Resonance®
Educating for better tomorrow

INDIAN OLYMPIAD QUALIFIER (IOQ) 2021-2022

INDIAN OLYMPIAD QUALIFIER IN PHYSICS (IOQP), 2022

QUESTIONS & SOLUTIONS (PART-A)

Sunday, March 13, 2022 | Time: 75 Minutes | Max. Marks : 120

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | **FAX No. :** +91-022-39167222

To Know more : sms **RESO** at **56677** | **Website :** www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN :** U80302RJ2007PLC024029

Toll Free : 1800 258 5555 7340010333 facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

INSTRUCTIONS

Write the question paper code mentioned above on YOUR OMR Answer Sheet (in the space provided), otherwise your Answer Sheet will NOT be evaluated. Note that the same Question Paper Code appears on each page of the question paper.

Instructions to Candidates:

1. Use of mobile phone, smart watch, and iPad during examination is STRICTLYPROHIBITED.
2. In addition to this question paper, you are given OMR Answer Sheet along with candidate's copy.
3. On the OMR sheet, make all the entries carefully in the space provided ONLY in BLOCK CAPITALS as well as by properly darkening the appropriate bubbles.
   Incomplete/ incorrect/ carelessly filled information may disqualify your candidature.
4. On the OMR Answer Sheet, use only BLUE or BLACK BALL POINT PEN for making entries and filling the bubbles.
5. Your fourteen-digit roll number and date of birth entered on the OMR Answer Sheet shall remain your login credentials means login id and password respectively for accessing your performance / result in Indian **Olympiad Qualifier in Physics 2020-21 (Part I).**
6. Question paper has two parts. In partA1 (Q. No.1 to 24) each question has four alternatives, out of which only one is correct. Choose the correct alternative and fill the appropriate bubble, as shown.

   In partA2 (Q. No. 25 to 32) each question has four alternatives out of which any number of alternative(s) (1, 2, 3,or 4) may be correct. You have to choose all correct alternative(s) and fill the appropriate bubble(s), as shown

7. For PartA1, each correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer. In Part A2, you get 6 marks if all the correct alternatives are marked and no incorrect. No negative marks in this part.
8. Rough work should be done only in the space provided. There are 08 printed pages in this paper.
9. Use of **non- programmable scientific** calculator is allowed.
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting answer paper, take away the question paper and candidate's copy of OMR for your reference

    Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the OMR answersheet.

    OMR answer sheets are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED. Scratching oroverwriting may result in a wrong score.

    DO NOTWRITE ONTHE BACK SIDE OFTHE OMRANSWER SHEET

Instructions to Candidates (Continued) :

You may read the following instructions after submitting the answer sheet.

12. Comments/Inquiries/Grievances regarding this question paper, if any, can be shared on the Inquiry/Grievance column on www.iaptexam.in on the specified format.
13. The answers/solutions to this question paper will be available on the website: www.iapt.org.in.
14. CERTIFICATES and AWARDS:
    Following certificates are awarded by IAPT to students, successful in the Indian Olympiad Qualifier in Physics 2021-22 (Part I)
    (i) "CENTRETOP10 %"
    (ii) "STATETOP1 %"
    (iii) "NATIONALTOP1 %"
    (iv) "GOLD MEDAL& MERITCERTIFICATE" to all students who attend OCSC-2022 at HBCSE Mumbai
15. All these certificates (except gold medal) will be downloadable from IAPT website : www.iapt.org.in.
16. List of students (with centre number and roll number only) having score above MAS will be displayed on the website: www.iapt.org.in. See the Minimum Admissible score Clause on the Student's brochure on the web.
17. List of Students eligible for evaluation of IOQP 2021-22 (Part II) shall be displayed on www.iapt.org.in

Physical constants you may need….

Mass of electron $m_e = 9.10\times10^{-31}$ kg	Magnitude of charge on electron $e = 1.60\times10^{-19}$ C
Mass of proton $m_p = 1.67\times10^{-27}$ kg	Permittivity of free space $\varepsilon_0 = 8.85\times10^{-12}\ C^2N^{-1}m^{-2}$
Acceleration due to gravity $g = 9.8\ ms^{-2}$	Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$
Universal gravitational constant $G = 6.67\times10^{-11}\ Nm^2\ Kg^{-2}$	Planck's constant $h = 6.625 \times 10^{-34}$ Js
Universal gas constant $R = 8.31\ Jmol^{-1}\ K^{-1}$	Faraday constant = 96,500 $Cmol^{-1}$
Boltzmann constant $k = 1.38\times10^{-23}\ JK^{-1}$	Rydberg constant $R = 1.097 \times 10^{7}\ m^{-1}$
Stefan's constant $\sigma = 5.67\times10^{-8}\ Wm^{-2}K^{-4}$	Speed of light in free space $c = 3 \times 10^{8}\ ms^{-1}$
Avogadro's constant $N_A = 6.023 \times 10^{23}\ mol^{-1}$	

