



INDIAN OLYMPIAD QUALIFIER (IOQ) 2023-2024
INDIAN OLYMPIAD QUALIFIER IN MATHEMATICS
(IOQM), 2023

QUESTION PAPER WITH SOLUTION

Sunday, September 03, 2023 |

Duration: 3 Hrs | Time: 10:00 AM to 1:00 PM






Max. Marks : 100

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INSTRUCTIONS

- Use of mobile phones, smartphones, iPads, calculators, programmable wrist watches is **STRICTLY PROHIBITED**. Only ordinary pens and pencils are allowed inside the examination hall.
- The correction is done by machines through scanning. On the OMR Sheet, darken bubbles completely with a **black or blue ball pen**. Please **DO NOT use a pencil or a gel pen**. Darken the bubbles completely, only after you are sure of your answer; else, erasing may lead to the OMR sheet getting damaged and the machine may not be able to read the answer. .
- The name, email address, and date of birth entered on the OMR sheet will be your login credentials for accessing your score.
- Incompletely, incorrectly or carelessly filled information may disqualify your candidature.
- Each question has a one or two digit number as answer. The first diagram below shows improper and proper way of darkening the bubbles with detailed instructions. The second diagram shows how to mark a 2-digit number and a 1-digit number.

INSTRUCTIONS		Q. 1	Q. 2		
<ol style="list-style-type: none"> "Think before your ink". Marking should be done with Blue/Black Ball Point Pen only. Darken only one circle for each question as shown in Example Below. 					
<table border="1"> <thead> <tr> <th>WRONG METHODS</th> <th>CORRECT METHOD</th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> </tr> </tbody> </table> <ol style="list-style-type: none"> If more than one circle is darkened or if the response is marked in any other way as shown "WRONG" above, it shall be treated as wrong way of marking. Make the marks only in the spaces provided. Carefully tear off the duplicate copy of the OMR without tampering the Original. Please do not make any stray marks on the answer sheet. 				WRONG METHODS	CORRECT METHOD
WRONG METHODS	CORRECT METHOD				

- The answer you write on OMR sheet is irrelevant. The darkened bubble will be considered as your final answer.
- Questions 1 to 10 carry 2 marks each; questions 11 to 20 carry 3 marks each; questions 21 to 30 carry 5 marks each.
- All questions are compulsory.
- There are no negative marks.
- Do all rough work in the space provided below for it. You also have blank pages at the end of the question paper to continue with rough work.
- After the exam, you may take away the Candidate's copy of the OMR sheet.
- Preserve your copy of OMR sheet till the end of current Olympiad season. You will need it later for verification purposes.
- You may take away the question paper after the examination.

1. Let n be a positive integer such that $1 \leq n \leq 1000$. Let M_n be the number of integers in the set $X_n = \sqrt{4n+1}, \sqrt{4n+2}, \dots, \sqrt{4n+1000}$. Let $a = \max \{M_n : 1 \leq n \leq 1000\}$, and $b = \min \{M_n : 1 \leq n \leq 1000\}$. Find $a - b$

Ans. 22

Sol. $x_n = \{\sqrt{4n+1}, \sqrt{4n+2}, \dots, \sqrt{4n+1000}\}$

for $n = 1$

$$\sqrt{5}, \sqrt{6}, \dots, \sqrt{1004}$$

Number of perfect squares = 29 = a

⋮

⋮

for $n = 1000$

$$\sqrt{4001}, \sqrt{4002}, \dots, \sqrt{5000}$$

Number of perfect squares = 7 = b

Because when gap is same b/w two number the number of perfect squares b/w two smaller number will always be greater then or equal to the number of perfect squares b/w two bigger number.

$$\therefore a - b = 29 - 7$$

$$a - b = 22$$

2. Find the number of elements in the set $\{(a, b) \in \mathbb{N} : 2 \leq a, b \leq 2023, \log_a(b) + 6\log_b(a) = 5\}$

Ans. 54

Sol. $\log_a b + 6\log_b a = 5$

$$\Rightarrow \log_a + \frac{6}{\log_a b} = 5$$

$$\Rightarrow (\log_a b)^2 - 5(\log_a b) + 6 = 0$$

$$\Rightarrow (\log_a b - 3)(\log_a b - 2) = 0$$

$$\Rightarrow (\log_a b = 2) \text{ or } \log_a b = 3$$

$$\Rightarrow b = a^2 \text{ or } b = a^3$$

Now $(a, b) \in$ and $2 \leq a, b \leq 2023$

$$\Rightarrow b = a^2 = 2^2, 3^2, 4^2, 5^2, 6^2, \dots, 44^2$$

$$\text{Or } b = a^3 = 2^3, 3^3, 4^3, 5^3, \dots, 12^3$$

So, number of elements in the set = 43 + 11 = 54

3. Let α and β be positive integers such that $\frac{16}{37} < \frac{\alpha}{\beta} < \frac{7}{16}$.

Find the smallest possible value of β .]

Ans. 23

Sol. $\frac{16}{37} < \frac{\alpha}{\beta} < \frac{7}{16}$

$$16\beta < 37\alpha \quad 16\alpha < 7\beta$$

$$\beta < \frac{37}{16}\alpha \quad 7\beta > 16\alpha$$

$$\beta > \frac{16}{7}\alpha$$

$$\frac{16\alpha}{7} < \beta < \frac{37\alpha}{16}$$

For $\alpha = 1, 2, 3, \dots, 9$, $\beta \notin \mathbb{I}^+$

at $\alpha = 10$
 $22.8571 < \beta < 23.125$
 $\beta = 23$

4. Let x, y be positive integers such that $x^4 = (x-1)(y^3 - 23) - 1$.
 Find the maximum possible value of $x + y$.

Ans. 07

Sol. $x^4 - (x-1)(y^3 - 23) - 1$
 $x^4 - 1 = (x-1)(y^3 - 23) - 2$
 $(x-1)[(x+1)(x^2+1) - (y^3 - 23)] = -2$
 $= -1 \times 2 = 1 \times x - 2$
 If $x-1 = -1 \Rightarrow x = 0$ Rejected
 If $x-1 = 1 \quad (x+1)(x^2+1) - (y^3 - 23) = -2$
 $x = -2 \quad 3 \times 5 - y^3 + 23 = -2$
 $38 - y^3 = -2 \Rightarrow y^3 = 40$ Rejected
 If $x-1 = 2 \Rightarrow x = 3$
 $\Rightarrow 4 \cdot 10 - y^3 + 23 = -1 \Rightarrow y^3 = 64$
 $y = 4$
 So maximum possible value of $x + y = 3 + 4 = 7$

5. In a triangle ABC, let E be the midpoint of AC and F be the midpoint of AB. The medians BE and CF intersect at G. Let Y and Z be the midpoints of BE and CF respectively. If the area of triangle ABC is 480, find the area of triangle GYZ.

Ans. 10

Sol. By mid point theorem

$FE \parallel BC$ & $FE = \frac{1}{2}BC \rightarrow (1)$

\Rightarrow F ECB is a parallelogram

$\text{ar}(\Delta BGC) = \frac{1}{3} \text{ar}(\Delta ABC)$

$= \frac{1}{3} \times 480$

$= 160$

In trapezium FEGB, since Y & Z are mid-point of diagonal

$\Rightarrow YZ \parallel BC$ & $YZ \parallel EF$

& $YZ = \frac{1}{2}(BC - EF)$

$= \frac{1}{2}\left(BC - \frac{1}{2}BC\right)$

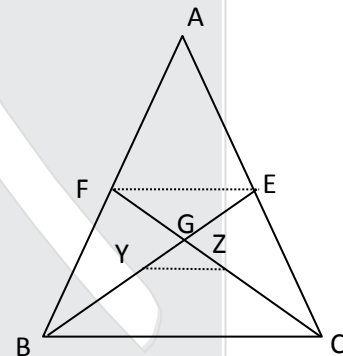
$YZ = \frac{1}{4}BC$

$\Delta GYZ \sim \Delta GBC$

$\frac{\text{ar}(\Delta GYZ)}{\text{ar}(\Delta GBC)} = \left(\frac{YZ}{BC}\right)^2$

$\frac{\text{ar}(\Delta GYZ)}{160} = \frac{1}{16}$

$\text{ar}(\Delta GYZ) = 10$



6. Let X be the set of all even positive integers n such that the measure of the angle of some regular polygon is n degrees. Find the number of elements in X .

Ans. 16

Sol. $n = \frac{180(P-2)}{P}$

Where P is number of sides of polygon $P \geq 3$ $\{p \in \mathbb{I}^+\}$

Total number of factors of 180

$$= (2 + 1) (2 + 1) (1 + 1)$$

$$= 3 \times 3 \times 2$$

$$= 18$$

Now

For $P \geq 3$

$$\text{Total number of factors of } 180 = 18 - 2 = 16$$

7. Unconventional dice are to be designed such that the six faces are marked with numbers from 1 to 6 with 1 and 2 appearing on opposite faces. Further, each face is colored either red or yellow with opposite faces always of the same color. Two dice are considered to have the same design if one of them can be rotated to obtain a dice that has the same numbers and colors on the corresponding faces as the other one. Find the number of distinct dice that can be designed.

