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CBSE 2021-22 (TERM-1)

DATE: 06-12-2021

Questions Paper

SERIES: SSJ/1 | CODE : 065/1/4 | SET-4

SUBJECT : MATHEMATICS

TIME ALLOWED: 90 MINUTES **MAXIMUM MARKS : 40**

NOTE

(i) Please check that this question paper contains 24 printed pages.

(ii) Please check that this question paper contains 50 multiple choice questions MCQs.)

(iii) QP Code given on the right hand side of the question paper should be written at the appropriate place of the OMR Sheet by the candidates.

(iv) 20 minute additional time has been allotted to read this question paper prior to actual time of commencement of examination.

General Instructions:

(i) This question paper contains 50 questions out of which 40 questions are to be attempted. All questions carry equal marks.

(ii) This question paper consists of three Sections — Section A, Section B and Section C.

(iii) Section–A – contains 20 questions. Attempt any 16 questions from Q. No. 1 to 20.

(iv) Section–B – also contains 20 questions. Attempt any 16 questions from Q. No. 21 to 40.

(v) Section–C – contains 10 questions including one Case Study. Attempt any 8 from Q. No. 41 to 50.

(vi) There is only one correct option for every Multiple Choice Question (MCQ). Marks will not be awarded for answering more than one option.

(vii) There is no negative marking.

SECTION - A

In this Section, attempt any **16** questions out of Questions **1-20**. Each question is of **One** mark.

1. A relation R is defined on N. Which of the following is the reflexive relation ?

(a) $R = \{(x,y) : x > y , x , y \in N\}$

(b) $R = \{(x,y) : x + y = 10 x , y \in N\}$

(c) $R = \{(x,y)$: xy is the square number, $x, y \in N\}$

(d) $R = \{(x,y) : x + 4y = 10 ; x, y \in N\}$

Ans. (c)

Sol. $R = \{(x,y)$: xy is the square number $x,y \in N\}$

Here Reflexive

$(x,x) \in R \quad \forall a \in N$

$x \times x \Rightarrow \quad x^2 \Rightarrow$ square number

It is true for $x \in N$

2. The function $f : R \to R$ defined by $f(x) = 4 + 3 \cos x$ is :

(a) bijective (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

Ans. (d)

Sol. $f : R \to R$

$f(x) = 4 + 3\cos x$

Let $f(x_1) = f(x_2)$

$\Rightarrow 4 + 3\cos x_1 = 4 + 3\cos x_2$

$\Rightarrow \cos x_1 = \cos x_2$

$\Rightarrow x_1, x_2 \in R$ hence many one functions

f(x) will not be onto function

3. If $y = \cot^{-1}x$, $x < 0$, then :

(a) $\frac{\pi}{2} < y \le \pi$ (b) $\frac{\pi}{2} < y < \pi$ (c) $-\frac{\pi}{2} < y < 0$ (d) $-\frac{\pi}{2} \le y < 0$

Ans. (b)

Sol. If $y = \cot^{-1}x$, $x < 0$

$\cot y = x, x < 0$

$\cot y < 0$

$\frac{\pi}{2} < y < \pi$

4. The number of functions defined from $\{1,2,3,4,5\} \to (a,b)$ which are one-one is :

(a) 5 (b) 3 (c) 2 (d) 0

Ans. (d)

Sol. $\{1,2,3,4,5\} \to \{a, b\}$

Number of one-one function will be 0

$\Rightarrow$ not possible

5. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then (A – 2I) (A – 3I) is equal to

(a) A (b) I (c) 5I (d) 0

Ans. (d)

Sol. $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow$ (A – 2I) (A – 3I)

$\Rightarrow \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)$

$\Rightarrow \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

6. If P is a 3 × 3 matrix such that P′ = 2P + I , where P′ is the transpose of P , then :

(a) P = I (b) P = – I (c) P = 2I (d) P = – 2I

Ans. **(b)**

Sol. P ′ = 2P + I ....(1)

(P′)′ = (2P + I)′

P = 2P′ + I

from equation (1)

P = 2(2P + I) + I

P = 4P + 2I + I

P = 4P + 3I

–3P = 3I

$\Rightarrow$ P = –I

7. If order of matrix A is 2 × 3, of matrix B is 3 × 2, and of matrix C is 3 × 3 , then which one of the following is not defined ?

