Resonance® Educating for better tomorrow

CBSE 2021-22 (TERM-1)

DATE: 06-12-2021

Questions Paper

SERIES: SSJ/2 | CODE : 065/2/4 | SET-4

SUBJECT : MATHEMATICS

TIME ALLOWED: 90 MINUTES **MAXIMUM MARKS : 40**

NOTE

(i) Please check that this question paper contains 24 printed pages.

(ii) Please check that this question paper contains 50 multiple choice questions MCQs.)

(iii) QP Code given on the right hand side of the question paper should be written at the appropriate place of the OMR Sheet by the candidates.

(iv) 20 minute additional time has been allotted to read this question paper prior to actual time of commencement of examination.

General Instructions:

(i) This question paper contains 50 questions out of which 40 questions are to be attempted. All questions carry equal marks.

(ii) This question paper consists of three Sections — Section A, Section B and Section C.

(iii) Section–A – contains 20 questions. Attempt any 16 questions from Q. No. 1 to 20.

(iv) Section–B – also contains 20 questions. Attempt any 16 questions from Q. No. 21 to 40.

(v) Section–C – contains 10 questions including one Case Study. Attempt any 8 from Q. No. 41 to 50.

(vi) There is only one correct option for every Multiple Choice Question (MCQ). Marks will not be awarded for answering more than one option.

(vii) There is no negative marking.

SECTION - A

In this Section, attempt any **16** questions out of Questions **1-20**. Each question is of **One** mark.

1. Differential of log [log(log x^5)] w.r.t. x is

(a) $\frac{5}{x\log(x^5)\log(\log x^5)}$ (b) $\frac{5}{x\log(\log x^5)}$

(c) $\frac{5x^4}{\log(x^5)\log(\log x^5)}$ (d) $\frac{5x^4}{\log x^5 \log(\log x^5)}$

Ans. **(a)**

Sol. y = log [log (logx^5)]

$\frac{dy}{dx} = \frac{1}{\log(\log x^5)} \times \frac{1}{\log x^5} \times \frac{1}{x^5} \times 5x^4$

$\Rightarrow \frac{5}{x\log x^5 \log(\log x^5)}$

2. The number of all possible matrices of order 2×3 with each entry 1 or 2 is

(a) 16 (b) 6 (c) 64 (d) 24

Ans. **(c)**

Sol. Order 2 × 3
total element = 2 × 3 = 6
$2^6 = 64$

3. A function f : R → R is defined as $f(x) = x^3 + 1$. Then the function has

(a) no minimum value (b) no maximum value

(c) both maximum and minimum values (d) neither maximum value nor minimum value

Ans. **(d)**

Sol. f : R → R
$f(x) = x^3 + 1$
$f'(x) = 3x^2$
critical point
$f'(x) = 0$
$3x^2 = 0$
x = 0
Again different $f''(x) = 6x$
$f''(0) = 0$
∴ Neither maximum nor minimum

4. If sin y = x cos (a + y), then $\frac{dx}{dy}$ is

(a) $\frac{\cos a}{\cos^2(a+y)}$ (b) $\frac{-\cos a}{\cos^2(a+y)}$ (c) $\frac{\cos a}{\sin^2 y}$ (d) $\frac{-\cos a}{\sin^2 y}$

Ans. **(a)**

Sol. If siny = x cos(a + y)

$\frac{dx}{dy} = ?$

differentiation with respect to x

$$\cos y\frac{dy}{dx} = x \times -\sin(a+y)\frac{dy}{dx} + \cos(a+y) \times 1$$

$$\frac{dy}{dx} = \frac{\cos(a+y)}{\cos y + x\sin(a+y)}$$

$$\frac{dx}{dy} = \frac{\cos y + x\sin(a+y)}{\cos(a+y)}$$

$$\frac{dx}{dy} = \frac{\cos y + \frac{\sin y}{\cos(a+y)}\sin(a+y)}{\cos(a+y)}$$

$$\{\because\ x = \frac{\sin y}{\cos(a+y)}\}$$

$$\frac{dx}{dy} = \frac{\cos y\cos(a+y) + \sin y\sin(a+y)}{\cos^2(a+y)}$$

$$\Rightarrow \frac{\cos a}{\cos^2(a+y)}$$

5. The points on the curve $\frac{x^2}{9} + \frac{y^2}{25} = 1$, where tangent is parallel to x-axis are

(a) (±5, 0) (b) (0, ±5) (c) (0, ±3) (d) (±3, 0)

Ans. **(b)**

Sol. $\frac{2x}{9} + \frac{2y}{25}\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-x}{9} \times \frac{25}{y}$$

$$= -\frac{25}{9}\frac{x}{y}$$

Parallel to x-axis

$$\frac{dy}{dx} = 0$$

$$-\frac{25}{9}\frac{x}{y} = 0$$

x = 0

$$\frac{0}{9} + \frac{y^2}{25} = 1$$

$y^2 = 25$

y = ± 5

∴ Point (0, ±5)