Ans. 96

Sol. Arrangement of 3, 4, 5, 6 can be done in $3!$ ways = 6 (By circular permutation)

Colouring can be done in $2 \times 2 \times 2 = 8$ ways

$$\text{Total design are} = 8 \times 6 \times 2 = 96 \text{ ways}$$

8. Given a 2×2 tile and seven dominoes (2×1 tile), find the number of ways of tiling (that is, cover without leaving gaps and without overlapping of any two tiles) a 2×7 rectangle using some of these tiles.

Ans. 59

Sol. **Case-I:-** If we use only dominoes

For $2 \times n$ rectangle we get

Recursion formula as $f(n) = f(n-1) + f(n-2)$

$$\text{Where } f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 5, f(5) = 8, f(6) = 13, f(7) = 21$$

Case-II:- When 2×2 tile is used

$$2 \times (f(5) + f(1) \times f(4) + f(2) \times f(3))$$

$$2 \times (8 + 1 \times 5 + 2 \times 3)$$

$$38$$

$$\text{Total} = 21 + 38 = 59$$

9. Find the number of triples (a, b, c) of positive integers such that

(a) ab is a prime;

(b) bc is a product of two primes;

(c) abc is not divisible by square of any prime and

(d) $abc \leq 30$.

Ans. 14

Sol. (i) ab is a prime

(ii) bc is product of two prime

(iii) abc is not divisible by square of any prime and

(iv) $abc \leq 30$

Case-1

clearly from (i) & (ii) $a = 1$ and b is prime

from (i) & (iii) $b \neq c$

hence by (i), (ii), (iii) & (iv) we have

$$a = 1,$$

$$\text{If } b = 2 \text{ then } c = 3, 5, 7, 11, 13$$

- b = 3 then c = 2, 5, 7
- b = 5 then c = 2, 3
- b = 7 then c = 2, 3
- b = 11 then c = 2
- b = 13 then c = 2

Hence number of triplets (a, b, c) are → 14

Case-2

When b = 1, a – Prime

b = 1 (a, b, c) = (2, 1, 15), (3, 1, 10), (5, 1, 6)

so total = 14 + 3 = 17 triples

10. The sequence $\{a_n\}_{n \geq 0}$ is defined by $a_0 = 1$, $a_1 = -4$ and $a_{n+2} = -4a_{n+1} - 7a_n$, for $n \geq 0$. Find the number of positive integer divisors of $a_{50}^2 - a_{49}a_{51}$.

Ans. 51

Sol.

given:-

$$a_0 = 1$$

$$a_1 = -4$$

$$a_{n+2} = -4a_{n+1} - 7a_n$$

$$x^2 + 4x + 7 = 0$$

Let x_1 and x_2 are roots

$$x_1 = -2 + \sqrt{3}i \quad x_1 + x_2 = -4$$

$$x_2 = -2 - \sqrt{3}i \quad x_1 + x_2 = -4$$

Let $a_n = p(x_1)^n + q(x_2)^n$

at $n = 0$ $a_0 = p + q = 1$

$$p + q = 1 \dots\dots (1)$$

at $n = 1$ $a_1 = p(x_1) + q(x_2)$

$$-4 = -2(p+q) + \sqrt{3}i(p-q)$$

$$p-q = \frac{2i}{\sqrt{3}} \dots\dots(1)$$

From eqⁿ (1) and (2)

$$p = \frac{1}{2} + \frac{i}{\sqrt{3}} \text{ and } q = \frac{1}{2} - \frac{i}{\sqrt{3}}$$

$$a_{50}^2 - a_{49} \cdot a_{51} = (p(x_1)^{50} + q(x_2)^{50})^2 - (p(x_1)^{49} + q(x_2)^{49})(p(x_1)^{51} + q(x_2)^{51})$$

$$= 2pq(x_1x_2)^{50} - pq(x_1^{49} \cdot x_2^{51}) - pqx_1^{51} \cdot x_2^{49}$$

$$= 2 \times \frac{7}{12} \times 7^{50} - \frac{7}{12}(x_1x_2)^{49}(x_2^2 + x_1^2)$$

$$= \frac{7^{51}}{6} - \frac{7}{12}(7)^{49} \times 2$$

$$\frac{7^{51} - 7^{50}}{6} = 5^{50}$$

Number of positive integer divisors = 51

11. A positive integer m has the property that m^2 is expressible in the form $4n^2 - 5n + 16$ where n is an integer (of any sign). Find the maximum possible value of $|m - n|$.

Ans. 14

Sol. $m^2 = 4n^2 - 5n + 16$

$$\Rightarrow 16m^2 = 64n^2 - 80n + 256$$

$$\Rightarrow (4m)^2 = (8n - 5)^2 + 231$$

$$\Rightarrow (4m)^2 - (8n - 5)^2 = 231$$

$$\Rightarrow (4m - 8n + 5)(4m + 8n - 5) = 231 = 3 \times 7 \times 11$$

$$\text{I} \quad \left. \begin{array}{l} 4m - 8n + 5 = 231 \\ 4m + 8n - 5 = 1 \end{array} \right\} \Rightarrow m = 29 \quad \text{but } n \notin \mathbb{I}$$

$$\text{II} \quad \left. \begin{array}{l} 4m - 8n + 5 = 1 \\ 4m + 8n - 5 = 231 \end{array} \right\} \Rightarrow m = 29, n = 15 \quad \Rightarrow (m, n) \equiv (29, 15)$$

$$\text{III} \quad \left. \begin{array}{l} 4m - 8n + 5 = -231 \\ 4m + 8n - 5 = -1 \end{array} \right\} \Rightarrow m = -29 \text{ Rej}$$

$$\text{IV} \quad \left. \begin{array}{l} 4m - 8n + 5 = -1 \\ 4m + 8n - 5 = -231 \end{array} \right\} \Rightarrow m = -29 \text{ Rej}$$

$$\text{V} \quad \left. \begin{array}{l} 4m - 8n + 5 = 3 \\ 4m + 8n - 5 = 77 \end{array} \right\} \Rightarrow m = 10; n \notin \mathbb{I}$$

$$\text{VI} \quad \left. \begin{array}{l} 4m - 8n + 5 = 77 \\ 4m + 8n - 5 = 3 \end{array} \right\} \Rightarrow m = 10, n = 4 \quad \Rightarrow (m, n) \equiv (10, -4)$$

$$\text{VII} \quad \left. \begin{array}{l} 4m - 8n + 5 = -3 \\ 4m + 8n - 5 = -77 \end{array} \right\} \Rightarrow m = -10, \text{ Rej}$$

$$\text{VIII} \quad \left. \begin{array}{l} 4m - 8n + 5 = -77 \\ 4m + 8n - 5 = -3 \end{array} \right\} \Rightarrow m = -10, \text{ Rej}$$

$$\text{IX} \quad \left. \begin{array}{l} 4m - 8n + 5 = 7 \\ 4m + 8n - 5 = 3 \end{array} \right\} \Rightarrow m = 5, n \notin \mathbb{I}$$

$$\text{X} \quad \left. \begin{array}{l} 4m - 8n + 5 = 33 \\ 4m + 8n - 5 = 7 \end{array} \right\} \Rightarrow m = 5, n = -1 \quad \Rightarrow (m, n) \in (5, -1)$$

$$\text{XI} \quad \left. \begin{array}{l} 4m - 8n + 5 = -33 \\ 4m + 8n - 5 = -7 \end{array} \right\} \Rightarrow m = -5, \text{ Rej}$$

$$\text{XII} \quad \left. \begin{array}{l} 4m - 8n + 5 = -7 \\ 4m + 8n - 5 = -33 \end{array} \right\} \Rightarrow m = -5, \text{ Rej}$$

$$\text{XIII} \quad \left. \begin{array}{l} 4m - 8n + 5 = 11 \\ 4m + 8n - 5 = 21 \end{array} \right\} \Rightarrow m = 4, n \notin \mathbb{I}$$

$$\text{XIV} \quad \left. \begin{array}{l} 4m - 8n + 5 = 21 \\ 4m + 8n - 5 = 11 \end{array} \right\} \Rightarrow m = 4, n = 0 \quad \Rightarrow (m, n) \equiv (4, 0)$$

$$\text{XV} \quad \left. \begin{array}{l} 4m - 8n + 5 = -21 \\ 4m + 8n - 5 = -11 \end{array} \right\} \Rightarrow m = -4, \text{ Rej}$$

$$\text{XVI} \quad \left. \begin{array}{l} 4m - 8n + 5 = -11 \\ 4m + 8n - 5 = -21 \end{array} \right\} \Rightarrow m = -4, \text{ Rej}$$

Hence maximum $|m - n| = 14$

12. Let $P(x) = x^3 + ax^2 + bx + c$ be a polynomial where a, b, c are integers and c is odd. Let p_i be the value of $P(x)$ at $x = i$. Given that $p_1^3 + p_2^3 + p_3^3 = 3p_1p_2p_3$, find the value of $p_2 + 2p_1 - 3p_0$.

