

SET-3**Series PPQQD/4****Code No. 30/4/3**

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Candidates must write the Q.P. Code on the title page of the answer-book.

- Please check that this question paper contains **13** printed pages.
- Q.P. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **14** questions.
- **Please write down the Serial Number of the questions in the answer-book before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS (STANDARD) THEORY

HINTS & SOLUTIONS

Time allowed: 2 hours**Maximum Marks: 40****General Instructions:**

- This Question paper contains **14** questions. **All** Questions are compulsory.
- This Question paper is divided into 3 Sections- **A, B** and **C**.
- Section-A** comprises of **6** questions (Q Nos. **1 to 6**) of **2** marks each. Internal choice has been provided in two questions.
- Section-B** comprises of **4** questions (Q Nos. **7 to 10**) of **3** marks each. Internal choice has been provided in **one** question.
- Section C** comprises of **4** questions (Q Nos. **11 to 14**) of **4** marks each. An Internal choice has been provided in one question. It also contains two case study based questions.
- There is no overall choice in the question paper. However, internal, choice has been provided in some questions. Attempt any one choice in such questions.
- Use of calculator is not permitted

SECTION –A

Question Numbers 1 to 6 carry 2 marks each.

1. (a) Find the value of 'k' for which the quadratic equation $2kx^2 - 40x + 25 = 0$ has real and equal roots. **2**
OR

(b) Solve for x : $\frac{5}{2}x^2 + \frac{2}{5} = 1 - 2x$

- Sol.** (a) Given equation : $2kx^2 - 40x + 25 = 0$
by Compare with standard form
 $a = 2k, \quad b = -40, \quad c = 25$

Since roots are real and equal
So discriminant $D = 0$

$$\begin{aligned} b^2 - 4ac &= 0 \\ (-40)^2 - 4 \times 2k \times 25 &= 0 \\ 1600 - 200k &= 0 \\ 1600 &= 200k \\ k &= \frac{1600}{200} \Rightarrow k = 8 \end{aligned}$$

(b) Given equation: $\frac{5}{2}x^2 + \frac{2}{5} = 1 - 2x$

$$\begin{aligned} \Rightarrow \frac{25x^2 + 4}{10} &= \frac{1 - 2x}{1} \\ \Rightarrow 25x^2 + 4 &= 10x(1 - 2x) \\ \Rightarrow 25x^2 + 4 &= 10 - 20x \\ \Rightarrow 25x^2 + 4 - 10 + 20x &= 0 \\ \Rightarrow 25x^2 + 20x - 6 &= 0 \end{aligned}$$

by Quadratic formula of shri Dharacharya
 $a = 25, b = 20, c = -6$

Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{aligned} &= \frac{-20 \pm \sqrt{(20)^2 - 4 \times 25 \times (-6)}}{2 \times 25} \\ &= \frac{-20 \pm \sqrt{400 + 600}}{50} \\ &= \frac{-20 \pm \sqrt{1000}}{50} \end{aligned}$$

$$= \frac{-20 \pm 10\sqrt{10}}{50}$$

$$x = 10 \left(\frac{-2 \pm \sqrt{10}}{50} \right)$$

$$x = \frac{-2 \pm \sqrt{10}}{5}$$

by taking (+) $x = \frac{-2 + \sqrt{10}}{5}$

by taking (-) $x = \frac{-2 - \sqrt{10}}{5}$

$$= -\frac{2 + \sqrt{10}}{5}$$

2. The mode of a grouped frequency distribution is 75 and the modal class is 65-80. The frequency of the class preceding the modal class is 6 and the frequency of the class succeeding the modal is 8. Find the frequency of the modal class. 2

Sol. (a) Given: Mode $Z = 75$
 Modal class 65 – 80
 lower limit of modal class $\ell = 65$ interval $h = 15$
 Freq. of class preceding the modal class $f_0 = 6$
 Freq. of class succeeding the modal class $f_2 = 8$
 Freq. of modal class $f_1 = ?$
 We know that