6. Three points P(2x, x + 3), Q (0, x) and R (x + 3, x + 6) are collinear, then x is equal to

(a) 0 (b) 2 (c) 3 (d) 1

Ans. **(d)**

Sol. $\Delta = 0$

$$\frac{1}{2}\begin{vmatrix} 2x & x+3 & 1 \\ 0 & x & 1 \\ x+3 & x+6 & 1 \end{vmatrix} = 0$$

Expansion along c_1

2x (x–x–6)–0 ( ) + (x+3) (x+3–x)= 0

$\Rightarrow$ –12x + 3x + 9 = 0

$\Rightarrow$ –9x + 9 = 0

x = 1

7. The principal value of $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{\pi}{12}$ (b) π (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Ans. **(a)**

Sol. $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{\pi}{3}-\frac{\pi}{4} = \frac{4\pi-3\pi}{12} = \frac{\pi}{12}$

8. If $(x^2 + y^2)^2 = xy$, then $\frac{dy}{dx}$ is

(a) $\frac{y+4x(x^2+y^2)}{4y(x^2+y^2)-x}$ (b) $\frac{y-4x(x^2+y^2)}{x+4(x^2+y^2)}$ (c) $\frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$ (d) $\frac{4y(x^2+y^2)-x}{y-4x(x^2+y^2)}$

Ans. **(c)**

Sol. $(x^2 + y^2)^2 = xy$

$= 2(x^2 + y^2)\left[2x+2y\frac{dy}{dx}\right] = x\frac{dy}{dx}+y\times 1$

$= 4x(x^2 + y^2) + 4y(x^2+y^2)\frac{dy}{dx} = x\frac{dy}{dx}+y$

$\frac{dy}{dx}\left[4y(x^2+y^2)-x\right] = y - 4x(x^2 + y^2)$

$\frac{dy}{dx} = \frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$

9. If a matrix A is both symmetric and skew symmetric, then A is necessarily a

(a) Diagonal matrix (b) Zero square matrix
(c) Square matrix (d) Identity matrix

Ans. **(b)**

Sol. A = A'
A = –A'
2A = 0
A = 0

10. Let sec X = {1,2,3} and a relation R is defined in X as : R = {(1,3), (2,2) (3,2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are

(a) {(1,1), (2,3), (1,2)} (b) {(3,3), (3,1), (1,2)}
(c) {(1,1), (3,3), (3,1), (2,3)} (d) {(1,1), (3,3), (3,1), (1,2)}

Ans. **(c)**

Sol. X = {1,2,3}
R {(1,3), (2,2) (3,2)}
to be reflexive and symmetric
{(1,1), (3,3), (3,1), (2,3)}

11. A linear Programming Problem is as follows :

Minimise $z = 2x + y$

subject to the constraints $x \geq 3,\ x \leq 9,\ y \geq 0$

$x - y \geq 0,\ x + y \leq 14$

The feasible region has

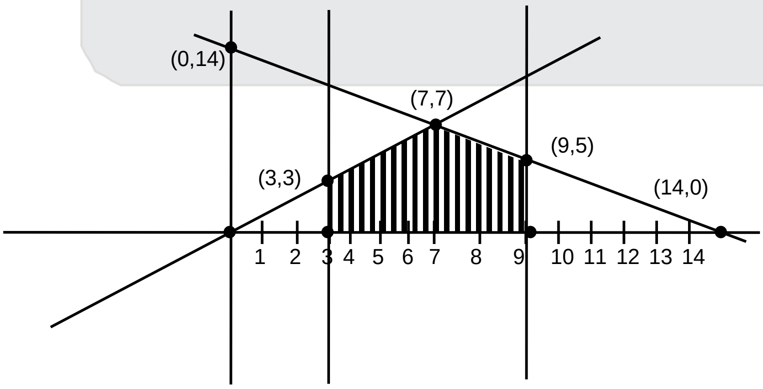
(a) 5 corner points including (0, 0) and (9,5) (b) 5 corner points including (7, 7) and (3, 3)
(c) 5 corner points including (14, 0) and (9, 0) (d) 5 corner points including (3, 6) and (9,5)

Ans. **(b)**

Sol. $z = 2x + y$

$x \geq 3,\ x \leq 9,\ y \geq 0$
$x - y \geq 0,\ x + y \leq 14$

$\Rightarrow$ x = 3 | x = 29 | x = y | (0.14)
(14,0)



Corner points (3,3), (3,0), (9,0), (9,5) and (7,7)

12. The function $f(x)=\begin{cases}\dfrac{e^{3x}-e^{5x}}{x}, & \text{if } x\neq 0\\ k, & \text{if } x=0\end{cases}$

is continuous at x = 0 for the value of k, as

(a) 3 (b) 5 (c) 2 (d) 8

Ans. **(d)**

Sol. $f(x)=\begin{cases}\dfrac{e^{3x}-e^{5x}}{x}, & \text{if } x\neq 0\\ k, & \text{if } x=0\end{cases}$

$\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{-5x}}{x}=k$ (using L hospital rule)

$\lim\limits_{x\to 0}\dfrac{3e^{3x}+5e^{-5x}}{1}=k$

3 + 5 = k

8 = k

13. If C_{ij} denotes the cofactor of element p_{ij} of the matrix $P=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & 2 & 4\end{bmatrix}$, then the value of $C_{31}.C_{23}$ is