(a) C (A + B′) (b) C(A + B′)′ (c) BAC (d) CB + A′

Ans. **(a)**

Sol. Order of matrix A $\Rightarrow$ 2 × 3

matrix B $\Rightarrow$ 3 × 2

matrix C $\Rightarrow$ 3 × 3

Order of matrix B′ = 2 × 3

So C [A + B′]

↓ ↓ ↓

3×3 2×3 2×3

8. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then the value of α is

(a) ± 1 (b) ± 2 (c) $\pm\sqrt{5}$ (d) $\pm\sqrt{7}$

Ans. **(d)**

Sol. $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$

$\Rightarrow |A| = \alpha^2 - 4$ and $|A|^3 = 27$

$\Rightarrow 3 = \alpha^2 - 4$ and $|A| = 3$

$\Rightarrow \alpha^2 = 7$

$\Rightarrow \alpha = \pm\sqrt{7}$

9. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is :

(a) 3 (b) 5 (c) 7 (d) 9

Ans. (d)

Sol. $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$

$\Rightarrow$ 5(–2x + 18) – 3(14 + 27) – 1(–42 – 9x) = 0

$\Rightarrow$ –10x + 90 – 123 + 42 + 9x = 0

$\Rightarrow$ –x + 9 = 0

$\Rightarrow$ x = 9

10. The inverse of $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$ is

(a) $\begin{bmatrix} -5 & 3 \\ 7 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}$

Ans. (b)

Sol. A = $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$

$A^{-1} = \frac{\text{adj}A}{|A|}$

adjA = $\begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}$

|A| = 20 – 21 = – 1

$\therefore\ A^{-1} = -1\begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$, then A^{-1}

(a) is A (b) is (–A) (c) is A^2 (d) Does not exist

Ans. (a)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$ given A^{-1} = ?

$A^{-1} = \frac{\text{adj}A}{|A|}$

adjA = $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -59 & -69 & 1 \end{bmatrix}$

and $A^{-1} = \frac{-1}{1}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -59 & -69 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix} = A$

12. If the function $f(x) = \begin{cases} 3x-8, \text{if} & x \le 5 \\ 2k, \text{if} & x > 5 \end{cases}$ is continuous, then the value of k is :

(a) $\frac{2}{7}$ (b) $\frac{7}{2}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

Ans. **(b)**

Sol. $f(x) = \begin{cases} 3x-8, & x \le 5 \\ 2k, & x > 5 \end{cases}$

continuous at x = 5

$\therefore$ LHL = RHL = f(5)

$\lim_{h\to 0} f(5-h) = \lim_{h\to 0} f(5+h)$

$\Rightarrow \lim_{h\to 0} 3(5-h) - 8, \quad \lim_{h\to 0} f(5+h)$

$\Rightarrow \lim_{h\to 0} 15 - 3h - 8, \quad 2k$

$\Rightarrow 7$

$\therefore$ 7 = 2k

$k = \frac{7}{2}$

13. The function f(x) = [x], where [x] is the greatest integer function that is less than or equal to x, is continuous at :

(a) 4 (b) –2 (c) 1.5 (d) 1

Ans. **(c)**

Sol. f(x) = [x] $\Rightarrow$ greatest integer function

$\Rightarrow$ It is continuous non-integer number

14. If $y = \tan^{-1}(e^{2x})$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{2e^{2x}}{1+e^{4x}}$ (b) $\frac{1}{1+e^{4x}}$ (c) $\frac{2}{e^{2x}+e^{-2x}}$ (d) $\frac{1}{e^{2x}-e^{-2x}}$

Ans. **(a)**

Sol. $y = \tan^{-1}(e^{2x})$

$\frac{dy}{dx} = \frac{1}{1+(e^{2x})^2} \times e^{2x} \times 2 \Rightarrow \frac{2e^{2x}}{1+e^{4x}}$

15. If $y^2(2-x) = x^3$, then $\left(\frac{dy}{dx}\right)_{(1,1)}$ is equal to :

(a) 2 (b) – 2 (c) 3 (d) $-\frac{3}{2}$

Ans. **(a)**

Sol. $y^2(2-x) = x^3$; $\left(\frac{dy}{dx}\right)_{(1,1)}$ equal to

$\therefore\ 2y\frac{dy}{dx}(2-x) + y^2(-1) = 3x^2$

$\Rightarrow\ 2y(2-x)\frac{dy}{dx} - y^2 = 3x^2$

$\frac{dy}{dx} = \frac{3x^2+y^2}{2y(2-x)}$

at (1, 1) $\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{3(1)^2+(1)^2}{2(1)(2-1)} \Rightarrow \frac{4}{2} = 2$

16. The angle between the tangents to the curves $y = x^2 - 5x + 5$ at , the points (2,0) and (3,0) is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) 0

Ans. **(a)**

Sol. Curve $y = x^2 - 5x + 6$

$\frac{dy}{dx} = 2x - 5$

Slope at point (2, 0)