Ans. 18

Sol. $P(x) = x^3 + ax^2 + 6x + c$

$$P(x_1) = x_1^3 + ax_1^2 + bx_1 + c = p_1$$

$$P(x_2) = x_2^3 + ax_2^2 + bx_2 + c = p_2$$

$$P(x_3) = x_3^3 + ax_3^2 + bx_3 + c = p_3$$

Now given $p_1^3 + p_2^3 + p_3^3 = 3p_1p_2p_3$

$$p_1 = p_2 = p_3 \quad \text{or} \quad p_1 + p_2 + p_3 = 0$$

case-1 $p_1 = p_2 = p_3$

$$\Rightarrow 3a + b + 7 = 0$$

$$\Rightarrow 1 + a + b + c = 8 + 4a + 2b + c = 27 + 9a + 3b + c$$

$$\Rightarrow 3a + b + 7 = 0$$

$$\Rightarrow a = -6, b = 11$$

$$5a + b + 19 = 0$$

$$\Rightarrow p(x) = x^3 - 6x^2 + 11x + c$$

$$= (x-1)(x-2)(x-3) + (c+6)$$

Hence $p_2 + 2p_1 - 3p_0 = (c+6) + 2(c+6) - 3(-6 + c+6)$

$$= 3(c+6) + 18 - 3(c+6) = 18$$

Case 2

If $p_1 + p_2 + p_3 = 0$

$$\Rightarrow 36 + 14a + 6b + 3c = 0$$

Which is not possible as c is odd

So Ans = 18

13. The ex-radii of a triangle are $10\frac{1}{2}$, 12 and 14. If the sides of the triangle are the roots of the cubic $x^3 - px^2 + qx - r = 0$, where p, q, r are integers, find the integer nearest to $\sqrt{p+q+r}$.

Ans. 58

Sol. $r_a = \frac{\Delta}{s-a}, r_b = \frac{\Delta}{s-b}, r_c = \frac{\Delta}{s-c}$

p, q, r are integer means

$a+b+c, ab+bc+ca, abc$ are also integer

$a, b, c \in \mathbb{I}$

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$\frac{1}{r} = \frac{2}{21} + \frac{1}{12} + \frac{1}{14}$$

$$\frac{1}{r} = \frac{24+21+18}{252} + \frac{63}{252} = \frac{1}{4}$$

$$r = 4$$

$$r = \frac{\Delta}{s} = 4$$

$$\Delta = 4s$$

$$r_a = \frac{\Delta}{s-a} = \frac{4s}{s-a}, \quad r_b = \frac{\Delta}{s-b} = \frac{4s}{s-b}, \quad r_c = \frac{\Delta}{s-c} = \frac{4s}{s-c}$$

$$\frac{21}{2} = \frac{4s}{s-a}, \quad 12 = \frac{4s}{s-b}, \quad 14 = \frac{4s}{s-c}$$

$$21(s-a) = 8s,$$

$$12s - 12b = 4s,$$

$$14s - 14c = 4s$$

$$21s - 21a = 8s,$$

$$8s = 12b,$$

$$10s = 14c$$

$$13S = 21a,$$

$$b = \frac{8S}{12} = \frac{2S}{3},$$

$$c = \frac{5S}{7}$$

$$a = \frac{13S}{21}$$

a, b, c are integers

$$a = \frac{13}{21}S, \quad b = \frac{2}{3}S, \quad c = \frac{5}{7}S$$

So S is multiple of LCM (2, 3, 7) so S is take S = 21

$$a = \frac{13}{21} \times 21 = 13$$

$$b = \frac{2}{3}S = \frac{2}{3} \times 21 = 14$$

$$c = \frac{5}{7}S = \frac{5}{7} \times 21 = 15$$

So a = 13, b = 14, c = 15

Now, $\sqrt{p+q+r}$

$$\sqrt{a+b+c+ab+bc+ca+abc}$$

$$\sqrt{(1+a)(1+b)(1+c)-1}$$

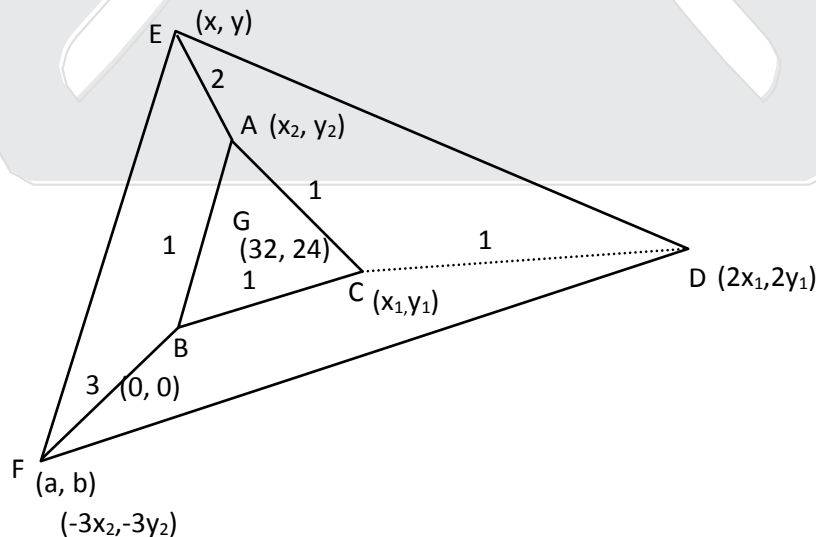
$$\sqrt{14 \times 15 \times 16 - 1}$$

$$\sqrt{3359} = 57.96$$

Nearest integers = 58

14. Let ABC be a triangle in the xy plane, where B is at the origin (0,0). Let BC be produced to D such that BC : CD = 1 : 1, CA be produced to E such that CA : AE = 1 : 2 and AB be produced to F such that AB : BF = 1 : 3. Let G(32,24) be the centroid of the triangle ABC and K be the centroid of the triangle DEF. Find the length GK.

Ans. 40
Sol.



By section formula

Coordinate of point D

D (2x₁ , 2y₁)

Let E (x , y)

$$x_2 = \frac{2x_1 + x}{3}$$

$$y_2 = \frac{2y_1 + y}{3}$$

$$3x_2 - 2x_1 = x$$

$$y = 3y_2 - 2y_1$$

$$E(3x_2 - 2x_1, 3y_2 - 2y_1)$$

Let $f(a, b)$

$$\frac{3x_2 + a}{4} = 0$$

$$0 = \frac{3y_2 + b}{4}$$

$$3x_2 + a = 0$$

$$0 = 3y_2 + b$$

$$a = -3x_2$$

$$b = -3y_2$$

$$\text{centroid of } \triangle DEF \text{ is } \left(\frac{2x_1 - 3x_2 + 3x_2 - 2x_1}{3}, \frac{3y_2 - 2y_1 - 3y_2 + 2y_1}{3} \right)$$

$$k(0, 0)$$

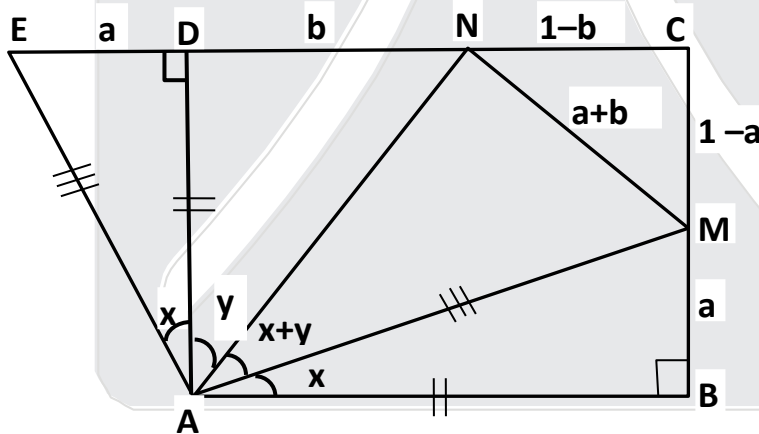
$$\text{Distance between GK} = \sqrt{(32-0)^2 + (24-0)^2}$$

$$= \sqrt{(32)^2 + (24)^2} = \sqrt{1024 + 576} = \sqrt{1600} = 40$$

15. Let ABCD be a unit square. Suppose M and N are points on BC and CD respectively such that the perimeter of triangle MCN is 2. Let O be the circumcentre of triangle MAN, and P be the circumcentre of triangle MON. If $\left(\frac{OP}{OA}\right)^2 = \frac{m}{n}$ for some relatively prime positive integers m and n, find the value of m + n.

