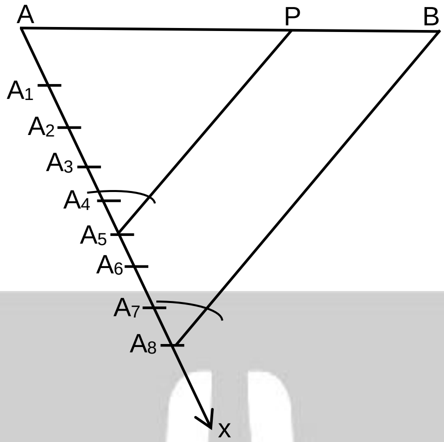
Question Numbers 7 to 10 carry 3 marks each.

7. Draw a line segment of length 7 cm and divide it in the ratio 5 : 3. **3**

Sol. **Given:**

Ratio is 5 : 3

Sum of ratio 5 + 3 = 8



Steps :

(1) Draw a line segment AB = 7 cm

(2) Draw a line AX making an acute angle with AB.

(3) Cut eight equal arcs such that $AA_1 = A_1A_2 = A_2A_3 = \ldots\ldots A_7A_8$

(4) Join A8 to B.

(5) Draw a line A_5P, parallel to A_8B, that intersect AB at point P.
So we get required ratio
AP : PB = 5 : 3

8. **(a)** If x = 3 is one root of the quadratic equation $2x^2 + px + 30 = 0$, find the value of p and the other root of the quadratic equation. **3**

OR

(b) The length of a rectangular park is 5 metres more than twice its breadth. If the area of the park is 250 sq m, find the length and breadth of the park. **3**

Sol. (a) Given

Quadratic equation :

$2x^2 + px + 30 = 0$

(x = 3) is a root of given quadratic equation, so it will satisfy the given quadratic equation.

$\Rightarrow 2(3)^2 + p(3) + 30 = 0$

$\Rightarrow 18 + 3p + 30 = 0$

$\Rightarrow 3p + 48 = 0$

$\Rightarrow 3p = -48$

$\Rightarrow 3p = -48/3$

$\Rightarrow p = -16$

so Quadratic equation is

$2x^2 - 16x + 30 = 0$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 5x - 3x + 15 = 0$

$\Rightarrow x(x - 5) - 3(x - 5) = 0$

$\Rightarrow (x - 5)(x - 3) = 0$

$\Rightarrow x - 5 = 0$ & $x - 3$

$\Rightarrow x = 5$ & $x = 3$

so the other root is (x = 5)

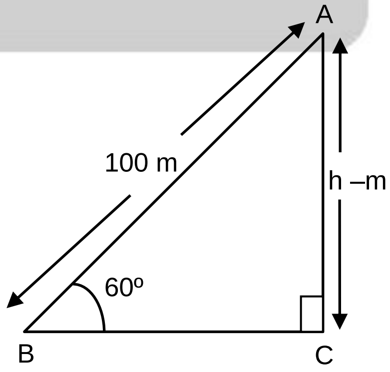
(b) Let the breadth of rectangular park is (x) meter
so length of rectangular park = (2x + 5) meter area of the park = length × breadth = 250 sq–m
$\Rightarrow$ (2x + 5) × x = 250 sq–m
$\Rightarrow 2x^2 + 5x = 250$
$\Rightarrow 2x^2 + 5x - 250 = 0$
$\Rightarrow 2x^2 + 25x - 20x - 250 = 0$
$\Rightarrow x(2x + 25) - 10(2x + 25) = 0$
$\Rightarrow (2x + 25)(x - 10) = 0$
$\Rightarrow 2x + 25 = 0 \ \& \ x - 10 = 0$
$\Rightarrow 2x = -25 \ \& \ x = 10$
$\Rightarrow x = -\frac{25}{2} \ \& \ x = 10$
so, the breadth of rectangular park = x = 10 m { $\because$ take positive value of x}
& length of rectangular park = 2x + 5 = 25 m

9. The sum of first n term of an AP is given by $S_n = 3n^2 + 2n$. Find the AP. **3**

Sol. Given $S_n = 3n^2 + 2n$
(n = 1) $S_1 = 3 \times 1^2 + 2 \times 1$
$\Rightarrow S_1 = 3 + 2 = 5$ for (n = 1)
$\Rightarrow a_1 = 5$ $S_1 = a_1$
(n = 2)
$\Rightarrow S_2 = 3 \times 2^2 + 2 \times 2$
$\Rightarrow S_2 = 12 + 4$
$\Rightarrow a_1 + a_2 = 16$ $(\because S_2 = a_1 + a_2)$
$\Rightarrow 5 + a_2 = 16$
$\Rightarrow a_2 = 16 - 5$ $\Rightarrow$ $a_2 = 11$
& $d = a_2 - a_1 = 11 - 5 = 6$
So third term $a_3 = a + 2d = 5 + 2 \times 6 = 17$
So required AP : 5, 11, 17.......