$\left(\frac{dy}{dx}\right)_{(2,0)} = 4 - 5 = -1$

Slope at point (3, 0)

$\left(\frac{dy}{dx}\right)_{(3,0)} = 6 - 5 = 1$

$m_1 \times m_2 = -1$

$-1 \times 1 = -1$

angle will be $\frac{\pi}{2}$

17. The interval , in which function $y = x^3 + 6x^2 + 6$ is increasing, is :

(a) $(-\infty, -4) \cup (0, \infty)$ (b) $(-\infty, 4)$ (c) $(-4, 0)$ (d) $(-\infty, 0) \cup (4, \infty)$

Ans. **(a)**

Sol. $y = x^3 + 6x^2 + 6$

$\frac{dy}{dx} = 3x^2 + 12x$

For increasing

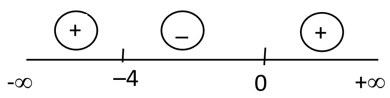
$\frac{dy}{dx} > 0$

$3x^2 + 12x + > 0$

$x^2 + 4x > 0$

$x(x + 4) > 0$

$x = 0$; $x = -4$



$x \in (-\infty, -4) \cup (0, \infty)$

18. The value of x for which $(x - x^2)$ is maximum, is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Ans. **(b)**

Sol. $y = x - x^2$

$\frac{dy}{dx} = 1 - 2x$

For maxima,

$\frac{dy}{dx} = 0$

$1 - 2x = 0$

$x = \frac{1}{2}$

Now $\frac{d^2y}{dx^2} = -2 < 0$

$x = \frac{1}{2}$ Local maxima .

19. If the corner points of the feasible region of an LPP are (0,3), (3,2) and (0,5) , then the minimum value of $Z = 11x + 7y$ is :

(a) 21 (b) 33 (c) 14 (d) 35

Ans. **(a)**

Sol. $z = 11x + 7y$

Corner point

$Z = 11 \times 0 + 7 \times 3$ at point (0,3)

$Z = 21$

$Z = 11 \times 3 + 7 \times 2$ at point (3,2)

$= 33 + 14$

$= 47$

$Z = 11 \times 0 + 7 \times 5$ at point (0,5)

$= 35$

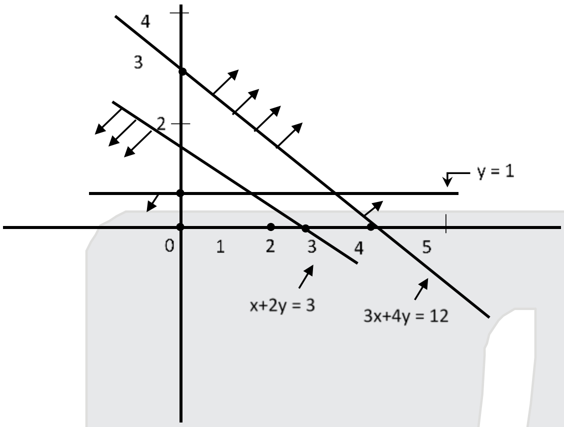
$\therefore$ minimum value of z = 21

20. The number of solutions of the system of inequations $x + 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$; $y \ge 1$

(a) 0 (b) 2 (c) finite (d) infinite

Ans. **(a)**

Sol. $x + 2y \le 3$; $3x + 4y \ge 12$; $x \ge 0$; $y \ge 1$
Drawing graph



Here No.
Feasible Region . so. No. of solution = 0

SECTION - B

In this Section, attempt any **16** questions out of Questions **21-40**. Each question is of **One** mark.

21. The number of equivalence relations in the set {1,2,3} containing the elements (1,2) and (2,1) is
(a) 0 (b) 1 (c) 2 (d) 3

Ans. **(c)**

Sol. {1,2,3}
No. of equivalence relation containing element (1,2) and (2,1)
R_1 { (1,1) (2,2) (3,3) (1,2) (2,1) } .
R_2 { (1,1) (2,2) (3,3) (1,2) (2,1) (1,3) (3,1) (2,3)(3,2)} .
$\therefore$ No. of equivalence relation = 2

22. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x}$, for all $x \in R$. Then , f is
(a) one-one (b) onto (c) bijective (d) not defined

Ans. **(d)**

Sol. $f : R \to R$
$F(x) = \frac{1}{x}$ for all $x < R$
Function is not defined as x = 0

23. The function $f : N \to N$ is defined by $f(n) = f(n) = \begin{cases} \frac{n+1}{2} & ,\text{if n is odd} \\ \frac{n}{2} & ,\text{if n is even} \end{cases}$, the function f is :
(a) bijective (b) one-one but not onto
(c) onto but not one-one (d) neither one-one nor onto

Ans. **(a)**

Sol. $f : N \to N$

$$f(n) = \begin{cases} \frac{n+1}{2} & : n \text{ add} \\ \frac{n}{2} & : n \text{ iseven} \end{cases}$$

for one-one : $f(n_1) = f(n_2)$ every n add.

$$\frac{n_1+1}{2} = \frac{n_2+1}{2}$$

$n_1 = n_2$ for every odd no.

for n = even no.