Question Paper Code: 64

PHYSICS 2021-22 (Part I) (NSEP 2021-22)

Time: 75 Minute **Max. Marks: 120**

Attempt All Thirty Two Questions

A-1

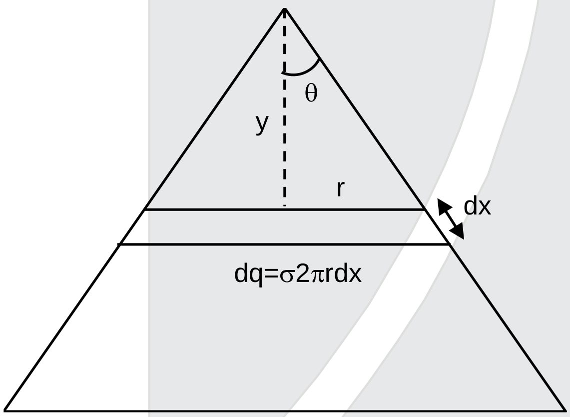
ONLY ONE OUT OF FOUR OPTIONS IS CORRECT. BUBBLE THE CORRECT OPTION.

1. A hollow non-conducing cone of base radius R = 50 cm and semi vertical angle of 15° has been uniformly charged on its curved surface up to three-fourth of its slant length from base with a surface charge density $\sigma = 2.5\ \mu C/m^2$. The electric field produced at the location of the vertex of the cone is

(a) $\frac{\sigma \ell n2}{2\varepsilon_0}$ (b) $\frac{\sigma \ell n2}{4\varepsilon_0}$ (c) $\frac{\sigma \ell n2}{8\varepsilon_0}$ (d) $\frac{\sigma \ell n2}{16\varepsilon_0}$

Ans. **(b)**

Sol.



$\frac{r}{x} = \frac{R}{\ell} \Rightarrow x = r\frac{\ell}{n}$ $dx = \frac{\ell}{n}dr$

$\frac{r}{y} = \tan\theta \Rightarrow y = r\cos\theta$

$$E = \int \frac{\sigma k(2\pi r dx)y}{(r^2+y^2)^{3/2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\int \frac{rdxy}{(x^2+y^2)^{3/2}}$$

$$= \frac{\sigma\ell}{2\varepsilon_0 R}\int \frac{rdr\cot\theta}{(r^2+y^2)^{3/2}} = \frac{\sigma\ell}{2\varepsilon_0 R}\int \frac{r^2 dr\cot\theta}{r^3\,\text{cosec}^3\theta}$$

$$= \frac{\sigma\ell}{2\varepsilon_0 R}\frac{\cot\theta}{\text{cosec}^3\theta}\int_{r/4}\frac{dr}{r} = \frac{\sigma\ell}{2\varepsilon_0 R}\frac{\cos\theta}{\sin\theta}\times\sin^3\theta(\ln r)^R$$

$$= \frac{\sigma\ell}{2\varepsilon_0 R}\sin^2\theta\cos\theta(2\ln 2) = \frac{\sigma}{2\varepsilon_0}\sin\theta\cos\theta)\ell n2$$

$$= \frac{\sigma\sin 2\theta}{4\varepsilon_0}\ell n2$$

$$\theta = 15^o = \frac{\sigma}{4\varepsilon_0}\ell n2$$

2. A freely falling spherical rain drop gathers moisture (maintaining its spherical shape all the way) from the atmosphere at a rate $\frac{dm}{dt} = kt^2$ where t is the time and m is the instantaneous mass of the drop, the constant k = 12 gm/s^3. If the drop, of initial mass m_0 = 2gm, starts falling from rest the instantaneous velocity of the drop exactly after 5 second shall be (ignore air friction and air buoyancy)

(a) 12.4 ms^{-1} (b) 49.0 ms^{-1} (c) 122.5 ms^{-1} (d) data insufficient

Ans. **(a)**

Sol. $mg - F_{th} = ma$

$$mg - \frac{dm}{dt}v = ma$$

$$a = g - \frac{kt^2 v}{m_0 + \frac{kt^3}{3}}$$

$$\frac{dv}{dt} = g - \frac{3kt^2}{3m_0 + kt^3}.v$$

$$\frac{dv}{dt} + \frac{3kt^2}{3m_0 + k + 3}v = g$$

$$\frac{dv}{dt} + Pv = Q$$

$$P = \frac{3kt^2}{3m_0 + kt^3}$$

$Q = g$

$$\int P dt = \int \frac{3kt^2}{3m_0 + kt^3} dt = \ln(3m_0 + kt^3)$$

multiplying with I.F.