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$\Rightarrow 75 = 65 + \left(\frac{f_1 - 6}{2f_1 - 6 - 8} \right) \times 15$$

$$\Rightarrow 75 - 65 = \left(\frac{f_1 - 6}{2f_1 - 6 - 8} \right) \times 15$$

$$\Rightarrow \frac{10}{15} = \frac{f_1 - 6}{2f_1 - 14}$$

$$\Rightarrow \frac{2}{3} = \frac{f_1 - 6}{2f_1 - 14}$$

$$\Rightarrow 4f_1 - 28 = 3f_1 - 18$$

$$\Rightarrow 4f_1 - 3f_1 = -18 + 28$$

$$\Rightarrow f_1 = 10$$

So freq. of modal class is 10.

3. (a) The curved surface area of a right circular cylinder is 176 sq cm and its volume is 1232 cu. Cm. What is the height of the cylinder? 2

OR

- (b) The largest sphere is carved out of the solid cube of side 21 cm. Find the volume of the sphere. 2

Sol. (a) Right circular cylinder:
 Given CSA of cylinder = 176 sq. cm
 $2\pi rh = 176$
 $r = \frac{176}{2\pi h} \dots(1)$

and

Volume of cylinder = 1232 cm³

$$\pi r^2 h = 1232$$

$$\Rightarrow \pi \left(\frac{176}{2\pi h} \right)^2 \times h = 1232 \quad [\text{Substitute value of } r \text{ from equation (1)}]$$

$$\frac{22}{7} \times \frac{176 \times 176}{2 \times 2 \times \frac{22}{7} \times \frac{22}{7} \times h^2} \times h = 1232$$

$$\frac{44 \times 176 \times 7}{22 \times h} = 1232$$

$$\frac{2 \times 176 \times 7}{1232} = h$$

$$h = 2$$

So, height of cylinder is 2 cm.

OR

(b) side of a cube $a = 21$ cm

Since the largest sphere is carved out of a solid cube.

So diameter of sphere = side of cube

$$d = 21 \text{ cm}$$

$$2 \times r = 21 \text{ cm}$$

$$R = \frac{21}{2} \text{ cm}$$

$$\text{Now volume of sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 11 \times 21 \times 21 = 4851 \text{ cm}^3$$

So volume of sphere is 4851 cubic cm.

4. Find the sum of all 11 terms of an A.P. whose 6th term is 30. 2

Sol. Arithmetic progression :

$$n^{\text{th}} \text{ term } a_n = a + (n-1)d$$

$$\text{Given 6}^{\text{th}} \text{ term } a_6 = a + (6-1)d = 30$$

$$\Rightarrow a + 5d = 30 \dots\dots(1)$$

Sum of first n terms

$$s_n = \frac{n}{2} [2a + (n-1)d]$$

$$(n=11) s_{11} = \frac{11}{2} [2a + (11-1)d]$$

$$= \frac{11}{2} [2a + 10d]$$

$$= \frac{11}{2} \times 2 (a+5d)$$

[Substitute value of (a+5d) from equation ... (1)]

$$= 11 \times 30$$

$$\Rightarrow s_{11} = 330$$

So sum of all 11 terms is 330

5. Find the median of the following distribution : 2

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of student	5	8	20	15	7	5

Sol. Median = ?

Marks	No. of studens(f)	cf
0-10	5	5
10-20	8	13
20-30	20	33
30-40	15	48
40-50	7	55
50-60	5	60

$$N = 60$$

$$\frac{N}{2} = \frac{60}{2} = 30$$

At here, cf just greater than 30, is 33. Whose corresponding class is 20-30.