Ans. 03

Sol. Since perimeter of $\triangle MCN = 2$



$$1-b+1-a+MN = 2$$

$$2 - a - b + MN = 2$$

$$MN = a+b$$

Extend CD to E

Such that $DE = BM = a$

$\triangle ABM \cong \triangle ADE$

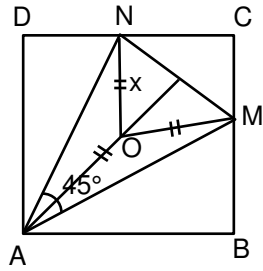
By SAS congruence criterion

By CPCT

$AE = AM$

$\angle MAB = \angle EAD = x$

Now $\triangle EAN$ and $\triangle MAN$



$EN = MN$ (a+b)
 $AE = MA$
 $AN = AN$

By SSS

$\triangle EAN \cong \triangle MAN$

So

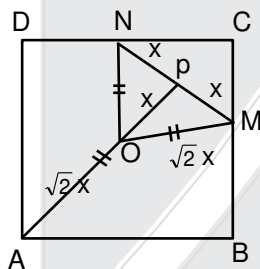
$ABCD$ is a square
 $y + x + y + x = 90$
 $2x + 2y = 90$
 $x + y = 45$

O is circumcentre so

$\angle MON = 2x \angle MAN = 2 \times 45 = 90^\circ$
 $\angle MON = 90^\circ$

So

$\triangle MON$ is a rigid \triangle
 So P (Circumcentre of $\triangle MON$)
 P is mid point of MN
 $OP = PM = PN = X$



Only one possibility for quad OMCN

So $OM = \sqrt{2}x$

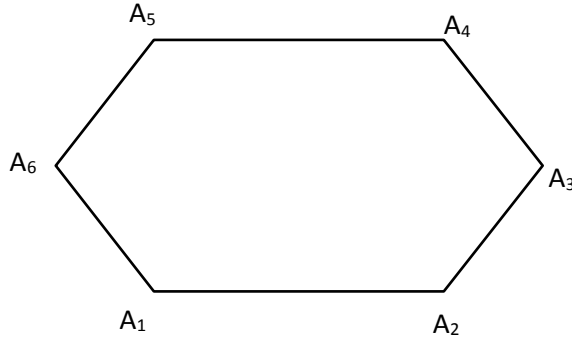
$OM = OB = OA = \sqrt{2}x$

$\left(\frac{OP}{OA}\right)^2 = \left(\frac{x}{\sqrt{2}x}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} = \frac{m}{n} = m + n = 1 + 2 = 3$

16. The six sides of a convex hexagon $A_1 A_2 A_3 A_4 A_5 A_6$ are colored red. Each of the diagonals of the hexagon is colored either red or blue. If N is the number of colorings such that every triangle $A_i A_j A_k$ where $1 \leq i < j < k \leq 6$, has at least one red side, find the sum of the squares of the digits of N.

Ans. 18

Sol. Number of diagonals = $\frac{n(n-3)}{2}$



$$= \frac{6(6-3)}{2}$$

$$= 3 \times 3$$

$$= 9$$

Case-1:-

when 2 sides red, 1 blue

Number of ways selecting two consecutive red sides = 6

So number of triangles = 6

Case-2:-

when 1 sides red, 2 blue

from side $A_1 A_2 \rightarrow$ (1) $A_1 A_4 A_2$

(2) $A_1 A_5 A_2$

From all sides = $6 \times 2 = 12$ triangles

Total triangle = 15

M-2 ${}^6C_3 - 2 [A_1 A_3 A_5 \text{ and } A_2 A_4 A_6] = 18$

17. Consider the set

$$S = \{(a, b, c, d, e) : 0 < a < b < c < d < e < 100\}$$

where a, b, c, d, e are integers. If D is the average value of the fourth element of such a tuple in the set, taken over all the elements of S , find the largest integer less than or equal to D .

Ans. 66

Sol. Total selection of a, b, c, d, e are ${}^{99}C_5$

When $d = 4$ then total ways = ${}^3C_3 \times {}^{95}C_1$

When $d = 5$ then total ways = ${}^4C_3 \times {}^{94}C_1$

.....
When $d = 98$, then total ways = ${}^{97}C_3 \times {}^1C_1$

$$\text{Average} = \frac{4 \times {}^3C_3 \times {}^{95}C_1 + 5 \times {}^4C_3 \times {}^{94}C_1 + \dots + 98 \times {}^{97}C_3 \times {}^1C_1}{{}^{99}C_5}$$

$$\text{Number for} = \sum_{r=3}^{97} (r+1)(100-(r+2)) {}^rC_3$$

$$= 100 \times 4 \sum_{r=3}^{97} \frac{r+1}{4} {}^rC_3 - 20 \sum_{r=3}^{97} \frac{(r+1)(r+2)}{4 \cdot 5} {}^rC_3$$

$$= 400 \sum_{r=3}^{97} {}^{r+1}C_4 - 20 \sum_{r=3}^{97} {}^{r+2}C_5$$

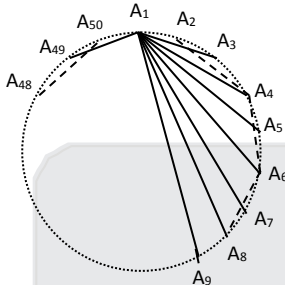
$$= 400 {}^{99}C_5 - 20 \times {}^{100}C_6$$

$$\text{Average} = 400 - 20 \times \frac{{}^{100}C_6}{{}^{99}C_5} = 400 - 20 \times \frac{100}{6} = 400 - \frac{1000}{3} = \frac{200}{3} = 66.6$$

18. Let P be a convex polygon with 50 vertices. A set F of diagonals of P is said to be minimally friendly if any diagonal $d \in F$ intersects at most one other diagonal in F at a point interior to P . Find the largest possible number of elements in a minimally friendly set F .

Ans. 71

Sol. Group of nonintersecting diagonals inside polygon are $A_1A_3, A_1A_4, A_1A_5, \dots, A_1A_{49}$, and $A_2A_4, A_4A_6, A_6A_8, \dots, A_{48}A_{50}$, but diagonal of first group intersect only one diagonal of second group. So total number of element in set F is $47 + 24 = 71$



19. For $n \in \mathbb{N}$, let $P(n)$ denote the product of the digits in n and $S(n)$ denote the sum of the digits in n . Consider the set

$A = \{n \in \mathbb{N} : P(n) \text{ is non-zero, square free and } S(n) \text{ is a proper divisor of } P(n)\}$.
Find the maximum possible number of digits of the numbers in A .

Ans. 92

Sol. $A = \{n \in \mathbb{N} ; p(n) \neq 0 \text{ is square free and } s(n) \text{ is proper divisor of } p(n)\}$ $p(n)$ is square free so number n can containing digit 1, 2, 3, 5, 7 or, 1, 5, 7, 6 $s(n)$ is proper divisor of $p(n)$

So maxi possible value of $s(n) = 3 \times 5 \times 7 = 105$

For making digit sum 105, n contain digit 2, 3, 5 and 7 one time and digit 1, 88 times

$s(n) = 2 + 3 + 5 + 7 + 1 \times 88 = 105$

$p(n) = 2 \times 3 \times 5 \times 7 \times 1 \dots \dots 1 = 210$

Maximum number of digits in $n = 88 + 4 = 92$

20. For any finite non empty set X of integers, let $\max(X)$ denote the largest element of X and $|X|$ denote the number of elements in X . If N is the number of ordered pairs (A, B) of finite non-empty sets of positive integers, such that $\max(A) \times |B| = 12$; and $|A| \times \max(B) = 11$ and N can be written as $100a + b$ where a, b are positive integers less than 100, find $a + b$.

Ans. 43

Sol. $\max(A) \times |B| = 12 = 1 \times 12 = 3 \times 4 = 2 \times 6$

$|A| \times \max(B) = 11 = 1 \times 11$

$|A| = 11 \quad 4 \max(B) = 1$

$|A| = 1 \text{ and } \max(B) = 11$

$\max(A) = 12 \text{ and } |B| = 1$

$\max(A) = 12 \quad |B| = 1$

$= 6 \quad = 2$

$= 4 \quad = 3$

$= 3 \quad = 4$

$= 2 \quad = 6$

Case-1 $|A| = 1, \max(A) = 12 \rightarrow$ number of set $A = 1$

$|B| = 1, \max(B) = 11 \rightarrow$ number of set $B = 1$

Case-2 $|A| = 1, \max(A) = 6 \rightarrow$ number of set $A = 1$

$|B| = 2, \max(B) = 11 \rightarrow$ number of set $B = {}^{10}C_1$

Case-3 $|A| = 1, \max(A) = 4 \rightarrow$ number of set $A = 1$

$|B| = 3, \max(B) = 11 \rightarrow$ number of set $B = {}^{10}C_2$

Case-4 $|A| = 1, \max(A) = 3 \rightarrow$ number of set $A = 1$

$$|B| = 4, \max(B) = 11 \rightarrow \text{number of set } B = {}^{10}C_3$$

Case-5 $|A| = 1, \max(A) = 2 \rightarrow \text{number of set } A = 1$
 $|B| = 6, \max(B) = 11 \rightarrow \text{number of set } B = {}^{10}C_5$

Case-5 $|A| = 11, \max(A) = 12 \rightarrow \text{number of set } A = {}^{11}C_{10}$
 $|B| = 1, \max(B) = 1 \rightarrow \text{number of set } B = 1$

$$N = 1 \times 1 + 1 \times {}^{10}C_7 + 1 \times {}^{10}C_2 + 1 + {}^{10}C_3 + 1 \times {}^{10}C_5 + 1 \times {}^{11}C_{10}$$

$$= 1 + 10 + 45 + 120 + 252 + 11 = 439$$

$$a = 4 \quad b = 39$$

$$a + b = 43$$

21. For $n \in \mathbb{N}$, consider non-negative integer-valued functions f on $\{1, 2, \dots, n\}$ satisfying $f(i) \geq f(j)$ for $i > j$ and $\sum_{i=1}^n (i + f(i)) = 2023$. Choose n such that $\sum_{i=1}^n f(i)$ is the least. How many such functions exist in that case?