$$(3m_0 + kt^3).v = \int g(3m_0 + kt^3)dt + C = g(3m_0 t + \frac{kt^4}{4}) + C$$

$t = 0, v = 0$

$3m_0 \times = g + C$

$\therefore c = 0$

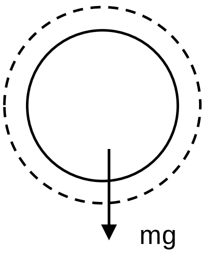
$$v = \frac{g\left[\left(3m_0 t + \frac{kt^4}{4}\right)\right]}{(3m_0 + kt^3)}$$

$$= \frac{9.8\left[3 \times 2 \times 10^{-3} \times 5 + \frac{12 \times 10^{-3} \times 625}{4}\right]}{3 \times 2 \times 10^{-3} + 12 \times 10^{-3} \times 125}$$

$$= 9.8\left[\frac{30 + 1875}{1506}\right]$$

= 12.4 m/s

Alternate Solution



$$mg - \frac{dmv}{dt} = m\frac{dv}{dt}$$

$$mg\int dt = \int d(mv)$$

$$= m_f v_i - m_i v_i$$

$$g\left(m_0 + \frac{kt^3}{3}\right)dt \ = mv_f$$

$$gm_0 t + \frac{kgt^4}{12} = \left(m_0 + \frac{kt^3}{3}\right)v_F$$

$$v_f = \frac{\left(m_0 + \frac{kt^3}{12}\right)gt}{m_0 + \left(\frac{kt^3}{3}\right)}$$

$$= \left(\frac{2 + \frac{12\times125}{12}}{2 + \frac{12\times125}{3}}\right)g\times5$$

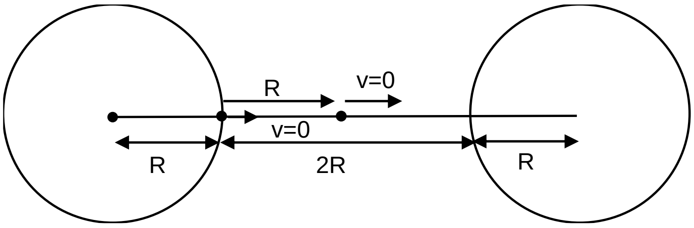
$$V_f = \left(\frac{127}{502}\right)5g \ = 12.39$$

3. Two planets, each of mass M and radius R are positioned (at rest) in space, with their centres a distance 4R apart. You wish to fire a projectile from the surface of one planet to the other. The minimum initial speed for which this may be possible is

(a) $\sqrt{\frac{2GM}{5R}}$ (b) $\sqrt{\frac{2GM}{3R}}$ (c) $\sqrt{\frac{4GM}{3R}}$ (d) $\sqrt{\frac{3GM}{2R}}$

Ans. (b)

Sol.



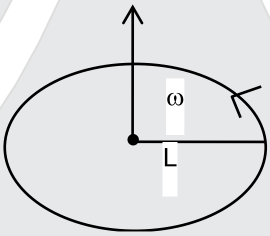
$$\frac{-GMm}{R}-\frac{GMm}{3R}+\frac{1}{2}mv^2=-\frac{GMm}{2R}\times 2+0$$

$$\frac{1}{2}mv^2=\frac{4}{3}\frac{GmM}{R}-\frac{GmM}{R}$$

$$\frac{1}{2}mv^2=\frac{1}{3}\frac{GmM}{R}$$

v = $\sqrt{\frac{2}{3}\frac{GM}{R}}$

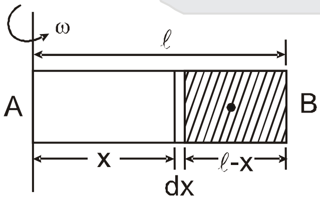
4. A thin uniform metallic rod of length L and radius R rotates with an angular velocity ω in a horizontal plane about a vertical axis passing through one of its ends. The density and the Young's modulus of the material of the rod are ρ and Y respectively. The elongation in its length is



(a) $\frac{\rho\omega^2L^3}{6Y}$ (b) $\frac{\rho\omega^2L^3}{3Y}$ (c) $\frac{\rho\omega^2RL^2}{2Y}$ (d) $\frac{\rho\omega^2L^3}{2Y}$

Ans. **(b)**

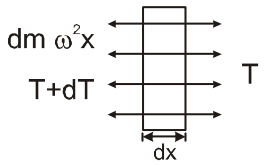
Sol.