$f(n_1) = f(n_2)$

$\Rightarrow n_1 = n_2$ for every even no.

hence f(n) will be many one function

For onto

Let $y = f(n)$ for n = odd no.

$$y = \frac{n+1}{2}$$

$\Rightarrow 2y = n + 1$

$\Rightarrow 2y - 1 = n$: $y = N$.

$\therefore$ Pre image exit for every odd no.

Let $y = f(n)$ when n = even no.

$$y = \frac{1}{2}$$

$2y = n$; $y = N$.

$\therefore$ Pre image exit for every even no.

function onto not one-one

24. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is

(a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

Ans. **(b)**

Sol. $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$

$$\Rightarrow \frac{\pi}{2} - \cos^{-1}\left[\cos\frac{13\pi}{5}\right]$$

$$\Rightarrow \frac{\pi}{2} - \cos^{-1}\left[\cos\left(2\pi + \frac{3\pi}{5}\right)\right]$$

$$\Rightarrow \frac{\pi}{2} - \cos^{-1}\left(\cos\frac{3\pi}{5}\right)$$

$$\Rightarrow \frac{\pi}{2} - \frac{3\pi}{5} \Rightarrow \frac{5\pi - 6\pi}{10} = \frac{-\pi}{10}$$

25. If $\sin^{-1}x > \cos^{-1}x$, then x should lie in the interval:

(a) $\left(-1, -\frac{1}{\sqrt{2}}\right)$ (b) $\left(0, \frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{1}{\sqrt{2}}, 1\right)$ (d) $\left(\frac{1}{\sqrt{2}}, 0\right)$

Ans. **(c)**

Sol. if $\sin^{-1}x > \cos^{-1}x$

$\Rightarrow \frac{\pi}{2} - \cos x > \cos^{-1}x$

$\Rightarrow \frac{\pi}{2} > 2\cos^{-1}x$

$\Rightarrow \frac{\pi}{4} > 2\cos^{-1}x$

$\Rightarrow \frac{1}{\sqrt{2}} > x$

$\therefore x \in \left(\frac{1}{\sqrt{2}}, 1\right)$

26. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and A+A' = I, then the value of α is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$

Ans. **(b)**

Sol. A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A + A^1 = I$

$\therefore \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & +\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore 2\cos\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

27. If A = $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$ is equal to :

(a) $k(3y+k^2)$ (b) $3y+k^3$ (c) $3y+k^2$ (d) $k^2(3y+k)$

Ans. **(d)**

Sol. $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

$\Rightarrow$ $C_1 \Rightarrow C_1 - C_2$ and $C_2 \Rightarrow C_2 - C_3$

$\Rightarrow \quad \begin{vmatrix} k & 0 & y \\ -k & k & y \\ 0 & -k & y+k \end{vmatrix}$

$\Rightarrow \quad k\,[ky + k^2 + ky] - 0 + y\,(k^2 - 0)$

$\Rightarrow \quad k^3 + 3k^2y$

$\Rightarrow \quad k^2\,(3y + k)$

28. If A = $\begin{vmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 4 & 2 & 0 \end{vmatrix}$ is the adjoint of a square matrix B, then B^{-1} is equal to :

(a) $\pm A$ (b) $\mp\sqrt{2}A$ (c) $\mp\frac{1}{\sqrt{2}}B$ (d) $\mp\frac{1}{\sqrt{2}}A$

Ans. **(d)**

Sol. A = $\begin{bmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 4 & 2 & 0 \end{bmatrix}$ { |A| = 2}.