Ans. 15

Sol. $\sum_{i=1}^n (i + f(i)) = 2023$

$$\sum_{i=1}^n i + \sum_{i=1}^n f(i) = 2023$$

$$\frac{n(n+1)}{2} + \left(\sum_{i=1}^n f(i) \right) = 2023$$

$$\left(\sum_{i=1}^n f(i) \right)_{\text{least}} = \left(2023 - \frac{n(n+1)}{2} \right)_{\text{least}} = 7$$

for $n = 63$

$$\text{So } \sum_{i=1}^n f(i) = 7$$

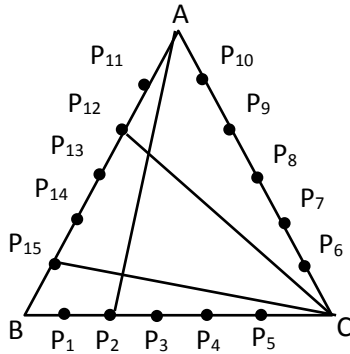
So number of such functions = 15 $f(i) \geq 0, f(i) \geq f(i+1)$

- (6, 1), (5, 2) (4, 3)
 (1, 2, 5), (1, 2, 4), (1, 3, 3), (2, 2, 3)
 (1, 1, 1, 4), (1, 1, 2, 3), (1, 2, 2, 2)
 (1, 1, 1, 1, 3), (1, 1, 1, 2, 2)
 (1, 1, 1, 1, 1, 2), (1, 1, 1, 1, 1, 1)

22. In an equilateral triangle of side length 6, pegs are placed at the vertices and also evenly along each side at a distance of 1 from each other. Four distinct pegs are chosen from the 15 interior pegs on the sides (that is, the chosen ones are not vertices of the triangle) and each peg is joined to the respective opposite vertex by a line segment. If N denotes the number of ways we can choose the pegs such that the drawn line segments divide the interior of the triangle into exactly nine regions, find the sum of the squares of the digits of N .

Ans. 77

Sol. Case-I



As per given condition we need to divide triangle into exactly 1 region, and for flu's to happen 3 line must be concurrent as shown in the above figure.

(i.e. in a way we are choosing 3 points on those sides, such that three lines from those points are concurrent)

So basically this is ideal solution of Ceva's theorem in which product of three different ratio leads to 1.

Possible ratio on side AB, BC and CA will be of the form $\frac{m}{n}, \frac{n}{m}$ and 1.

$$\text{i.e. } \frac{m}{n} \times \frac{n}{m} \times 1 = 1$$

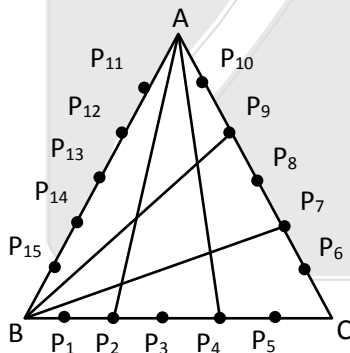
Now, we can choose the ratio 1:1 in 3 ways for all three sides and other ratio can be choose in 4 ways other two sides.

i.e. there are a $3 \times 4 + 1 = 13$ ways

Now, fourth point can be choose in ${}^{12}C_2$ ways

\therefore Total such possibilities = $12 \times 13 = 156$ ways

Case-II:-



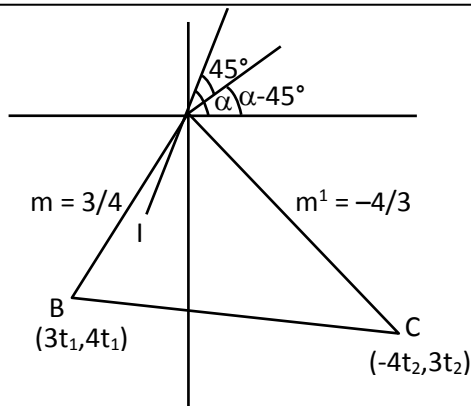
Seating any two points on any two sides, no of ways: = $3 \times {}^5C_2 \times {}^5C_2 = 300$

Total case possible = $300 + 156 = 456$

Some of square of digit = $4^2 + 5^2 + 6^2 = 16 + 25 + 36 = 77$

23. In the coordinate plane, a point is called a lattice point if both of its coordinates are integers. Let A be the point (12,84). Find the number of right angled triangles ABC in the coordinate plane where B and C are lattice points, having a right angle at the vertex A and whose in center is at the origin (0,0).

Ans. 18
Sol.



Using concept of shifting co-ordinate slope of AI = $\frac{84}{12} = 7$

$$\text{Slope of BC} = \tan(\alpha - 45) = \frac{7 - 1}{1 + 7} = \frac{3}{4}$$

$$\text{Radius of in circle is } \frac{AI}{\sqrt{2}} = \frac{\sqrt{12^2 + 84^2}}{\sqrt{2}} = 60$$

$$r = 60$$

$$\text{Now } \frac{AB + AC - BC}{2} = r$$

$$(BC)^2 = (AC)^2 + (AB)^2$$

$$(BC)^2 = (AC + AB - 2r)^2$$

$$(5t_1 + 5t_2 - 2 \times 5 \times 12)^2 = 25 (t_1^2 + t_2^2)$$

$$(t_1 + t_2 - 2 \times 12)^2 = t_1^2 + t_2^2$$

$$\text{Put } t_1 = x + 12$$

$$t_2 = y + 12$$

Equation become

$$(x - 12)(y - 12) = 2 \times 12^2$$

$$\geq 0 \quad \geq 0$$

Total number of triangle is equal to pair of (x, y)

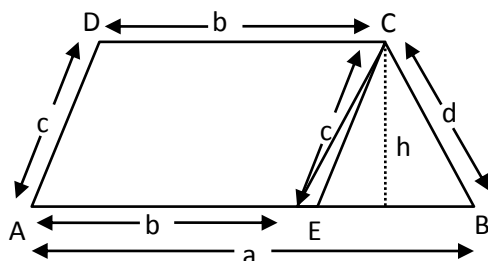
$$(x - 12)(y - 12) = 2^5 \times 3^2$$

$$\text{number of pair (x, y) = } 6 \times 3 = 18$$

24. A trapezium in the plane is a quadrilateral in which a pair of opposite sides are parallel. A trapezium is said to be non-degenerate if it has positive area. Find the number of mutually non-congruent, non-degenerate trapeziums whose sides are four distinct integers from the set (5,6,7,8,9,10).

Ans. 31

Sol.



$$[BEC] = \frac{1}{2}h(a - b) \dots\dots\dots (1)$$

$$\text{total area} = bh + [BEC]$$

by $eg^n(1)$

$$h = \frac{2[BEC]}{a-b}$$

$$\text{So total area} = \left(\frac{2[BEC]}{a-b}\right)b + [BEC]$$

$$= [BEC] \left(\frac{a+b}{a-b}\right)$$

So total area is non-zero if [BEC] has non-zero area we want to find $a, b, c, d \in \{5, 6, 7, 8, 9, 10\}$ such that [BEC] has non-zero area, and are pairwise non-congruent.

note that $\{c, d, a-b\}$ for a non-degenerate triangle if semi perimeter $>$ all side

$$\text{i.e. } \frac{c+d+(a-b)}{2} > c, d, a-b$$

Notice that since $a, b, c, d \in \{5, 6, 7, 8, 9, 10\}$

$c, d \geq 5 \geq a-b$, so we need to check

$$\frac{c+d+a-b}{2} > c, d \text{ this means } |c-d| < a-b$$

Since exchanging c, d lead to a congruent trapezium.