(a) 5 (b) 24 (c) –24 (d) –5

Ans. **(a)**

Sol. $P=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & 2 & 4\end{bmatrix}$

$C_{31}=(-1)^{3+1}\begin{vmatrix}-1 & 2\\ 2 & -3\end{vmatrix}$ $C_{ij}=(-1)\,p_{ij}$

$C_{31}=3-4=(-1)$

$C_{2-3}=(-1)^{2+3}\begin{vmatrix}-1 & 1\\ 3 & 2\end{vmatrix}$

$=-1\,(2+3)=-5$

Then $c_{31}\times c_{23}\Rightarrow -1\times -5=5$

14. The function $y = x^2e^{-x}$ is decreasing in the interval

(a) (0, 2) (b) (2, ∞) (c) (–∞, 0) (d) (–∞, 0) ∪ (2, ∞)

Ans. **(d)**

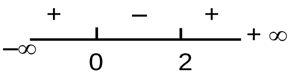
Sol. $y = x^2e^{-x}$

$\dfrac{dy}{dx}=-x^2e^{-x}+e^{-x}\times 2x$

$\frac{dy}{dx} = 0 \;\; e^{-x}\left[2x - x^2\right] = 0$

$\Rightarrow \;\; e^{-x}\alpha \;\; x\left[2 - x\right] = 0$

X = 0 and x= 2



$f'(x) < 0$

$-e^{-x}$ x [x–2] < 0

e^{-x} x (x–2) >0

$\therefore \;\; x \in (-\infty, 0) \cup (2, \infty)$

15. If R = {(x, y) : x, y $\in$ Z, $x^2 + y^2 \le 4$} is a relation in set Z, then domain of R is

(a) {0,1,2} (b) {–2, –1, 0, 1, 2} (c) {0, –1, –2} (d) {–1, 0, 1}

Ans. **(b)**

Sol. R = {(x, y) : xy$\in$z) ; $x^2 + y^2 \le 4$} ..(b)

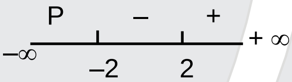
$y^2 \le 4 - x^2$

$y \le \sqrt{4 - x^2}$

$4 - x^2 \ge 0$

$-(x^2 - 4) \le 0$

$x^2 - 4 \le 0$



$(x-2)(x+2) \le 0$ { domain {–2, –1, 0, 1, 2}

16. The system of linear equations

5x + ky = 5,

3x + 3y = 5;

will be consistent if

(a) $k \neq -3$ (b) k = –5 (c) k = 5 (d) $k \neq 5$

Ans. **(d)**

Sol. 5x+ky = 5

3x+3y = 5

Will be consistent

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{5}{3} \neq \frac{k}{3}$

$\frac{5}{3} \neq \frac{k}{3} \;\; \Rightarrow \;\; k \neq 5$

17. The equation of the tangent to the curve y $(1 + x^2)$ = 2 – x where it crosses the x-axis is

(a) x – 5y = 2 (b) 5x – y = 2 (c) x + 5y = 2 (d) 5x + y = 2

Ans. **(c)**

Sol. $y(1+x^2) = 2-x$

$(1+x^2)\frac{dy}{dx} + y \times 2x = -1$

$(1+x^2)\frac{dy}{dx} = -1 - 2xy$

$\frac{dy}{dx} = \frac{-1-2xy}{1+x^2}$

Equation of tan equation

It crosses to x-axis

Points ⇒ (x, 0)
$y(1+x^2) = 2 - x$
x = 2
Point (2, 0)

$\therefore$ slope $\left(\frac{dy}{dx}\right)_{(2,0)} = \frac{-1}{5}$

$\therefore$ Equation of tangent
$\Rightarrow y - y_1 = m(x - x_1)$
$\Rightarrow y - 0 = \frac{-1}{5}(x-2)$
5y = –x+2
x + 5y = 2

18. If $\begin{bmatrix} 3c+6 & a-b \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of ab – cd is

(a) 4 (b) 16 (c) –4 (d) –16

Ans. **(a)**

Sol. $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$

3c+b=12…(i) ⇒ 2–3b=4–4…(iv)
a–d=2…(ii) ⇒ –3b=–6
a+d= – 8…(iii) ⇒ b = 2
equation (ii) and (iii) c= 2
2a = –6 ⇒ $\therefore$ ab – cd
a = –3 ⇒ $\therefore -3 \times 2 - 2 \times -5$
d= –5 ⇒ –6+10 ⇒ 4

19. The principal value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$ is

(a) $\frac{\pi}{8}$ (b) $\frac{3\pi}{8}$ (c) $-\frac{\pi}{8}$ (d) $-\frac{3\pi}{8}$

Ans. **(a)**

Sol. $\tan^{-1}\left[\tan\frac{9\pi}{8}\right]$

$\tan^{-1}\left[\tan\left(\pi+\frac{\pi}{8}\right)\right]$

$\tan^{-1}\left[\tan\frac{\pi}{8}\right] = \frac{\pi}{8}$

20. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ P – Q is

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

Ans. **(b)**

Sol. $p = \begin{bmatrix} 3 & 2 \\ -1 & 1 \\ 0 & \end{bmatrix} \quad Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$\therefore\ Q = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$\therefore$ P –Q

$\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

SECTION - B

In this Section, attempt any **16** questions out of Questions **21-40**. Each question is of **One** mark.