So median class 20-30

$$\text{Now } l = 20, h = 10, \frac{N}{2} = 30, cf = 13, f = 20$$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$

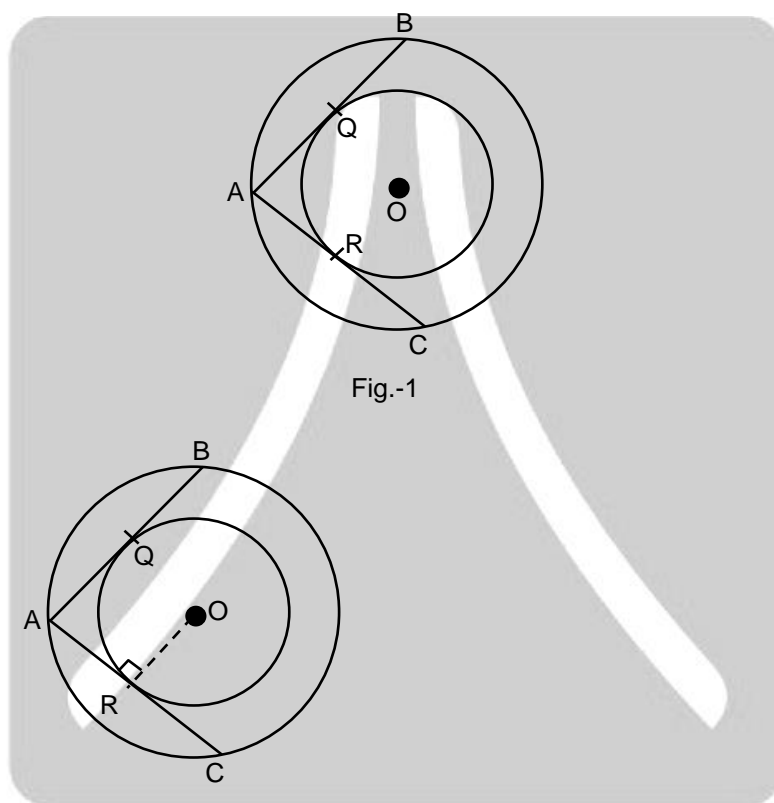
$$= 20 + \left(\frac{30 - 13}{20} \right) \times 10$$

$$= 20 + \frac{17}{2}$$

$$= 20 + 8.5 = 28.5$$

So median is 28.5

6. In Fig. 1, there are two concentric circle with centre O. If ARC and AQB are tangents to the smaller circle from the point. A lying on the larger circle, find the length of AC, if AQ = 5cm. 2



Sol.

Concentric circles having centre O.

Given AQ = 5cm

AR = AQ = 5cm [Tangent drawn from an exterior point are equal]

join O to R

So $\angle ORA = 90^\circ$ (tangent form angle of 90° with radius)

For big circle AC is chord

We know that perpendicular drawn from radius to chord bisects the chord.

So AR = RC = 5cm

Now AC = AR + RC

= 5 + 5 = 10cm

So AC = 10 cm

SECTION –B

Question Numbers 7 to 10 carry 3 marks each.

7. An aeroplane at altitude of 200 metres observes the angle of depression of opposite points on the two banks of a river to be 45° and 60°. Find the width of the river. (Use $\sqrt{3} = 1.732$) **3**

Sol. Altitude of Aeroplane AD = 200 m
B & C are two banks of a river.