$\therefore\ B^{-1} = \frac{1}{|B|}\,\text{adj}(B)$ $\left\{\begin{array}{l} \text{adj}B = A \\ |\text{adj}B| = |A| = |B|^2 \end{array}\right.$

$\left\{\begin{array}{l} \Rightarrow |B|^2 = 2 \\ \Rightarrow |B| = \pm\sqrt{2} \end{array}\right.$

$= \frac{1}{|B|}\ A$

$B^{-1} = \pm\ \frac{1}{\sqrt{2}}\ A$

29. If $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$. Then $A^5 - A^4 - A^3 + A^2$ is equal to :

(a) 2A (b) 3A (c) 4A (d) O

Ans. **(d)**

Sol. A = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

$\therefore \Rightarrow A^5 - A^4 - A^3 + A^2$ $\left\{\begin{array}{l} A^3 = A^5 \\ \text{and } A^2 = A^4 \end{array}\right.$

$\therefore \Rightarrow 0$

30. If $y = e^{-x}$, then $\frac{d^2y}{dx^2}$ is equal to :

(a) –y (b) y (c) x (d) –x

Ans. **(b)**

Sol. $y = e^{-x}$

$\frac{dy}{dx} = -e^{-x}$

$\frac{d^2y}{dx^2} = e^{-x}$

$\frac{d^2y}{dx^2} = y$

31. If $x = t^2 + 1$, $y = 2at$, then $\frac{d^2y}{dx^2}$ at t = a is :

(a) $-\frac{1}{a}$ (b) $-\frac{1}{2a^2}$ (c) $\frac{1}{2a^2}$ (d) 0

Ans. **(b)**

Sol. If $x = t^2 + 1$; $y = 2at$

$\frac{d^2y}{dx^2}$ at t = a is

$\therefore \frac{dx}{dt} = 2t$ ------- (1)

$\therefore \frac{dy}{dt} = 2a$ ---------- (2)

Equation (2) ÷ Equation (1)

$\frac{dy}{dx} = \frac{2a}{2t} = \frac{a}{t}$

Again differentiate with respect to x

$\frac{d^2y}{dx^2} = a \times -\frac{1}{t^2}\frac{dt}{dx}$

$= -\frac{1}{t^2}\frac{dt}{dx}$

$= -\frac{a}{t^2} \times \frac{1}{2t}$

$\frac{d^2y}{dx^2}$ at t = a

$-\frac{a}{a^2} \times \frac{1}{2a}$

$\Rightarrow -\frac{1}{2a^2}$

32. The function $f(x) \begin{cases} x^2 \text{ for } x < 1 \\ 2 - x \text{ for } x \geq 1 \end{cases}$ is :

(a) not differentiable at x = 1

(b) differentiable at x= 1

(c) not continuous at x = 1

(d) neither continuous nor differentiable at x= 1

Ans. **(a)**

Sol. $f(x) = \begin{cases} x^2 ; x < 1 \\ 2 - x ; x \geq 1 \end{cases}$

Here LHD ≠ RHD, at x = 1.

∴ f (x) is not differentiable at

x = 1

33. The curve $x^2 - xy + y^2 = 27$ has tangents parallel to x-axis at

(a) (3, 6) and (–3, –6) (b) (3, –6) and (–3, 6)

(c) (–3, –6) and (3, –6) (d) (–3, 6) and (–3, –6)

Ans. **(a)**

Sol. Curve $x^2 - xy + y^2 = 27$

differentiate equation wr to x - - - - -

$$\Rightarrow 2x - \left(x\frac{dy}{dx} + y \times 1\right) + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow 2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(-x + 2y) = y - 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-2x}{2y-x}$$

tangent parallel to x-axis.

$$\therefore \frac{dy}{dx} = 0$$

$$\frac{y-2x}{2y-x} = 0$$

$y = 2x$ $\therefore$ Option (a) Satisfies given condition.

34. A wire of length 20 cm is bent in the form of a sector of a circle. The maximum area that can be enclosed by the wire is :

(a) 20 sq cm (b) 25 sq cm (c) 10 sq cm (d) 30 sq cm

Ans. **(b)**

Sol. $2r + L = 20$ cm.

$$\text{Area} = \frac{1}{2}r^2 \times \frac{l}{r}$$

$$A = \frac{1}{2}lr$$

$$\text{Area} = \frac{1}{2}r^2\theta$$

$$\frac{dA}{dr} = \frac{1}{2}r(20 - 2r)$$

$\Rightarrow$ $10 - 2r = 0$

$r = 5$ cm.

$l = 20 - 10$

$l = 10$ cm.

$$\text{Area} = \frac{1}{2} \times 10 \times 5$$

$= 25\ cm^2$

Option (b)

35. The function (x–sinx) decreases for :

(a) all x (b) $x < \frac{\pi}{2}$ (c) $0 < x < \frac{\pi}{4}$ (d) no value of x

Ans. **(d)**

Sol. $f(x) = (x - \sin x)$

$f'(x) = 1 - \cos x$

decrease

$f'(x) < 0$

$1 - \cos x < 0$

$1 < \cos x$

No value possible.

36. If θ is the angle of intersection between the curves $y^2 = 4ax$ and $ay = 2x^2$ at (a, 2a) then the value of tan θ is :

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{2}{5}$

Ans. **(a)**

Sol. $y^2 = 4ax$ and $ay = 2x^2$.