21. The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval

(a) $(-\infty, 2) \cup (3, \infty)$ (b) $(-\infty, 2)$ (c) $(-\infty, 2] \cup [3, \infty)$ (d) $[3, \infty)$

Ans. **(a)**

Sol. $f(x) = 2x^3 - 15x^2 + 36x + 6$

$f'(x) = 6x^2 - 30x + 36$

$f'(x) \geq 0$

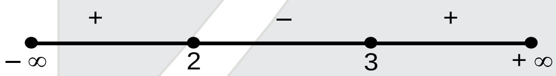
$6x^2 - 30x + 36 \geq 0$

$x^2 - 5x + 6 \geq 0$

$x^2 - 3x - 2x + 6 \geq 0$

$x(x-3) - 2(x-3) \geq 0$

$x = 3 ; 2$



$x \in (-\infty, 2] \cup [3, \infty)$

22. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then $\frac{dy}{dx}$ is

(a) $\frac{\cos\theta + \cos 2\theta}{\sin\theta - \sin 2\theta}$ (b) $\frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$ (c) $\frac{\cos\theta - \cos 2\theta}{\sin\theta - \sin 2\theta}$ (d) $\frac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta}$

Ans. **(b)**

Sol. $x = 2\cos\theta - \cos 2\theta$

$\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$

$y = 2\sin\theta - \sin 2\theta$

$\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\frac{dy}{dx} = \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$

23. What is the domain of the function $\cos^{-1}(2x - 3)$?

(a) $[-1, 1]$ (b) $(1, 2)$ (c) $(-1, 1)$ (d) $[1, 2]$

Ans. **(d)**

Sol. $\cos^{-1}(2x - 3)$

domain.

$\Rightarrow \quad -1 \le 2x - 3 \le 1$

$\Rightarrow \quad 2 \le 2x \le 4$

$\Rightarrow \quad 1 \le x \le 2$

$[1, 2]$

24. A matrix $A = [a_{ij}]_{3\times 3}$ is defined by

$$a_{ij} = \begin{cases} 2i + 3j, i < j \\ 5 \quad , i = j \\ 3i - 2j, i > j \end{cases}$$

The number of elements in A which are more than 5, is

(a) 3 (b) 4 (c) 5 (d) 6

Ans. **(b)**

Sol. $A = [a_{ij}]_{3\times 3}$

$$a_{ij} \begin{bmatrix} 2i + 3j & i < j \\ 5 & i = j \\ 3i - 2j & i > j \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{12} \quad \Rightarrow \quad 2 \times 1 + 3 \times 2$

$\quad \Rightarrow \quad 2 + 6 \Rightarrow 8$

$a_{13} \quad \Rightarrow \quad 2 \times 1 + 3 \times 3$

$\quad \Rightarrow \quad 2 + 9 \Rightarrow 11$

$a_{21} \quad \Rightarrow \quad 3 \times 2 - 2 \times 1$

$\quad = \quad 4$

$a_{23} \quad \Rightarrow \quad 2 \times 2 + 3 \times 3$

$\quad = \quad 4 + 9$

$\quad = \quad 13$

a_{31} Þ $3 \times 3 - 2 \times 1$

$\quad = \quad 9 - 3$

$\quad = \quad 7$

$a_{32} \quad \Rightarrow \quad 3 \times 3 - 2 \times 2$

$\quad = \quad 9 - 4$

$\quad = \quad 5$

$$A = \begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$$

25. If function f defined by

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases}$$

is continuous at $x=\frac{\pi}{2}$, then the value of k is

(a) 2 (b) 3 (c) 6 (d) – 6

Ans. **(c)**

Sol. $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & x=\dfrac{\pi}{2}\\ 3, & x=\dfrac{\pi}{2}\end{cases}$

$$\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x} = 3$$

$$\lim_{x\to\frac{\pi}{2}}\frac{-k\cos x}{-2}=3$$

$$\frac{-k}{-2}=3$$

k = 6

26. For the matrix $X=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$, (X^2-X) is

(a) 2I (b) 3I (c) I (d) 5I

Ans. **(a)**

Sol. $x=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$

x^2-x

$$\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}-\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix}-\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}\Rightarrow\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}\Rightarrow 2I$$

27. Let $X=\{x^2 : x\in N\}$ and the function $f : N\to X$ is defined by $f(x)=x^2$, $x\in N$. Then this function is

(a) injective only (b) not bijective (c) surjective only (d) bijective

Ans. **(d)**

Sol. $x=\{x^2 : x\in N\}=\{1, 4, 9 \ldots\ldots\}$

$f : N\to X$

$f(x) = x^2$; $x \in N$
one one $f(x_1) = f(x_2)$
$\Rightarrow x_1^2 = x_2^2$
$\Rightarrow x_1 = \pm x_2$
$x_1 = x_2$ one - one
onto
$y = x^2$
$x = \sqrt{y}$ $\qquad y \in \{1,4,9\ldots\}$
$y \in$ co-domain
exist pre- in ………..
$\therefore$ onto
$\therefore$ function bijective