$$\text{In } \triangle ABD, \tan 60^\circ = \frac{AD}{BD}$$

$$\Rightarrow BD = \frac{AD}{\tan 60^\circ} = \frac{200}{\sqrt{3}} \text{ m.}$$

Now, In $\triangle ACD$,

$$\tan 45^\circ = \frac{AD}{DC}$$

$$DC = \frac{AD}{\tan 45^\circ} = \frac{200}{1}$$

$$\therefore DC = 200 \text{ m.}$$

Hence, Width of the river = BC = BD + DC

$$= \frac{200}{\sqrt{3}} + 200 = \frac{200 + 200\sqrt{3}}{\sqrt{3}}$$

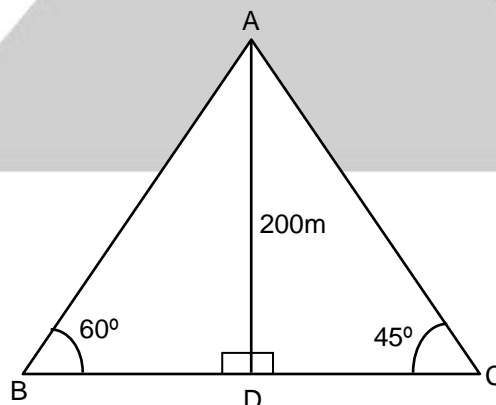
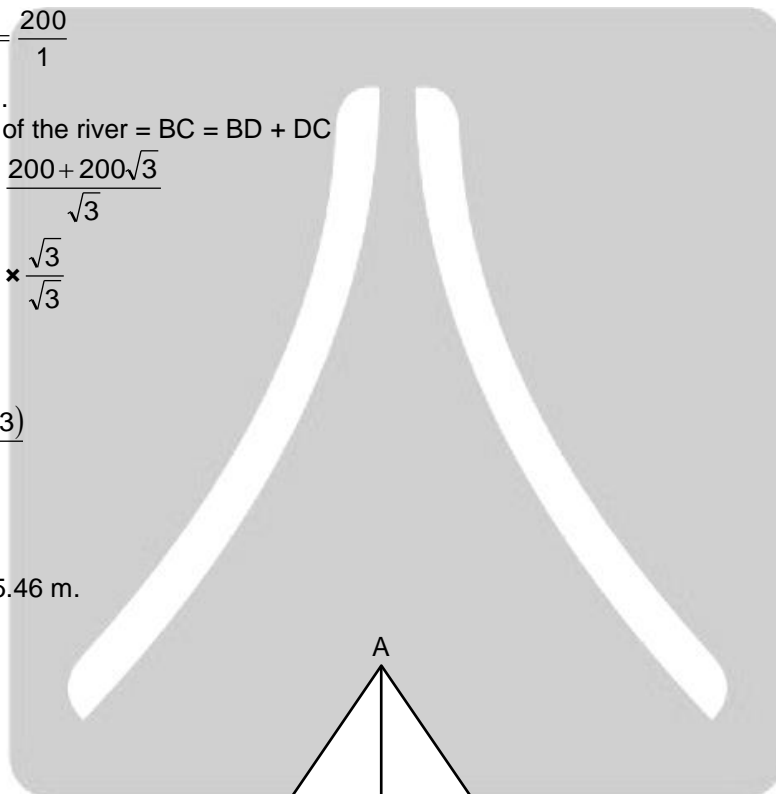
$$= 200 \left(\frac{1 + \sqrt{3}}{\sqrt{3}} \right) \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{200(\sqrt{3} + 3)}{3}$$

$$= \frac{200(1.732 + 3)}{3}$$

$$= \frac{200 \times 4.732}{3}$$

$$= \frac{946.4}{3} = 315.46 \text{ m.}$$



8. The sum of the first three terms of an A.P. is 33. If the product of first and third term exceeds the second term by 29, find the A.P. **3**

Sol. Let three terms of A.P. are $a - d$, a , $a + d$.