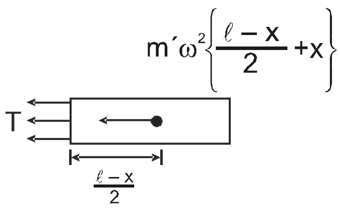


mass of shaded portion

$m' = \frac{m}{\ell}(\ell - x)$ [where m = total mass = $\rho A\lambda$]

$T = m'\omega^2\left[\frac{\ell-x}{2}+x\right] \Rightarrow T = \frac{m}{\ell}(\ell-x)\omega^2\left(\frac{\ell+x}{2}\right)$ $T = \frac{m\omega^2}{2\ell}(\ell^2-x^2)$





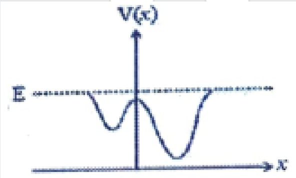
this tension will be maximum at A $\left(\frac{m\omega^2\ell}{2}\right)$ and minimum at 'B' (zero), elongation in element of width 'dx' = $\frac{Tdx}{AY}$

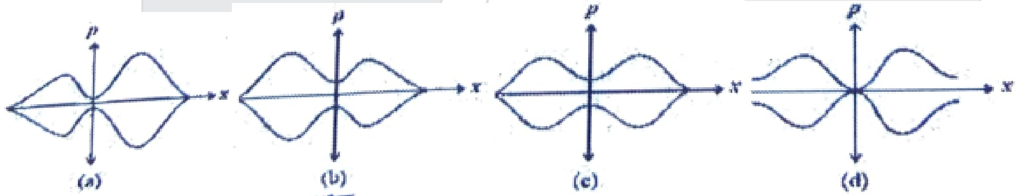
Total elongation $\delta = \int \frac{Tdx}{AY} = \int_0^{\ell} \frac{m\omega^2(\ell^2 - x^2)}{2\ell AY} dx$

$$\delta = \frac{m\omega^2}{2\ell \quad AY}\left[\ell^2 x - \frac{x^3}{3}\right]_0^{\ell} = \frac{m\omega^2 \times 2\ell^3}{2\ell AY \times 3} = \frac{m\omega^2\ell^2}{3AY} = \frac{\rho A\ell\omega^2\ell^2}{3AY}$$

$$\delta = \frac{\rho\omega^2\ell^3}{3y}$$

5. Consider a particle of mass m with a total energy E moving in a one dimensional potential field. The potential V(x) is plotted against x in the figure beside. The plot of momentum – position graph of this particle is qualitatively best represented by :



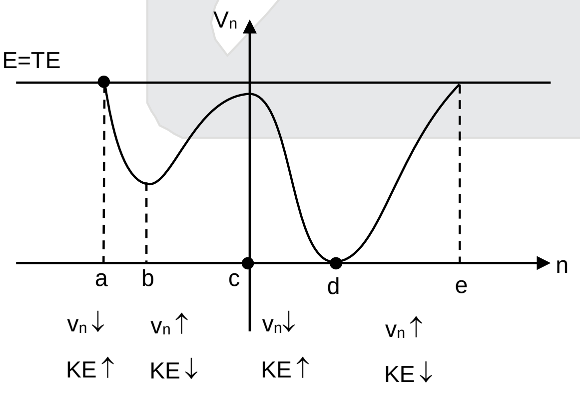


All plots are symmetrical about x-axis

(a) Figure (a) (b) Figure (b) (c) Figure (c) (d) Figure (d)

Ans. **(a)**

Sol.



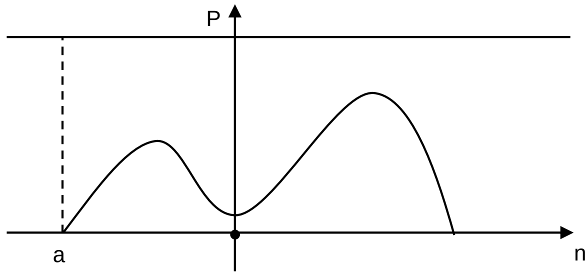


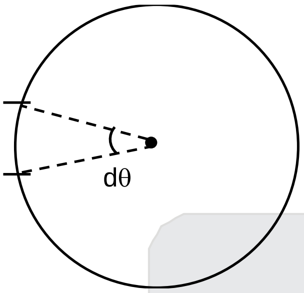
Fig. (a)

6. Knowing that the parallel currents attract, the inward pressure on the curved surface of a thin walled, long hollow metallic cylinder of radius R = 50 cm carrying a current of i = 2 amp parallel to its axis distributed uniformly over the entire circumference, is

(a) 2.05×10^{-1} Nm^{-2} (b) 2.55×10^{-3} Nm^{-2} (c) 2.05×10^{-5} Nm^{-2} (d) 2.55×10^{-7} Nm^{-2}

Ans. (d)

Sol.