Let $c > d$, since they are distinct, so $0 < c-d < a-b$ is are condition

Now $a-b$ can range from 1 to 5

Case-I:

$$a-b=1 \quad 0 < c-d < 1 \quad \text{no solutions}$$

Case-II:

$$a-b=2 \quad 0 < c-d < 2 \quad \begin{aligned} c-d &= 1 \\ c &= d+1 \end{aligned}$$

$$(a, b) = (7, 5) \quad d = 8 \text{ or } 9$$

$$(a, b) = (8, 6) \quad d = 9$$

$$(a, b) = (9, 7) \quad d = 5$$

$$(a, b) = (10, 8) \quad d = 5 \text{ or } 6 \quad \text{6 Solutions}$$

Case-III:

$$a-b=3 \quad 0 < c-d < 3$$

$$c-d=1 \quad \text{or} \quad c-d=2$$

$$c=d+1 \quad \text{or} \quad c=d+2$$

$$(a, b) = (8, 3) \quad \begin{aligned} d &= 6, c = 7 \\ d &= 7, c = 9 \\ d &= 8, c = 10 \end{aligned}$$

$$(a, b) = (9, 6) \quad \begin{aligned} d &= 5, c = 7 \\ d &= 7, c = 8 \\ d &= 8, c = 10 \end{aligned}$$

$$(a, b) = (10, 7) \quad \begin{aligned} d &= 5, c = 6 \\ d &= 6, c = 8 \\ d &= 8, c = 9 \end{aligned}$$

Total = 9 solutions

Case-IV:

$$a-b=4 \quad 0 < c-d < 4$$

$$c-d=1 \quad c-d=2 \quad c-d=3$$

$$c=d+1 \quad c=d+2 \quad c=d+3$$

$$(a, b) = (9, 5) \quad \begin{aligned} d &= 6, c = 7, 8 \\ d &= 7, c = 8, 9 \\ d &= 8, c = 10 \end{aligned}$$

$$(a, b) = (10, 6) \quad \begin{aligned} d &= 5, c = 7, 8 \\ d &= 7, c = 8, 9 \\ d &= 8, c = 9 \end{aligned}$$

Total = 10 solutions

Case-V:

$$\begin{array}{llll}
 a - b = 5 & & 0 < c - d < 5 & \\
 c - d = 1 & c - d = 2 & c - d = 3 & c - d = 4 \\
 c = d + 1 & c = d + 2 & c = d + 3 & c = d + 4 \\
 (a, b) = (10, 5) & d = 6, c = 7, 8, 9 & & \\
 & d = 7, c = 8, 9 & & \\
 & d = 8, c = 9 & &
 \end{array}$$

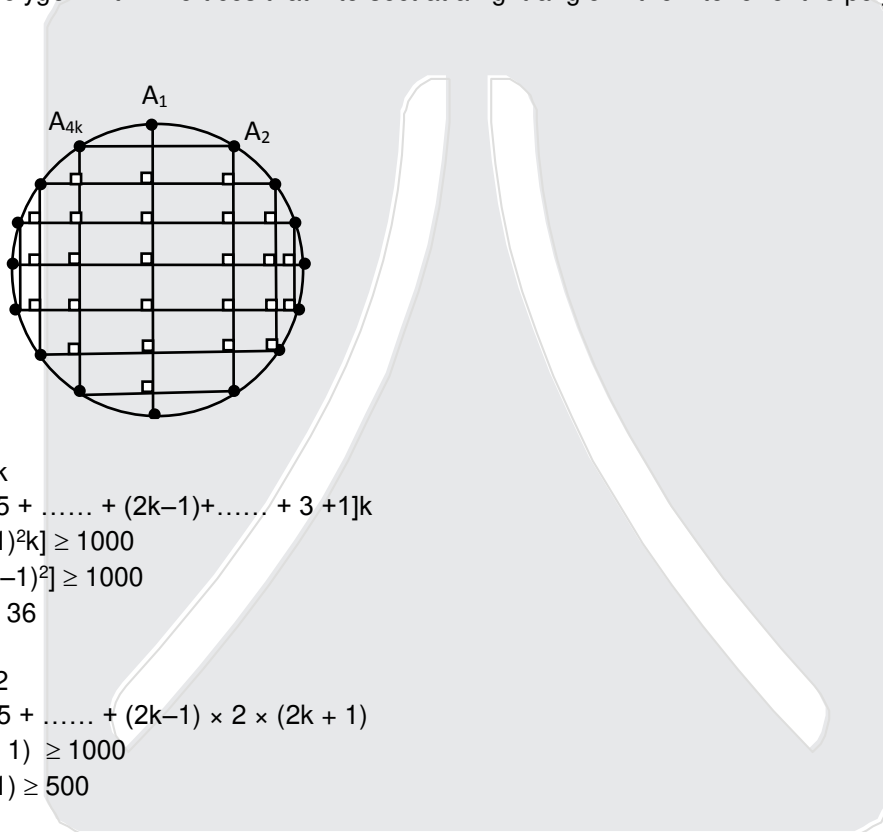
Total = 6 solutions

Total = 0 + 6 + 9 + 10 + 6 = 31 trapezium

25. Find the least positive integer n such that there are at least 1000 unordered pairs of diagonals in a regular polygon with n vertices that intersect at a right angle in the interior of the polygon.

Ans. 30

Sol.



Case-I

Let $n = 4k$

$$[1 + 3 + 5 + \dots + (2k-1) + \dots + 3 + 1]k$$

$$[k^2 + (k-1)^2]k \geq 1000$$

$$k(k^2 + (k-1)^2) \geq 1000$$

$$k \geq 9 \quad n \geq 36$$

Case-II

$n = 4k + 2$

$$(1 + 3 + 5 + \dots + (2k-1)) \times 2 \times (2k + 1)$$

$$2k^2(2k + 1) \geq 1000$$

$$k^2(2k + 1) \geq 500$$

$$k^2 \geq 7$$

$$n \geq 30$$

Final Answer = 30

26. In the land of Binary, the unit of currency is called Ben and currency notes are available in denominations 1, 2, 2², 2³, ... Bens. The rules of the Government of Binary stipulate that one can not use more than two notes of any one denomination in any transaction. For example, one can give a change for 2 Bens in two ways: 2 one Ben notes or 1 two Ben note. For 5 Ben one can give 1 one Ben note and 1 four Ben note or 1 one Ben note and 2 two Ben notes. Using 5 one Ben notes or 3 one Ben notes and 1 two Ben notes for a 5 Ben transaction is prohibited. Find the number of ways in which one can give change for 100 Bens, following the rules of the Government.

Ans. 19

Sol. 2⁰(1), 2¹, 2², 2³, = 1, 2, 4, 8, 16, 32, 64

$$64 + 32 + 4$$

$$64 + 32 + 2 + 2$$

$$64 + 32 + 2 + 1 + 1$$

$$\begin{aligned}
 64 + 16 + 16 + 4 &= 32 + 32 + 16 + 16 + 4 \\
 64 + 16 + 16 + 2 + 2 &= 32 + 32 + 16 + 16 + 2 + 2 \\
 64 + 16 + 16 + 2 + 1 + 1 &= 32 + 32 + 16 + 16 + 2 + 1 + 1 \\
 64 + 16 + 8 + 8 + 4 &= 32 + 32 + 16 + 8 + 8 + 4 \\
 64 + 16 + 8 + 8 + 2 + 2 &= 32 + 32 + 16 + 8 + 8 + 2 + 2 \\
 64 + 16 + 8 + 8 + 2 + 1 + 1 &= 32 + 32 + 16 + 8 + 8 + 2 + 1 + 1 \\
 64 + 16 + 8 + 4 + 4 + 2 + 2 &= 32 + 32 + 16 + 8 + 4 + 4 + 2 + 2 \\
 64 + 16 + 8 + 4 + 4 + 2 + 1 + 1 &= 32 + 32 + 16 + 8 + 4 + 4 + 2 + 1 + 1
 \end{aligned}$$

Sol. (Incomplete)

$$2^0(1), 2^1, 2^2, 2^3$$

100B

$$100 = 2^6 + 2^5 + 2^2 (64 + 32 + 4)$$

$$= 2^6 + 2^5 + 2^1 + 2^1$$

$$= 2^6 + 2^5 + 2^1 + 2^0 + 2^0$$

$$= 2^6 + 2 \times 2^4 + 2 \cdot 2^1$$

$$= 2 \times 2^5 + 2 \times 2^4 + 2^2$$

$$= 2 \times 2^5 + 2 \cdot 2^4 \times 2 \cdot 2^1$$

$$= 2^6 + 2^5 + 2 + 2 \cdot 2^0$$

$$= 2^6 + 2 \cdot 2^4 + 2^1 + 2 \cdot 2^0$$

$$= 2 \times 2^5 + 2 \times 2^4 + 2^1 + 2 \times 2^0$$

$$64 + 16 + 8 + 8 + 4$$

$$2^6 + 2^4 + 2 \times 2^3 + 2^2$$

$$2^6 + 2^4 + 2 \times 2^3 + 2 \cdot 2^1$$

$$2 \times 2^5 + 2^4 + 2 \times 2^3 + 2 \times 2^2$$

$$2 \times 2^5 + 2^4 + 2 \times 2^3 + 2 \times 2^1$$

$$64 + 16 + 8 + 4 + 4 + 2 + 2$$

$$2^6 + 2^4 + 2^3 + 2 \times 2^2 + 2 \times 2^1$$

$$2 \times 2^5 + 2^4 + 2^3 + 2 \times 2^2 + 2 \cdot 2^1$$

Total cases = 15

27. A quadruple (a, b, c, d) of distinct integers is said to be balanced if $a + c = b + d$. Let S be any set of quadruples (a, b, c, d) where $1 \leq a < b < d < c \leq 20$ and where the cardinality of S is 4411. Find the least number of balanced quadruples in S.