ATQ, Sum of three terms = 33

$$a - d + a + a + d = 33$$

$$3a = 33$$

$$a = \frac{33}{3} = 11$$

$$\Rightarrow a = 11 \quad \dots (i)$$

and product of first and third term = second term + 29

$$(a - d) \times (a + d) = a + 29$$

$$\Rightarrow a^2 - d^2 = a + 29$$

Put the value of a,

$$(11)^2 - d^2 = 11 + 29$$

$$121 - d^2 = 40$$

$$d^2 = 121 - 40$$

$$d^2 = 81$$

$$\therefore d = \pm 9$$

$$d = 9$$

$$\therefore a - d = 11 - 9 = 2$$

$$a = 11$$

$$a + d = 11 + 9 = 20$$

$$\therefore \text{AP is } 2, 11, 20 \dots\dots\dots$$

9. (a) Find the value of 'p' for which the quadratic equation $p(x-4)(x-2) + (x-1)^2 = 0$ has real and equal roots. **3**

OR

(b) Had Aarush scored 8 more marks in a Mathematics test, out of 35 marks, 7 times these marks would have been 4 less than square of his actual marks. How many marks did he get in the test? **3**

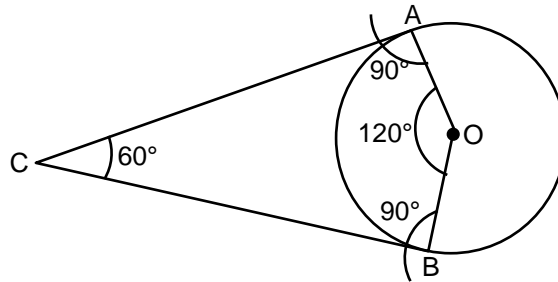
Sol.

(a) $p(x-4)(x-2) + (x-1)^2 = 0$
 Given QE has real and equal roots
 so $D = 0$
 Discriminant ($b^2 - 4ac = 0$)
 $P(x^2 - 6x + 8) + x^2 - 2x + 1 = 0$
 $(P + 1)x^2 - (6P + 2)x + 8P + 1 = 0$
 $D = 0$
 $\Rightarrow b^2 - 4ac = 0$
 $\Rightarrow [- (6P + 2)]^2 - 4 (P + 1) (8P + 1) = 0$
 $36P^2 + 4 + 24P - 4 (8P^2 + 9P + 1) = 0$
 $36P^2 - 32P^2 + 24P - 36P + 4 - 4 = 0$
 $\Rightarrow 4P^2 - 12P = 0$
 $\Rightarrow P^2 - 3P = 0$
 $\Rightarrow P(P - 3) = 0$
 $\Rightarrow P = 0 \text{ \& } P - 3 = 0$
 $\Rightarrow P = 0 \text{ \& } P = 3$

OR

(b) Let the actual marks scored = x
 New scored marks = x + 8
 ATQ
 $7(x + 8) = x^2 - 4$
 $\Rightarrow 7x + 56 = x^2 - 4$
 $\Rightarrow x^2 - 7x - 60 = 0$
 $x^2 - 12x + 5x - 60 = 0$
 $x(x - 12) + 5(x - 12) = 0$
 $(x - 12)(x + 5) = 0$
 $\Rightarrow x - 12 = 0 \qquad x + 5 = 0$
 $x = 12 \qquad x = -5 \text{ (Not possible)}$
 So, he scored 12 marks in test.

10. Construct a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60° 3
- Sol. Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P. PA and PB are the required tangents at an angle of 60°



Justification

The construction can be justified by proving that $\angle ACB = 60^\circ$

By our construction

$$\angle OAC = 90^\circ$$

$$\angle OBC = 90^\circ$$

$$\text{And } \angle AOB = 120^\circ$$

We know that the sum of all interior angles of a quadrilateral = 360°

$$\angle OAC + \angle AOB + \angle OBC + \angle ACB = 360^\circ$$

$$90^\circ + 120^\circ + 90^\circ + \angle ACB = 360^\circ$$

$$\angle ACB = 60^\circ$$

This justifies the construction

Steps of construction :

- (i) Draw a circle of radius 5 cm.
- (ii) Draw an angle $\angle AOB = 120^\circ$ at centre.
- (iii) Draw a perpendicular at A and B.
- (iv) These lines are the tangents which meet at C and are inclined to each other at 60° .

SECTION C

Question Numbers 11 to 14 carry 4 marks each.