28. The corner points of the feasible region for a Linear Programming problem are P(0,5), Q(1,5), R (4,2) and S(12,0) . The minimum value of the objective function Z = 2x + 5y is at the point

A) P (b) Q (c) R (d) S

Ans. **(c)**

Sol. $z = 2x + 5y$
points $P(0, 5) \Rightarrow z = 25$
$Q (1, 5) \Rightarrow z = 2 \times 1 + 25 = 27$
$R (4, 2) \Rightarrow z = 8 + 10 = 18$
$S (12, 0)$ $z = 24 + 0 = 24$

29. The equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3) is

(a) $2y - 3mx + am^3 = 0$ (b) $2x + 3my - 3am^4 - am^2 = 0$
(c) $2x + 3my + 3am^4 - 2am^2 = 0$ (d) $2x + 3my - 3am^4 - 2am^2 = 0$

Ans. **(d)**

Sol. $ay^2 = x^3$ point (am^2, am^3)

$$= a \times 2y\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ya}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(am^2, am^3)} = \frac{3 \times a^2m^4}{2 \times am^3 \times a} = \frac{3}{2}m$$

$$\text{Slope f normal} = \frac{-1}{\left(\frac{dy}{dx}\right)} = \frac{-2}{3m}$$

$$\therefore \mathbf{y - y_1 =} \frac{-1}{\left(\frac{dy}{dx}\right)}(x - x_1)$$

$$y - am^3 = \frac{-2}{3m}(x - am^2)$$

$3my - 3am^4 = -2x + 2am^2$
$2x + 3my - 3am^4 - 2am^2 = 0$

30. If A is a square matrix of order of 3 and |A| = –5, then |adj A| is

(a) 125 (b) – 25 (c) 25 (d) ± 25

Ans. **(c)**

Sol. |A| = – 5

|adj A| = $|A|^{n-1}$

= $|A|^{3-1}$

= $(-5)^2 \Rightarrow 25$

31. The simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ is

(a) $\frac{\pi}{4}-\frac{x}{2}$ (b) $\frac{\pi}{4}+\frac{x}{2}$ (c) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$ (d) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$

Ans. **(c)**

Sol. $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$

put x = cos 2θ

$\Rightarrow \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$

$\Rightarrow \tan^{-1}\left[\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right]$

$\Rightarrow \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right]$

$\Rightarrow \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$

$\Rightarrow \frac{\pi}{4}-\theta$ $\left\{\begin{array}{l} x=\cos 2\theta \\ \frac{1}{2}\cos^{-1}x=\theta \end{array}\right.$

$\Rightarrow \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

32. If for the matrix $A=\begin{bmatrix}\alpha & -2\\ -2 & \alpha\end{bmatrix}$. $|A^3| = 125$, then the value of α is

(a) ± 3 (b) – 3 (c) ±1 (d) 1

Ans. **(a)**

Sol. $A=\begin{bmatrix}\alpha & -2\\ -2 & \alpha\end{bmatrix}$

$|A^3| = 125$

$\Rightarrow |A^3| = 125$

$|A| = 5$

$\alpha^2 - 4 = 5$

$\alpha^2 = 9$

$\alpha = \pm 3$

33. If y = sin (m $\sin^{-1}$ x), then which one of the following equations is true ?

(a) $(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+m^2y=0$
(b) $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$
(c) $(1+x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$
(d) $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2x=0$

Sol. **(b)**

y = sin (m $\sin^{-1}$ x)

$$\frac{dy}{dx}=\cos(m\sin^{-1}x)\times m\times\frac{1}{\sqrt{1-x^2}}$$

$\Rightarrow$ $\sqrt{1-x^2}\ \frac{dy}{dx}=m\cos[m\sin^{-1}x]$

$\Rightarrow$ $\sqrt{1-x^2}\ \frac{d^2y}{dx^2}+\frac{dy}{dx}\times\frac{1}{2\sqrt{1-x^2}}\times -2x$ = m × –sin [m $\sin^{-1}$x] x m $\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow$ $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$ = – m^2 sin [m $\sin^{-1}$x]

$\Rightarrow$ $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$ = – m^2y

34. The principal value of $[\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})]$ is

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

Sol. **(b)**

$\left[\tan^{-1}\sqrt{3}-\cos^{-1}\left(-\sqrt{3}\right)\right]$

$\Rightarrow$ $\frac{\pi}{3}-\left\{\pi-\cos^{-1}\sqrt{3}\right\}$

$\Rightarrow$ $\frac{\pi}{3}-\left\{\pi-\frac{\pi}{6}\right\}$

= $\frac{\pi}{3}-\left\{\frac{5\pi}{6}\right\}$

= $\frac{2\pi-5\pi}{6}=\frac{-3\pi}{6}=\frac{-\pi}{2}$

35. The maximum value of $\left(\frac{1}{x}\right)^x$ is

(a) $e^{1/e}$ (b) e (c) $\left(\frac{1}{e}\right)^{1/e}$ (d) e^e

Sol. **(a)**

$f(x)=\left(\frac{1}{x}\right)^x$

$y=\left(\frac{1}{x}\right)^x$

$\log y = x \log\left(\frac{1}{x}\right)$

$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{\left(\frac{1}{x}\right)} \times \frac{-1}{x^2} + \log\left(\frac{1}{x}\right) \times 1$