$2y\frac{dy}{dx} = 4a$ $a\frac{dy}{dx} = 4x$

$\frac{dy}{dx} = \frac{2a}{y}$ $\frac{dy}{dx} = \frac{4x}{a}$

Slope at point (a, 2a) slope at point at (a, 2a)

$\left(\frac{dy}{dx}\right)_{(a,2a)} = \frac{2a}{2a} = 1$ $\left(\frac{dy}{dx}\right)_{(a,2a)} = \frac{4a}{a} = 4$

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{1-4}{1+1\times 4}\right|$

$\tan\theta = \left|\frac{-3}{5}\right|$

$\tan q = \frac{3}{5}$

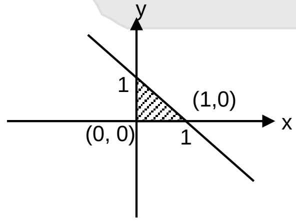
37. The maximum value of Z = 3x + 4y subject to the constraints x ≥ 0, y ≥ 0, and x + y ≤ 1 is :

(a) 7 (b) 4 (c) 3 (d) 10

Ans. **(b)**

Sol. Z = 3x + 4y

Constraint $x \geq 0$: $y \geq 0$ and $x + y \leq 1$



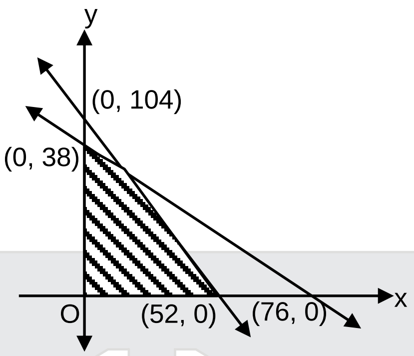
Corner point (0, 0) (1,0) (0, 1)

(0, 0) ⇒ $Z = 0$

(1, 0) ⇒ $Z = 3$

(0, 1) ⇒ $Z = 4$ maximum value

38. The feasible region of an LPP is given in the the following figure:



Then the constraint of LPP are $x \geq 0$, $y \geq 0$ and

(a) $2x + y \leq 52$ and $x + 2y \leq 76$ (b)) $2x + y \leq 104$ and $x + 2y \leq 76$

(c)) $x + 2y \leq 104$ and $2x + y \leq 76$ (d)) $x + 2y \leq 104$ and $2x + y \leq 38$

Ans. **(b)**

Sol. corner point of feasible region are (0, 0), (52, 0) and (0, 38) then constraint will be

$2x + y \leq 104$ and $x + 2y \leq 76$

39. If the minimum value of an objective function $Z = ax + by$ occurs at two points (3, 4) and (4, 3), then :

(a) a+b=0 (b) a = b (c) 3a = b (d) a = 3b

Ans. **(b)**

Sol. $z = ax + by$

Minimum value occurs at two

Point (3, 4) and (4,3)

$\therefore Z = 3a + 4b$ and $z = 4a + 3b$

$\Rightarrow 3a + 4b = 4a + 3b \Rightarrow a = b$

40. For the following LPP

Maximize $Z = 3x + 4y$

Subject to constraints

$x - y \geq -1$, $x \leq 3$

$x \geq 0$, $y \geq 0$

the maximum value is :

(a) 0 (b) 4 (c) 25 (d) 30

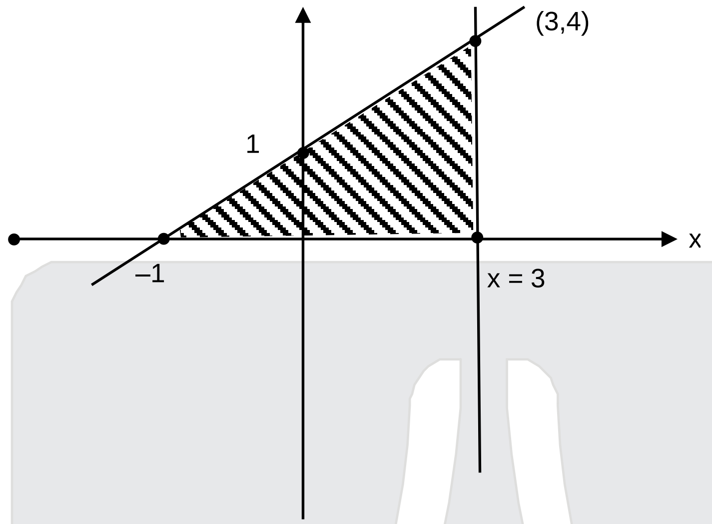
Ans. **(c)**

Sol. Maximize $z = 3x + 4y$

subject to constraints

$x - y \geq -1$; $x \leq 3$

$x \geq 0$; $y \geq 0$



Corner points

(–1,0) (3,0) and (3,4) z = 3x + 4y

(–1,0) $\rightarrow$ – 3

(3,0) $\rightarrow$ 9

(3,4) $\rightarrow$ 25 (Maximum)

Option (C)

SECTION - C

In this Section, attempt any **8** questions out of Questions **41-50**. Each question is of **One** mark.