11. From the top of an 8 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower. (Take $\sqrt{3} = 1.732$) 4

Sol.

In $\triangle ABC$

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{8}{BC} = 1$$

$$BC = 8 \text{ m}$$

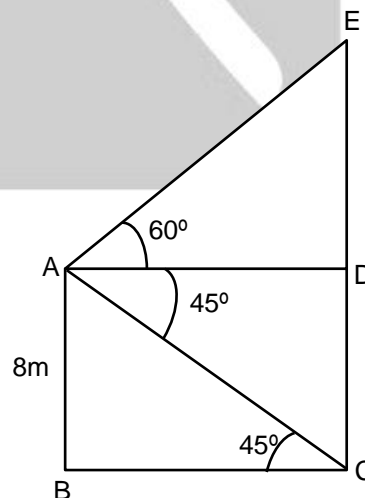
$$AD = BC = 8 \text{ m}$$

In $\triangle ADE$

$$\frac{DE}{AD} = \tan 60^\circ$$

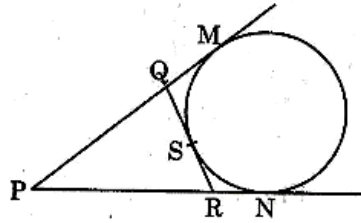
$$\frac{DE}{8} = \sqrt{3}$$

$$DE = 8\sqrt{3} \text{ m}$$



$$\begin{aligned} \text{Height of cable tower} &= ED + DC \\ &= 8\sqrt{3} + 8 \quad (\text{AB} = \text{DC}) \\ &= 8(\sqrt{3} + 1) \text{ m} \\ &= 8(1.732 + 1) \\ &= 8(2.732) = 21.856 \text{ m} \end{aligned}$$

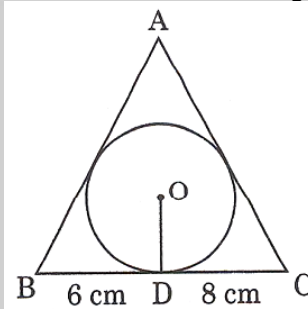
12. (a) In figure 2, if a circle touches the side QR of $\triangle PQR$ at S and extended sides PQ and PR at M and N, respectively. 4



Prove that $PM = \frac{1}{2} (PQ + QR + PR)$

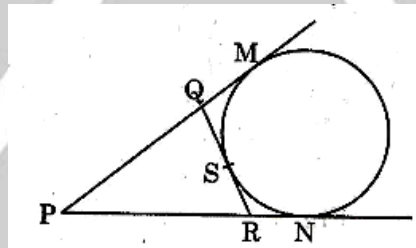
OR

- (b) In figure 3, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. If the area of $\triangle ABC$ is 84 cm^2 , find the length of sides AB and AC. 4



- Sol. (a) To prove :

$$PM = \frac{1}{2} (PQ + QR + PR)$$



R.H.S.

$$\frac{1}{2} (PQ + QR + PR)$$

$$= \frac{1}{2} (PQ + QS + SR + PR)$$

$$= \frac{1}{2} (PQ + QM + RN + PR)$$

$$[QS = QM, SR = RN]$$

Tangent drawn from an external point to a circle are equal.

$$= \frac{1}{2} (PM + PN)$$

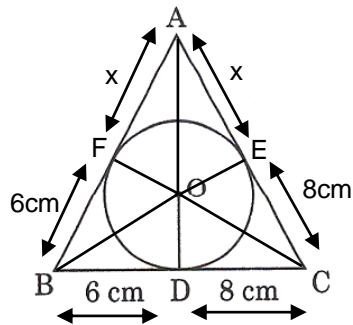
$$= \frac{1}{2} (PM + PM)$$

$$= \frac{1}{2} (2PM)$$

$$= PM$$

[PM = PN tangents drawn from an external point to a circle are equal]