$\frac{dy}{dx} = y\left[-1 + \log\left(\frac{1}{x}\right)\right]$

$= \left(\frac{1}{x}\right)\left[-1 + \log\left(\frac{1}{x}\right)\right]$

Critical points

$\frac{dy}{dx} = 0$

$-1 + \log x^{-1} = 0$

$\Rightarrow$ $-1 - \log x = 0$

$\log x = -1$

$x = e^{-1}$

$x = \frac{1}{e}$

hence maximum value

$y = \left(\frac{1}{\left(\frac{1}{e}\right)}\right)^{\frac{1}{e}}$

$y = (e)^{\frac{1}{e}}$

36. Let matrix X = $[x_{ij}]$ is given by $X = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$. Then the matrix Y = $[m_{ij}]$, where m_{ij} = Minor of x_{ij}, is

(a) $\begin{bmatrix} 7 & -5 & -3 \\ 19 & 1 & -11 \\ -11 & 1 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -19 & -11 \\ 5 & -1 & -1 \\ 3 & 11 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 19 & -11 \\ -3 & 11 & 7 \\ -5 & -1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

Sol. (d)

$x = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$

y = $[m_{ij}]$ where m_{ij} = minor of x_{ij} is

M_{11} minor of 1 = $\begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix}$ = 12 – 5 = 7

M_{12} = $\begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} \Rightarrow$ 9 + 10 = 19

M_{13} = $-3 - 8 \Rightarrow -11$

M_{21} = $-3 + 2 \Rightarrow -11$

M_{22} = $3 - 4 = -1$
M_{23} = $-1 + 2 = 1$
M_{31} = $5 - 8 = -3$
M_{32} = $-5 - 6 = -11$
M_{33} = $4 + 3 = 7$

$\therefore \quad y = \begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

37. A function f : R → R defined by $f(x) = 2 + x^2$ is
(a) not one-one (b) one-one
(c) not onto (d) neither one-one nor onto

Ans. **(d)**

Sol. $f : R \to R$
$f(x) = 2 + x^2$
one – one
$f(x_1) = f(x_2)$
$\Rightarrow \quad 2 + x_1^2 = 2 + x_2^2$
$\Rightarrow \quad x_1 = \pm x_2$ many one
onto let $y = f(x)$
$y = 2 + x^2$
$x^2 = y - 2$
$x = \sqrt{y-2}$ $y \in R$ (co-domain)
If y = 1
$x = \sqrt{-1} \notin R$ domain
Neither one-one nor onto

38. A Linear Programming Problem is as follows:
Maximise / Minimise objective function $Z = 2x - y + 5$
Subject to the constraints
$3x + 4y \le 60$
$x + 3y \le 30$
$x \ge 0, y \ge 0$
If the corner points of the feasible region are A (0, 10), B(12, 6), C(20, 0) and 0(0, 0), then which of the following is true?
(a) Maximum value of Z is 40
(b) Minimum value of Z is — 5
(c) Difference of maximum and minimum values of Z is 35
(d) At two corner points, value of Z are equal

Ans. **(b)**

Sol. $Z = 2x - y + 5$
corners points A (0,10)
$z \Rightarrow -10 + 5$
$\Rightarrow -5$ min.
B(12, 6) $z \Rightarrow 24 - 6 + 5$
$\Rightarrow 18 + 5 = 23$
C (20,0) $Z = 40 - 0 + 5$
$Z = 45$ max.
O(0,0) $Z = 5$

39. If x = – 4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is

(a) 4 (b) – 3 (c) 2 (d) 5

Ans. **(a)**

Sol. $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$

$\Rightarrow x(x^2 - 2) - 2(x - 3) + 3(2 - 3x) = 0$

$\Rightarrow x^3 - 2x - 2x + 6 + 6 - 9x = 0$

$\Rightarrow x^3 - 13x + 12 = 0$

x = – 4 is one root

(x + 4) is factor

$x^2 - 4x + 3$

$$\begin{array}{r|l} & x^2 - 4x + 3 \\ x + 4 & x^3 - 13x + 12 \\ & x^3 + 4x^2 \\ & - \quad - \\ \hline & -4x^2 - 13x + 12 \\ & -4x^2 - 16x \\ & + \quad + \\ \hline & 3x + 12 \\ & 3x + 12 \\ \hline & 0 \end{array}$$

Factorization

$\Rightarrow x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x - 3) - 1(x - 3) = 0$

$(x - 3)(x - 1) = 0$

x = 3, 1

sum of roots $\Rightarrow 3 + 1 = 4$

40. The absolute maximum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in the interval $\left[-2, \frac{9}{2}\right]$ is