41. A relation R is defined on Z as :

aRb if and only if $a^2 - 7ab + 6b^2 = 0$

Then R, is

(a) reflexive and symmetric
(b) symmetric but not reflexive
(c) transitive but not reflexive
(d) reflexive but not symmetric

Ans. **(d)**

Sol. aRb if and only is

$a^2 - 7ab + 6b^2 = 0$

reflexive $(a, a) \in R\ \forall\ a \in Z$

$\Rightarrow a^2 - 7a^2 + 6a$

$\Rightarrow -6a^2 + 6a^2 = 0$

$\therefore$ reflexive

Symmetric : $(a, b) \in R \Rightarrow (b, a) \in R$

$\Rightarrow a^2 - 7ab + 6b^2 = 0$

For (b, a)

$\Rightarrow b^2 - 7ba + 6a^2 \neq 0$

Not symmetric

42. The value of $\begin{vmatrix} 1! & 2! & 3! \\ 2.2! & 3.3! & 4.4! \\ 3! & 4! & 5! \end{vmatrix}$ is

(a) 12 (b) –12 (c) 24 (d) –24

Ans. **(d)**

Sol. $\begin{vmatrix} 1! & 2! & 3! \\ 2!2 & 3!3 & 4!4 \\ 3! & 4! & 5! \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 2 & 6 \\ 4 & 18 & 96 \\ 6 & 24 & 120 \end{vmatrix} = 2\times 6 \begin{vmatrix} 1 & 2 & 6 \\ 2 & 9 & 48 \\ 1 & 4 & 20 \end{vmatrix}$

$\Rightarrow 12[1(180-192)-2(40-48)+6(8-9)]$

$\Rightarrow 12 \times -2 = -24$

43. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then :

(a) a = 1 = b (b) a = cos 2θ , b = sin2θ

(c) a = sin 2θ , b = cos2θ (d) a = cos θ , b = sinθ

Ans. **(b)**

Sol. $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \times \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1-\tan^2\theta & 0 \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} \times \cos^2\theta$

$\Rightarrow \cos^2\theta \begin{bmatrix} \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} & 0 \\ \frac{2\sin\theta}{\cos\theta} & \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\therefore \cos^2\theta - \sin^2\theta = a$ 2sinθ cosθ = b

cos2θ = a b = sin2θ

44. The normal to the curve $3y = 6x - 5x^3$ at the point $\left(1, \frac{1}{3}\right)$ passes through the point:

(a) (3, 1) (b) (3, 2) (c) (2, 3) (d) (1, 1)

Ans. **(a)**

Sol. Curve $3y = 6x - 5x^3$

At point $\left(1, \frac{1}{3}\right)$

differentiate w.r.t. to x …

$3\frac{dy}{dx} = 6 - 15x^2$

$\frac{dy}{dx} = \frac{6-15x^2}{3} = \frac{3(2-5x^2)}{3}$

$\frac{dy}{dx} = 2 - 5x^2$

Slope of tangent at point $\left(1,\frac{1}{3}\right)$ = 2– 5× (1)²

$\left(\frac{dy}{dx}\right)_{\left(1,\frac{1}{3}\right)} = 2 - 5 = -3$

Slope of normal $= \frac{-1}{-3} = \frac{1}{3} \Rightarrow -\frac{1}{\left(\frac{dy}{dx}\right)}$

∴ equation of normal

$y - y_1 = \frac{1}{3}(x - x_1)$

$y - \frac{1}{3} = \frac{1}{3}(x - 1)$

$3y - 1 = x - 1$

$x - 3y = 0$

$x = 3y$

45. If $y = \sin((2\sin^{-1}x)$, then $(1-x^2)y_2$ is equal to :

(a) $-xy_1 + 4y$ (b) $-xy_1 - 4y$ (c) $xy_1 - 4y$ (d) $xy_1 + 4y$

Ans. **(c)**

Sol. $y = \sin[2\sin^{-1}x]$

∴ diff. with respect to x

$y_1 = \cos(2\sin^{-1}x) \times 2 \times \frac{1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\, y_1 = 2\cos(2\sin^{-1}x)$