(b) Given :
 BD = 6 cm
 CD = 8 cm
 ar (ΔABC) = 84 cm²
 radius = 4 cm



BD = BF
 CD = CE
 AF = AE
 [Tangents drawn from an external point to a circle are equal]

Construction :

Join OB , OC, OA and OE, OF

Let AF = AE = x cm

OD \perp BC

OE \perp AC

OF \perp AB

[The line joining the point of contact to the centre is perpendicular to the tangent]

ar (ΔABC) = ar (AOB) + ar (BOC) + ar (AOC)

$$84 = \frac{1}{2} \times AB \times OF + \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OE$$

$$84 = \frac{1}{2} \times (6 + x) \times 4 + \frac{1}{2} \times 14 \times 4 + \frac{1}{2} \times (8 + x) \times 4$$

$$84 = 12 + 2x + 28 + 16 + 2x$$

$$84 = 4x + 56$$

$$84 = 4(x + 14)$$

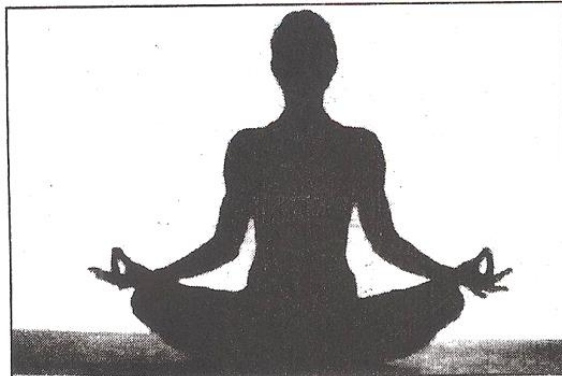
$$12 - 14 = x$$

$$7 \text{ cm} = x$$

$$\text{Now } AB = 6 + x = 6 + 7 = 13 \text{ cm}$$

$$AC = 8 + x = 8 + 7 = 15 \text{ cm}$$

13. Yoga is an ancient practice which is a form of meditation and exercise. By practicing yoga, we not even make our body healthy but also achieve inner peace and calmness. The international Yoga Day is celebrated on 21st of June every year since 2015. To promote Yoga, Green park society in Pune organised a 7 –day Yoga camp in their society. The number of people of different age groups who enrolled for this camp is given as follows :



Age Group	15-25	25-35	35-45	45-55	55-65	65-75	75-85
Number of People	8	10	15	25	40	24	18

Based on the above, find the following :

2

(A) Find the median age of people enrolled for the camp.

(B) If x more people of age group 65 – 75 had enrolled for the camp, the mean age would have been 58. Find the value of x.

2

Sol. (a)

Age Group	No. of People	C.F.
15-25	8	8
25-35	10	18
35-45	15	33
45-55	25	58
55-65	40	98
65-75	24	122
75-85	18	140
	N = 140	

$$\frac{N}{2} = \frac{140}{2} = 70$$

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{N}{2} - \text{C.F.}}{f} \right) \times h \\ &= 55 + \left(\frac{70 - 58}{40} \right) \times 10 \\ &= 55 + \frac{12}{4} = 58 \end{aligned}$$

(b)

Age Group	No. of People (Fi)	Xi	FiXi
15-25	8	20	160
25-35	10	30	300
35-45	15	40	600
45-55	25	50	1250
55-65	40	60	2400
65-75	(24 + x)	70	1680 + 70x
75-85	18	80	1440
	$\sum f_i = 140 + x$		$\sum f_i x_i = 7830 + 70x$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$58 = \frac{7830 + 70x}{140 + x}$$