(a) 8 (b) 9 (c) 6 (d) 10

Ans. **(a)**

Sol. $f(x) = 4x - \frac{1}{2}x^2 \quad x \in \left[-2, \frac{9}{2}\right]$

$f'(x) = 4 - \frac{1}{2} \times 2x$

$f'(x) = 4 - x = 0$

$\Rightarrow x = 4$

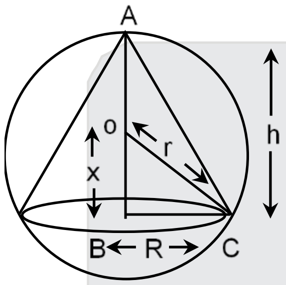
SECTION - C

In this Section, attempt any **8** questions out of Questions **41-50**. Each question is of **One** mark.

41. In a sphere of radius r, a right circular cone of height h having maximum curved surface area is inscribed. The expression for the square of curved surface of cone is

(a) $2\pi^2 rh\,(2rh + h^2)$ (b) $\pi^2 hr\,(2rh + h^2)$ (c) $2\pi^2 r\,(2rh^2 - h^3)$ (d) $2\pi^2 r^2\,(2rh - h^2)$

Ans. **(c)**



Let the radius of cone = R

height = h

radius of sphere = r

Let OB = x m.

∴ curve surface area of cone = $\pi R\ell \Rightarrow$ $\ell\ \sqrt{R^2 + h^2}$

$A = \pi R\ell$

$A = \pi\ R\sqrt{R^2 + h^2}$

squaring both side

$A^2 = \pi^2 R^2\,(R^2 + h^2)$-------(1)

$A^2 = \pi^2\,(r^2 - x^2)\left[r^2 - x^2 + h^2\right]$

$A^2 = \pi^2\,(r^2 - x^2)\left[r^2 - x^2 + r^2 + x^2 + 2rx\right]$

$A^2 = \pi^2\,(r^2 - x^2)\left[2r^2 + 2rx\right]$

deff. wr to x –

$$\frac{dA^2}{dx} = \pi^2\left[2r^3 - 6rx^2 - 4xr^2\right]$$

for maximum $\frac{dA^2}{dx} = 0$

$\Rightarrow$ $2r\ \left[r^2 - 3x^2 - 2xr\right] = 0$

$\Rightarrow$ $(x + r)\ (3x - r) = 0$

$x = -r$ or $r = 3x$

Not possible

at point $x = \frac{r}{3}$

$A^2 = 2\pi^2 r\left[2rh^2 - h^3\right]$

we know

$h = r + x$

and ΔOBC $r^2 = x^2 + R^2$

$R^2 = r^2 - x^2$

$h = r + x \Rightarrow h^2 = r^2 + x^2 + 2rx$

42. The corner points of the feasible region determined by a set of constraints (linear inequalities) are P(0,5), Q (3,5), R(5,0) and S(4,1) and the objective function is Z = ax + 2by where a, b > 0 . The condition on a and b such that the maximum Z occurs at Q and S is

(a) a – 5b = 0 (b) a – 3b = 0 (c) a – 2b = 0 (d) a – 8b = 0

Ans. **(d)**

Sol. z = ax + 2by

corner points p(0,5) Q(3,5) R(5, 0) S(4,1)

z maxi occurs at point Q and S

Z_{max} = ax + 2by at Q(3,5)

= 3a + 10 yb

Z_{max} = 4a + 2b at S(4,1)

$\Rightarrow$ 3a + 10b = 4a + 2b

$\Rightarrow$ 8b = a

$\Rightarrow$ a – 8b = 0

43. If curves $y^2 = 4x$ and xy = c cut at right angles then the value of c is

(a) $4\sqrt{2}$ (b) 8 (c) $2\sqrt{2}$ (d) $-4\sqrt{2}$

Sol. **(a)**

curve $y^2 = 4x$ and

xy = c

cut right angle

then $m_1 \times m_2 = -1$

$y^2 = 4x$ xy = c

$2y\frac{dy}{dx} = 4$ $x\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = \frac{2}{y}$ $\frac{dy}{dx} = \frac{-y}{x}$

$\therefore$ $\frac{2}{y} \times \frac{-y}{x} = -1$

x = 2

$\therefore$ $y^2 = 4x$

$y^2 = 4 \times 2$

$\Rightarrow$ $y^2 = 8$

$y = \pm 2\sqrt{2}$

hence c = xy = $2\sqrt{2}$

44. The inverse of the matrix $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(a) $24\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$ (b) $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