10. The string of a kite is 100 metres long and it makes an angle of 60º with the horizontal. Find the height of the kite, assuming that there is no slack in the string.[Use $\sqrt{3} = 1.73$] **3**

Sol. Length of string AB = 100 m angle $\angle ABC = 60^o$
Let height of kite is AC = h – m
In ΔABC



$$\sin\theta = \frac{AC}{AB}$$

$$\sin 60^o = \frac{h}{100}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{100}$$

$$\Rightarrow 2h = 100\sqrt{3}$$

$$h = \frac{100 \times 1.73}{2}$$

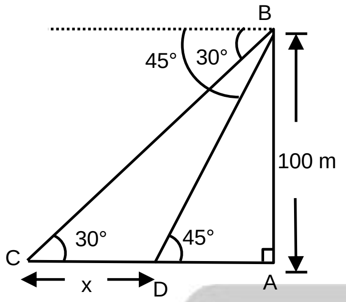
h = 86.5 m
So height of kite is 86.5 m

SECTION C

Question Numbers 11 to 14 carry 4 marks each.

11. As observed from the top of a 100 m high lighthouse from the sea-level, the angles of depression of two ships are found to be 30º and 45º. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. [Use $\sqrt{3}$ = 1.732] **4**

Sol.



In ΔBAD,

$\tan 45° = \frac{AB}{AD}$

$\Rightarrow \quad 1 = \frac{100}{AD}$

$\Rightarrow \quad AD = 100$ m

Now In ΔABC,

$\tan 30º = \frac{AB}{AC} = \frac{AB}{CD + AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x + 100}$

$\Rightarrow x + 100 = 100\sqrt{3}$

$\Rightarrow x = 100\sqrt{3} - 100$

$\Rightarrow x = 100(\sqrt{3} - 1)$m $= 100(1.732 - 1) = 73.2$m

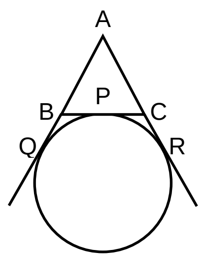
Distance between the two ships is 73.2 m

12. **(a)** Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. **4**

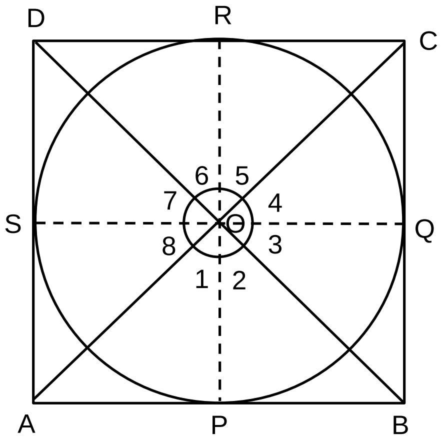
OR

(b) If a circle is touching the side BC of ΔABC at P and is touching AB and AC produced at Q and R respectively (see the figure).

Prove that AQ = $\frac{1}{2}$ (perimeter of ΔABC).



Sol. **(a)**



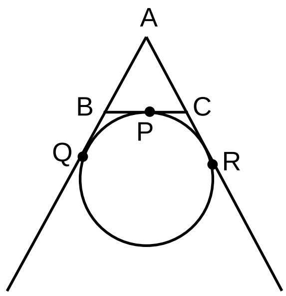
Given : ABCD is a quadrilateral and a circle inside it.
Constant : Join OA, OB, OC, OD.
To prove :
(1) $\angle AOB + \angle COD = 180°$
(2) $\angle BOC + \angle AOD = 180°$
Proof :
P, Q, R, S are point of contact of tangent AB, BC, CD , DA
Join OP, OQ, OR, OS
in Δ AOS and ΔAOP.
AO = AO (Common)
AS = AP (tangent drawn from an exterior point are equal)
OS = OP (Radius of circle)
So $\Delta AOS \cong \Delta AOP$ (by sss congruency)
So $\angle 8 = \angle 1$ …(1) (by CPCT)
Similarly $\Delta BOP \cong \Delta BOQ$
$\Rightarrow \angle 2 = \angle 3$ …(2) (by CPCT)
$\Delta COQ \cong \Delta COR$
$\Rightarrow \angle 4 = \angle 5$ …(3) (by CPCT)
$\Delta DOR \cong \Delta BOS$
$\Rightarrow \angle 6 = \angle 7$ …(4) (by CPCT)
Now by adding all angles :
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
(Replace $\angle 3$, $\angle 4$, $\angle 7$, $\angle 8$ by using above equations)
$\angle 1 + \angle 2 + \angle 2 + \angle 5 + \angle 5 + \angle 6 + \angle 6 + \angle 1 = 360°$
$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$
$2[\angle 1 + \angle 2 + \angle 5 + \angle 6] = 360°$

$$\angle AOB + \angle COD = \frac{360}{2} = 180°$$

Similarly, we can prove : $\angle BOC + \angle AOD = 180°$
Therefore opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