$$8120 + 58x = 7830 + 70x$$

$$8120 - 7830 = 70x - 58x$$

$$290 = 12x$$

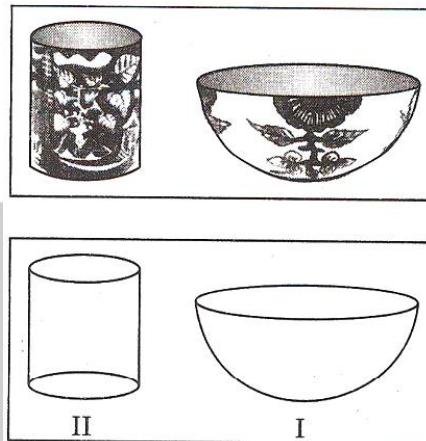
$$\frac{290}{12} = x$$

$$24.16 = x$$

$$\text{Approx} = 24 \text{ People}$$

Case Study – 2

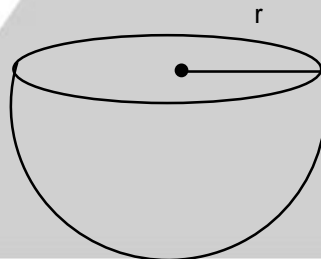
14. Khurja is a city in the Indian state of Uttar Pradesh famous for the pottery. Khurja pottery is traditional Indian pottery work which has attracted Indians as well as foreigners with a variety of tea-sets, crockery and ceramic tile works. A huge portion of the ceramics used in the country is supplied by Khurja and is also referred as 'The Ceramic Town'.
One of the private schools of Bulandshahr organised an Educational Tour for class 10 students to Khurja. Students were very excited about the trip.
Following are the few pottery objects of Khurja.



Students found the shapes of the objects very interesting and they could easily relate them with mathematical shapes viz sphere, hemisphere, cylinder etc. Maths teacher who was accompanying the students asked following questions:

- (a) The internal radius of hemispherical bowl (filled completely with water) in I is 9 cm and radius and height of cylindrical jar in II is 1.5 cm and 4 cm respectively. If the hemispherical bowl is to be emptied in cylindrical jars, then how many cylindrical jars are required? **2**
- (B) If in the cylindrical jar full of water, a conical funnel of same height and same diameter is immersed, then how much water will flow out of the jar? **2**

Sol. (a) $r = 9$ cm



Radius of cylinder

$$R = 1.5 \text{ cm}$$

$$H = 4 \text{ cm}$$

Number of cylindrical jars required

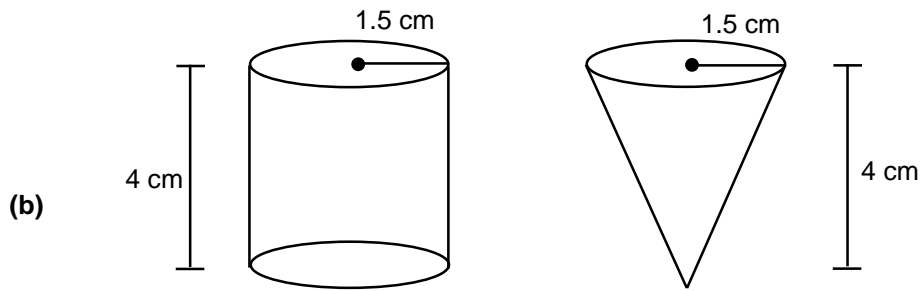
$$= \frac{\text{Vol of hemisphere}}{\text{Vol. of cylinder}}$$

$$= \frac{\frac{2}{3} \pi r^3}{\pi R^2 H}$$

$$= \frac{2}{3} \times \frac{9 \times 9 \times 9}{1.5 \times 1.5 \times 4}$$

$$= \frac{2}{3} \times \frac{9 \times 9 \times 9}{6 \times 1.5}$$

$$= 54.$$



The amount of water that will flow

= Volume of one

$$= \frac{1}{3} \pi R^2 H$$

$$= \frac{1}{3} \times \frac{22}{7} \times 1.5 \times 1.5 \times 4$$

$$= \frac{66}{7} = 9.42 \text{ cm}^3$$