Ans. 91

Sol. At first find maximum cardinality of quadruple (a, b, c, d) ignoring the balanced one, the that is simply ${}^{20}C_4 = 4845$

But cardinality of s {(a, b, c, d)} is given to be 4411.

So leaving out maximum possible balanced quadruple in set $S = \{(a, b, c, d)\}$ with $1 \leq a < b < d < c \leq 20$

Now, counting balanced quadruples is s, we have

$$a + c = b + d = 5 \Rightarrow (1, 4), (2, 3) \rightarrow {}^2C_2$$

$$a + c = b + d = 6 \Rightarrow (1, 5), (2, 4) \rightarrow {}^2C_2$$

$$a + c = b + d = 7 \Rightarrow (1, 6), (2, 5), (3, 4) \rightarrow {}^3C_2$$

$$a + c = b + d = 8 \Rightarrow (1, 7), (2, 6), (3, 5) \rightarrow {}^3C_2$$

$$a + c = b + d = 9 \Rightarrow (1, 8), (2, 8), (3, 7), (4, 5) \rightarrow {}^4C_2$$

$$a + c = b + d = 10 \Rightarrow (1, 9), (2, 8), (3, 7), (4, 6) \rightarrow {}^4C_2$$

⋮

⋮

$$a + c = b + d = 36 \Rightarrow (16, 20), (17, 19), \rightarrow {}^2C_2$$

$$a + c = b + d = 37 \Rightarrow (17, 20), (18, 19), \rightarrow {}^2C_2$$

$$\text{Total such case are} = 4({}^2C_2 + {}^3C_2 + \dots + {}^9C_2) + {}^{10}C_2$$

$$= 480 + 45$$

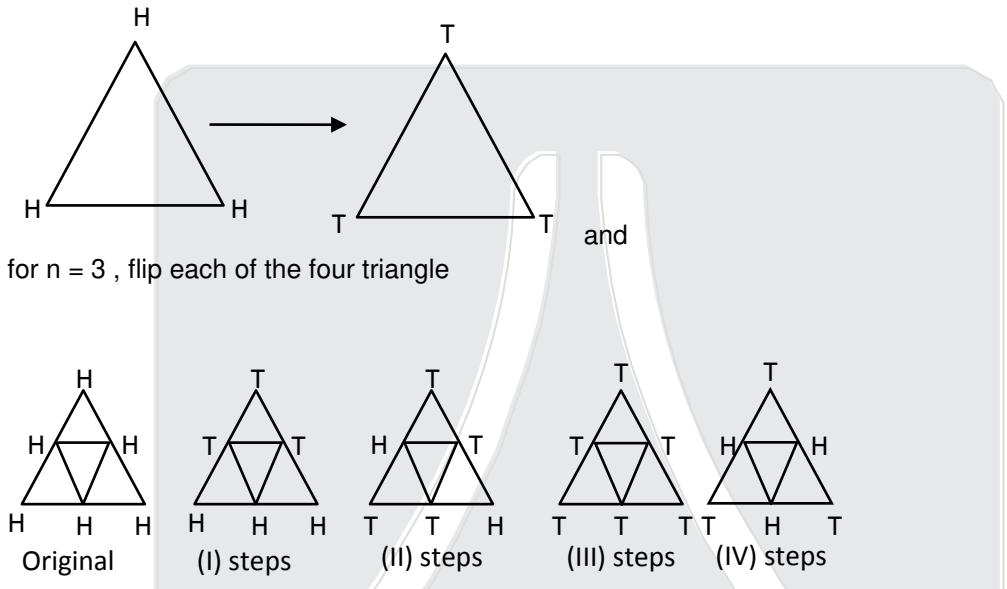
$$= 525 \text{ balanced quadruples}$$

$$\text{Remaining will be } 525 - 434 = 91$$

28. On each side of an equilateral triangle with side length n units, where n is an integer, $1 \leq n \leq 100$, consider $n - 1$ points that divide the side into n equal segments. Through these points, draw lines parallel to the sides of the triangle, obtaining a net of equilateral triangles of side length one unit. On each of the vertices of these small triangles, place a coin head up. Two coins are said to be adjacent if the distance between them is 1 unit. A move consists of flipping over any three mutually adjacent coins. Find the number of values of n for which it is possible to turn all coins tail up after a finite number of moves.

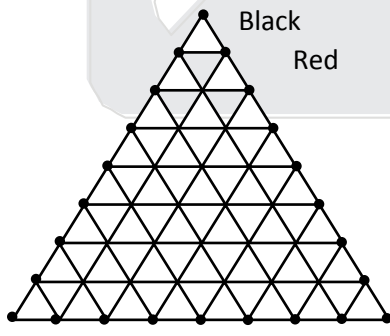
Ans. 67

Sol. This can be done for $n \equiv 0, 2 \pmod 3$. below by a triangle, we will mean three coins which are mutually adjacent for $n = 2$, clearly it can be done.

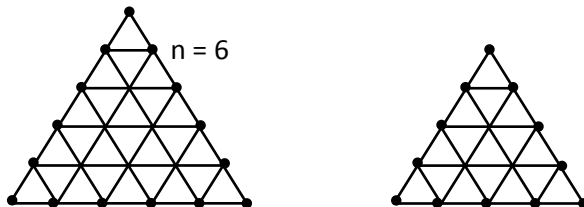


for $n \equiv 0, 2 \pmod 3$
and $n > 3$ flip every triangle. Then the coins at the corners are flipped one. The coins on the sides (not comers) are flipped three times each. So all these coins will have tails up. The interior coins flipped six times each and have heads up. Since the interior coins have side length $n - 3$ by the induction step all of them (an be flipped so to have tails up.

$n \equiv 0 \pmod 3$



$n = 4$

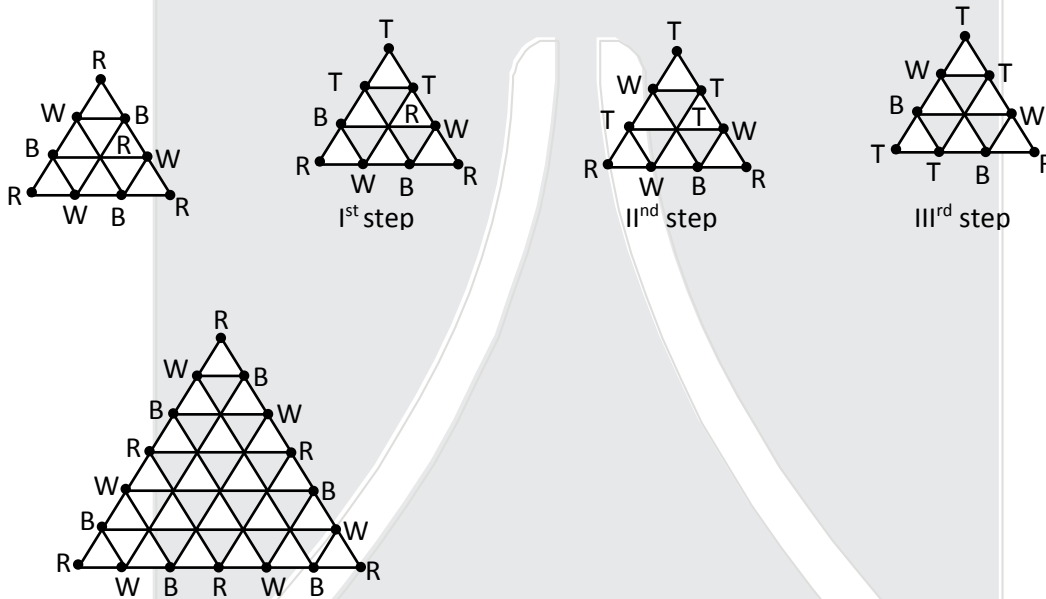


reduces to $n = 6 \rightarrow$ reduces $n = 3$ point

Next suppose $r = 1 \pmod 3$ colour the heads of each coin red, white and blue so that adjacent coins

different colour and any three coins in a row have different colours. then the coins in the corner have the same colour say red [$\therefore n = 3k + 1$] A simple count shows that there are one more red coin than white or blue coins, so the (odd or even) parities of the red and white coins increase by 1 or (b) both by 1 or (c) one increases by 1 and other decreases by 1. So the parities of the red and white coins stay different. In the case all coins are flipped the number of red and white coins could be zero and the parities would be the same so this cannot happen $n \equiv 1 \pmod 3$ case number of coins

R	W	step
4	3	original
3	2	(I) - 1
2	3	(II) ± 1
1	2	(III) - 1



so it is possible for $r = 33$
 $n = 1, 2, 3, 5, 6, 8, 9, \dots, 98, 99$
 for $n = 67$
 possible value it can be done

29. A positive integer $n > 1$ is called beautiful if n can be written in one and only one way as $n = a_1 + a_2 + \dots + a_k = a_1, a_2, \dots, a_k$ for some positive integers a_1, a_2, \dots, a_k , where $k > 1$ and $a_1 \geq a_2 \geq \dots \geq a_k$. (For example 6 is beautiful since $6 = 3 + 2 + 1$, and this is unique. But 8 is not beautiful since $8 = 4 + 2 + 1 + 1 = 4 \cdot 2 \cdot 1 \cdot 1$ as well as $8 = 2 + 2 + 2 + 1 + 1 = 2 \cdot 2 \cdot 2 \cdot 1 \cdot 1$, so uniqueness is lost.) Find the largest beautiful number less than 100.