(b) A circle is touching side BC of ΔABC at P and touching AB an AC when produced at Q and R respectively
To prove :

AQ = $\frac{1}{2}$ (perimeter of ΔABC)



Proof :
AQ = AR .... (i)
BQ = BP .... (ii)
CP = CR .... (iii)
[Tangents drawn from an external point to a circle are equal]
Now, perimeter of ΔABC = AB + BC + CA
= AB + (BP+PC)+CA
= (AB+BQ)+ (CR+CA) [From (ii) & (iii)]
= AQ + AR = AQ + AQ [From (i)]
$\Rightarrow$ Perimeter of ΔABC = 2AQ
$\Rightarrow$ AQ = $\frac{1}{2}$ (Perimeter of ΔABC)

Hence proved

CASE STUDY - 1

13. Electric buses are becoming popular nowadays. These buses have the electricity stored in a battery. Electric buses could have a range of approximately 280 km with just one charge. Electric buses are superior to diesel buses as they reduce brake wear and also reduce pollution. Transport department of a city wants to buy some electric buses for the city. So, the department wants to know the distance travelled by existing public transport buses in a day.

The following data shows the distance travelled by 50 existing public transport buses in a day.

Daily distance travelled (in km)	100-120	120-140	140-160	160-180	180-200
Number of buses	12	14	8	6	10

(a) Find the 'median' distance travelled by a bus. **2**

(b) Find the 'mean (average)' distance travelled by a bus. **2**

Sol. (a)

x	Number of buses (f)	C.F.
100 – 120	12	
120 – 140	[14] $\rightarrow$ f	[26] $\rightarrow$ c
140 – 160	8	34
160 – 180	6	40
180 – 200	10	50
	Σf = 50	

$\frac{N}{2} = \frac{50}{2}$ = 25

Median class = (120 – 140)

Here ℓ = 120, C = 12, F = 14, h = 20

Median = $\ell + \left(\frac{\frac{N}{2} - C}{f}\right) \times h$

$= 120 + \left(\frac{25-12}{14}\right) \times 20$

$= 120 + \left[\frac{13}{14}\right] \times 20$

$= 120 + \left(\frac{260}{14}\right)$

= 120 + 18.57

= 138.57

(b)

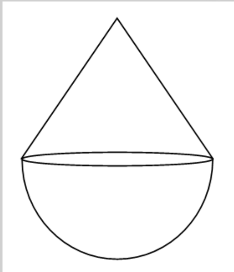
Dist	Number of buses (f)	(x_i) Class– Mark	f_ix_i
100 – 120	12	110	1320
120 – 140	14	130	1820
140 – 160	8	150	1200
160 – 180	6	170	1020
180 – 200	10	190	1900
	$\Sigma f_i = 50$		$\Sigma f_ix_i = 7260$

Mean (average) distance travelled by bus

$$= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{7260}{50} = 145.2 \text{ km}$$

CASE STUDY – 2

14. A company deals in casting and moulding of metal on orders received from its clients.

In one such order, company is supposed to make 50 toys in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of hemisphere. If the radius of the base of the cone is 21 cm and height is 28 cm, then

(a) Find the volume of 50 toys. **2**

(b) Find the ratio of the volume of hemisphere to the volume of cone. **2**

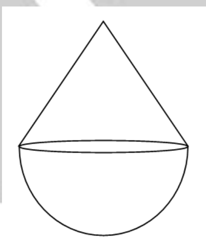
Sol. (a)

r = 21 cm

h = 28 cm

(i) volume of 1 toy =

Volume of cone + volume of hemisphere



$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi [r^2h + 2r^3]$

$= \frac{1}{3}\pi r^2 [h + 2r]$

$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 [28 + 2 \times 21]$

$= 22 \times 21 (28 + 42) = 22 \times 21 \times 70$

$= 32340 \text{ cm}^3$

So, volume of 50 toys $= 50 \times 32340 \text{ cm}^3 = 1617000 \text{ cm}^3$

(b) Volume of hemisphere $= \frac{2}{3}\pi r^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

So, $\frac{\text{Volume of hemisphere}}{\text{Volume of cone}} = \frac{\frac{2}{3}\pi r^3}{\frac{1}{3}\pi r^2 h} = \frac{2r}{h} = \frac{2 \times 21}{28} = \frac{3}{2} = 3:2$