Ans. **(d)**

Sol. $x = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$x^{-1} = \frac{adj x}{|x|}$

$$|x| = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix}$$

$|x| = 2(12 - 0) - 0 + 0$

$|x| = 24$ copaertor or 2

$$\text{adj}x = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}^T \qquad \begin{cases} C_{11} \;\; (-1)^{1+1} m_{11} \\ C_{11} = \;\; 12 \\ C_{22} = (-1)^4 m_{22} \end{cases}$$

$$\text{adj}x = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

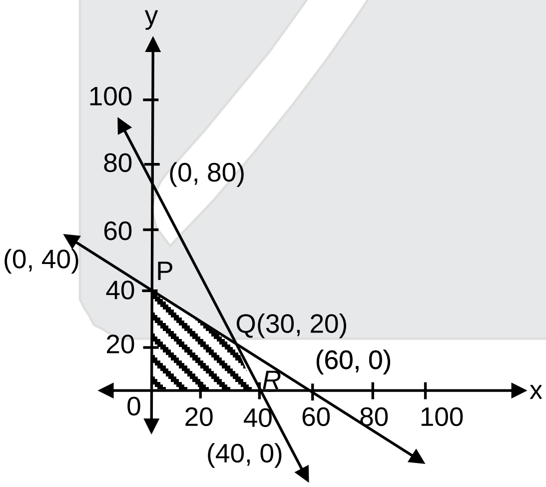
$C_{22} = 8$

$C_{33} = (-1)^6 m_{33}$
$C_{33} = 6$

$$\therefore x^{-1} = \frac{1}{24}\begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$x^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

45. For an L.P.P. the objective function is Z = 4x + 3y, and feasible region determined by a set of constraints (linear in equations) is shown in the graph.



Which one of the following statements is true?

(a) Maximum value of Z is at R
(b) Maximum value of Z is at Q
(c) Value of Z at R is less than the value P
(d) Value of Z at Q is less than the value of R.

Ans. **(b)**

Sol. z = 4x + 3y

Corner point : z = 4x + 3y

R (40,0) $\Rightarrow$ z = 160
Q (30,20) $\Rightarrow$ z = 180 max
P (0,40) $\Rightarrow$ z = 120
O(40,0) $\Rightarrow$ z = 0

Case Study

In a residential society comprising of 100 house there were 60 children between the ages of 10-15 years. They were inspired by their teachers to start composting to ensure that biodegradable waste is recycled. For the purpose, instead of each child doing it for only his/her house, children convinced the Residents welfare association to do it as a society initiative. For this they identified a square area in the local park. Local authorities charged amount of Rs. 50 per square metre for space so that there is no misuse of the space and Resident welfare association takes it seriously. Association hired a labourer for digging out 250 m^3 and he charged ` 400 × $(\text{depth})^2$. Association will like to have minimum cost.

Compost Pit

Based on this information, answer the any 4 of the following question.

46. Let side of square plot is x m and its depth is h metres, then cost c for the pit is

(a) $\frac{50}{h}+400h^2$ (b) $\frac{12500}{h}+400h^2$ (c) $\frac{250}{h}+h^2$ (d) $\frac{250}{h}+400h^2$

Ans. **(b)**

Sol. Local authority charged amount 50 Rs/m^2. Labourer for digging one 250 m^3 and charged 400 Rs × $(\text{depth})^2$ side of square plot = x m.

depth = h meter.

Volume = x × x × h = 250

$x^2 \times h = 250$

$$x^2 = \frac{250}{h}$$

Total cost:

$$C=\frac{250}{h}\times 50\text{Rs}+400\times(h)^2 \quad =\frac{12500}{h}+400h^2$$

47. Value of h (in m) for which $\frac{dc}{dh}=0$

(a) 1.5 (b) 2 (c) 2.5 (d) 3

Ans. **(c)**

Sol. $\therefore C = \frac{12500}{h}+400h^2$

$$\frac{dc}{dh}=12500\times-\frac{1}{h^2}+800h$$

$$\frac{dc}{dh}=0$$

$$12500\times-\frac{1}{h^2}+800h=0$$

$$h^3=\frac{125}{8} \Rightarrow \quad h=\frac{5}{2}\Rightarrow 2.5\text{m}.$$

48. $\frac{d^2c}{dh^2}$ is given by

(a) $\frac{25000}{h^3}+800$ (b) $\frac{500}{h^3}+800$ (c) $\frac{100}{h^3}+800$ (d) $\frac{500}{h^3}+2$

Ans. **(a)**

Sol. $\frac{dc}{dh}=\frac{-12500}{h^2}+800h$

Again differentiate

$\frac{d^2c}{dh^2}=\frac{2500}{h^3}+800$

49. Value of x (in m) for minimum cost is

(a) 5 (b) $10\sqrt{\frac{5}{3}}$ (c) $5\sqrt{5}$ (d) 10

Ans. **(d)**

Sol. $\therefore x^2 = \frac{250}{h}$

$h=\frac{5}{2}$

$x^2=\frac{250}{5/2}\Rightarrow\frac{500}{5}=100$

$x^2 = 100$

$x=\sqrt{100}=10m$

50. Total minimum cost of digging the pit (in Rs.) is

(a) 4,100 (b) 7,500 (c) 7,850 (d) 3,220

Ans. **(b)**

Sol. $C=\frac{12500}{h}+400h^2$

putting

$h=\frac{5}{2}$

$C=\frac{12500}{5/2}+400\times\frac{25}{4}$

$\Rightarrow 2500 \times 2 + 2500$

$\Rightarrow 5000 + 2500$

$\Rightarrow 7500$ Rs.

------*-----*-----*-----*-----