Ans. 95

Sol. $95 = 19 \times 5 \times (1 \times 1 \times 1 \dots 71 \text{ times})$
 $95 = 19 + 5 + (1 + 1 + 1 \dots 71 \text{ times})$

30. Let $d(m)$ denote the number of positive integer divisors of a positive integer m . If r is the number of integers $n \leq 2023$ for which $\sum_{i=1}^n d(i)$ is odd, find the sum of the digits of r

Ans. 18

Sol. As m is a positive integer and If m is perfect square then number of its divisor is odd otherwise

number of divisor is even and $\sum_{i=1}^n d(i) = \text{odd}$

So for sum to be odd, number of perfect square to be odd number of times

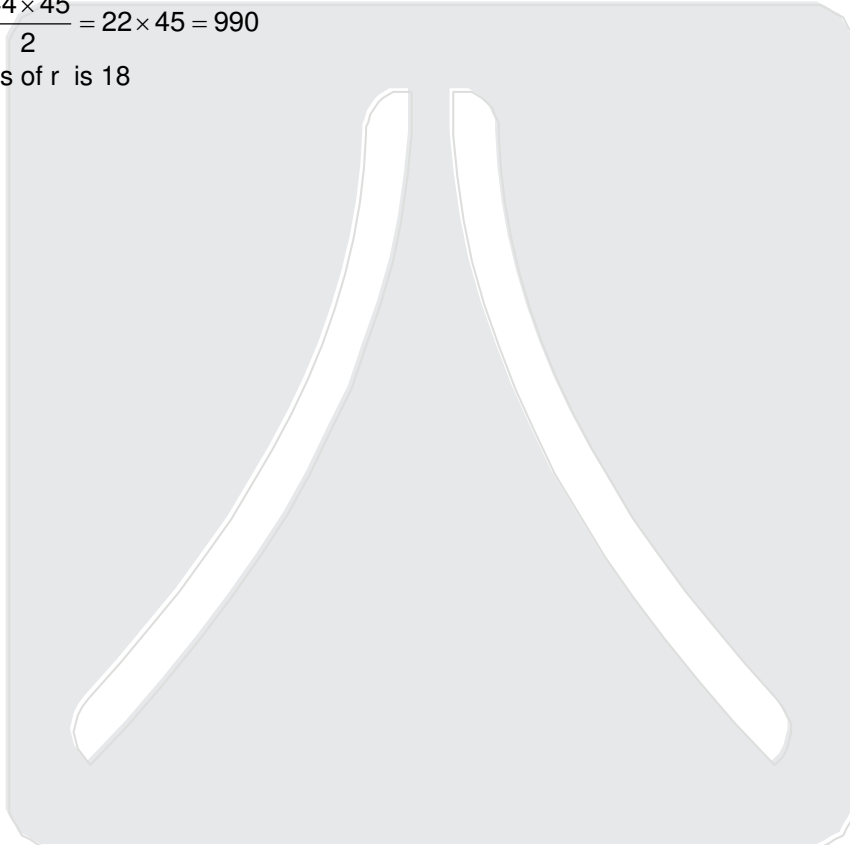
Hence number of interfere $n \leq 2023$ for which $\sum d(i) = \text{odd}$ is equal to

$$r = (2^2 - 1^2) + (4^2 - 3^2) + (6^2 - 5^2) + \dots + (44^2 - 43^2)$$

$$\Rightarrow r = 1 + 2 + 3 + 4 + \dots + 44$$

$$= \frac{44 \times 45}{2} = 22 \times 45 = 990$$

Sum of digits of r is 18



From Excellence to Prominence..

AIR 22



AIR SC-1

DESHANK PRATAP SINGH
Reso Roll No.: 22235068

AIR 26



MAYANK SONI
Reso Roll No.: 22741545

AIR 29



TANISHQ M MANDHANE
Reso Roll No.: 22235023

AIR 32



KRITIN GUPTA
Reso Roll No.: 22235049

AIR 33



NAMAN GOYAL
Reso Roll No.: 22235036

AIR 99



AIR SC-1

MD SAHIL AKHTAR
Reso Roll No.: 22235069

STUDENTS IN
TOP-100
All India Ranks (AIRs)

15

All AIRs are in Common Rank List (CRL)


AIR 7



BIKKINA A. CHOWDARY
Reso Roll No.: 22235070


All Students are from
Our Offline/Online Classroom Programs

AIR 37



S S SUMEDH
Reso Roll No.: 22235059

AIR 92



APURVA SAMOTA
Reso Roll No.: 22235055

AIR 44



KAUSHAL VIJAYVERGIYA
Reso Roll No.: 22741546

AIR 69



KRISH GUPTA
Reso Roll No.: 22235051

AIR 66



KEYAAN K RAJESH
Reso Roll No.: 21214207

AIR 63



AIR PWD-1

DIPEN SOJITRA
Reso Roll No.: 22235054

AIR 61



RIDAM JAIN
Reso Roll No.: 22235057

AIR 54



BHAVYA BANSAL
Reso Roll No.: 22235065

Special Achievers

The Power of **SHE**
ADITI SINGH
Reso Roll No.: 22235053
GIRLS' Topper
IIT-Bombay Zone
AIR 17
(BSC-MCU)
AIR (CRL) 104

The Power of **SHE**
ANUSHKA SINGHAL
Reso Roll No.: 21153204
AIR (CRL) 291

हिन्दी माध्यम के विजेता
बिप्लिन मीणा
Reso Roll No.: 21218129
AIR (BTD) 61

Distance Learning
SANJAY P MALLAR
Reso Roll No.: 23405290
AIR (CRL) 86

Total Selections

1200
(Classroom: 920 | Distance: 280)
Eligible for Counselling

Congratulations...!!!

To all the Selected Students & their Proud Parents



22 वर्षों से लगातार... श्रेष्ठ शिक्षण, श्रेष्ठ परिणाम...

KAUSHAL VIJAYVERGIYA | Course: iVIETA
AIR **5**
300/300 Marks

FREDIE GEORGE ROBIN | Course: DLP
AIR **98**

SOHAM DAS | Course: DLP
AIR **26**
100%ile

HARSHAL LASOD | Course: VIETA
AIR **50**
100%ile (Maths)

7
AIRs (CRL) IN TOP-100

ASHIK STENNY | Course: DLP
AIR **29**
100%ile

MAYANK SONI | Course: VIETA
AIR **34**
100%ile

KRISH GUPTA | Course: iVIETA
AIR **31**
100%ile

RESULT HIGHLIGHTS

20
AIRs (Category) in Top - 100

9228
Students Qualified for JEE (Advanced) 2023

Subject Wise 100%ile
19 | 3 | 3
Physics | Chemistry | Maths

Classroom Students (Offline/Online) in Top-1000 AIRs (CRL)

AIR 115 RAMKRISHNA GENA	AIR 258 ABHISHEK GUPTA	AIR 378 KAVYA AGRAWAL	AIR 390 APOORV SHARMA	AIR 407 KRITIN GUPTA	AIR 454 ARHAN M. VORA	AIR 465 ARJUN KRISHNASWAMY	AIR 481 SANYAM AGRAWAL	AIR 538 ARYA SAMEER JOSHI	AIR 551 KEYAAN K. RAJESH	AIR 564 SUJIT ADIGA	AIR 584 AYUSH GUPTA
AIR 641 UTKARSH GUPTA	AIR 642 CEZAN V. DAMANIA	AIR 657 ANUSHKA SINGHAL	AIR 668 DEVANSH JAIN	AIR 716 HARSH AGRAWAL	AIR 757 DARSHIT MITTAL	AIR 765 PRAJWAL YADAV	AIR 768 PRATYUSH DASH	AIR 853 MOHIT AGARWAL	AIR 854 SHAURYA SINGHAL	AIR 860 ANIRUDH BHARDWAJ	AIR 867 PARTH GUPTA

*Result Received so far

Congratulations to all the Qualified Students & their Parents