Again diff. equation

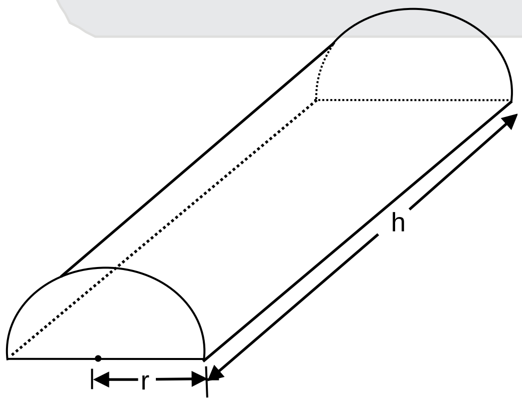
$\sqrt{1-x^2}\, y_2 + y_1 \times \frac{1}{2\sqrt{1-x^2}} \times (-2x) = -2\sin(2\sin^{-1}x) \times 2 \times \frac{1}{\sqrt{1-x^2}}$

$(1-x^2)\, y_2 - xy_1 = -4\sin(2\sin^{-1}x)$

$(1-x^2)\, y_2 = -4y + xy_1$

46. Some young entrepreneurs started an industry "young achievers " for casting metal into various shapes. They put up an advertisement online stating the same and expending order to cast metal for toys, sculptures, decorative pieces and more

A group of friends wanted to make innovative toys and hence contacted the "young achievers" to order them to cast metal into solid half cylinders with rectangular base and semi-circular ends.



Based on the above information, answer the following question :

46. The volume (V) of the casted half cylinder will be:

(a) $\pi r^2 h$ (b) $\frac{1}{3}\pi r^2 h$ (c) $\frac{1}{2}\pi r^2 h$ (d) $\pi r^2 (r+h)$

Ans. (c)

Sol. Vol. of the casted half cylinder

$$V = \frac{\pi r^2 h}{2} \Rightarrow \frac{1}{2}\pi r^2 h$$

47. The total surface area (S) of the casted half cylinder will be :

(a) $\pi rh + 2\pi r^2 + rh$ (b) $\pi rh + \pi r^2 + 2rh$ (c) $2\pi rh + \pi r^2 + 2rh$ (d) $\pi rh + \pi r^2 + rh$

Ans. (b)

Sol. Total surface area (s)

$s \Rightarrow \pi rh + \pi r^2 + 2rh$

48. The total surface area S can be expressed in terms of V and r as :

(a) $2\pi r + \frac{2V(\pi+2)}{\pi r}$ (b) $\pi r + \frac{2V}{\pi r}$ (c) $\pi r^2 + \frac{2V(\pi+2)}{\pi r}$ (d) $2\pi r^2 + \frac{2V(\pi+2)}{\pi r}$

Ans. (c)

Sol. $\because$ $V = \frac{1}{2}\pi r^2 h$

and $s = \pi rh + \pi r^2 + 2rh$

$\therefore$ $h = \frac{2V}{\pi r^2}$

$\therefore$ $s = \pi r\left(\frac{2V}{\pi r^2}\right) + \pi r^2 + 2r\left(\frac{2V}{\pi r^2}\right)$

$s = \pi r^2 + \frac{2V(\pi+2)}{\pi r}$

49. For the given half cylinder of volume V, the total surface area S is minimum, when

(a) $(\pi+2)V = \pi^2 r^3$ (b) $(\pi+2)V = \pi^2 r^2$ (c) $2(\pi+2)V = \pi^2 r^3$ (d) $(\pi+2)V = \pi^2 r$

Ans. (d)

Sol. $s = \pi r^2 + \frac{2V(\pi+2)}{\pi r}$

diff. wr. to r....

$\frac{ds}{dr} = 2\pi r + \frac{2v}{\pi}(\pi+2) \times \frac{-1}{r^2}$

For minimum

$\frac{ds}{dr} = 0$

$\Rightarrow$ $2\pi r - \frac{2v}{\pi}(\pi+2) \times \frac{1}{r^2} = 0$

$\Rightarrow$ $2\pi r = \frac{2v}{\pi}(\pi+2) \times \frac{1}{r^2}$

$\Rightarrow$ $r^3 = \frac{v(\pi+2)}{\pi^2}$

$\Rightarrow$ $(\pi + 2)v = \pi^2 r^3$

50. The ration h: 2r for S to be minimum will be equal to :

(a) $2\pi : \pi + 2$ (b) $2\pi : \pi + 1$ (c) $\pi : \pi + 1$ (d) $\pi : \pi + 2$

Ans. **(d)**

Sol. Ration $\frac{h}{2r} = \frac{\pi}{\pi + 2}$

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