$\frac{\mu_0 I}{2\pi R}$ $B_1 = 0$

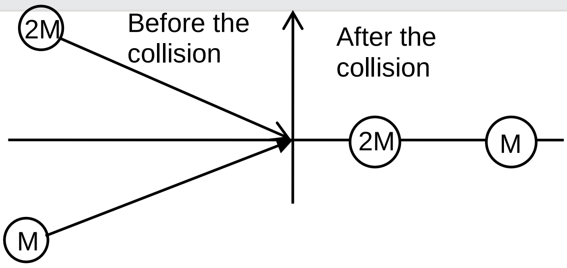
This field is covered by the element Rdθ and remaining portion = $\frac{\mu_0 I}{2\pi R}\Big/2 = \frac{\mu_0 I}{4\pi R}$

Force on rdθ

$$dF = \left(\frac{\mu_0 I}{4\pi R}\right)\left(\frac{I}{2\pi}d\theta\right)\ell$$

$$\text{Pressure } P = \frac{dF}{dA} = \frac{dF}{(Rd\theta)\ell} = \frac{dF}{dA} = \frac{dF}{(Rd\theta)\ell} = \frac{\mu_0 I^2}{8\pi^2 R^2} = 2.55 \times 10^{-7}\ N/m$$

7. Two masses move on a collision path as shown. Before the collision the object with mass 2M moves with a speed v making an angle $\theta = \sin^{-1}\frac{3}{5}$ to the x-axis while the object with mass M moves with a speed $\frac{3}{2}v$ making an angle $\phi = \sin^{-1}\frac{4}{5}$ with the x-axis. After the collision the object of mass 2M is observed to be moving to the right along the x-axis with a speed of $\frac{4}{5}v$. There are no external forces acting during the collision. The correct option is



(a) The velocity of mass M, after the collision, is zero

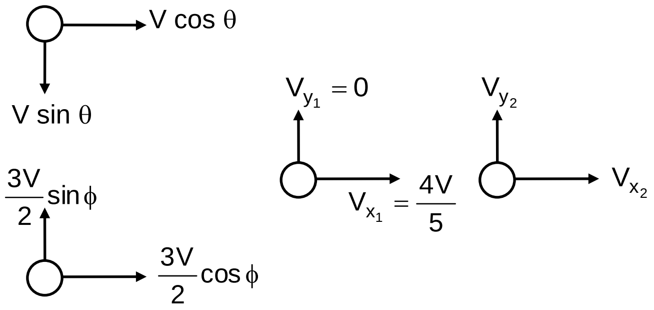
(b) The centre of mass is moving along x-axis before the collision

(c) The velocity of centre of mass after the collision is $\frac{5}{2}v$

(d) The total linear momentum of the system before the collision along x-axis is $\frac{5}{6}Mv$

Ans. (b)

Sol.



$P_i = P_f$ Along y-axis

$$M\frac{3}{2}V\sin\phi - 2MV\sin\theta = MV_{y_2}$$

$$\frac{3MV}{2}\frac{4}{5} - 2MV\frac{3}{5} = MV_{y_2}$$

$$\frac{6V}{5} - \frac{6V}{5} = V_{y_2}$$

$$V_{y_2} = 0$$

$P_i = P_f$ along x-axis

$$2MV\cos\theta + M\frac{3}{2}V\cos\phi = 2M\left(\frac{4V}{5}\right) + MV_{x_2}$$

$$2MV.\frac{4}{5} + \frac{3}{2}MV\frac{3}{5} = \frac{8MV}{5} + MV_{x_2}$$

$$\frac{25}{5\times 2}MV - \frac{8MV}{5} = MV_{x_2}$$

$$\frac{5}{2}V - \frac{8V}{5} = V_{x_2}$$

$$\frac{9V}{10} = V_{x_2}$$

Velocity of COM,
along x-axis

$$V_{C_m} = \frac{2MV\cos\theta + \frac{3}{2}\cos\phi}{2M + m} = \frac{2V\cos\theta + \frac{3}{2}V\cos\phi}{3}$$

$$= \frac{2V}{3}\times\frac{4}{5} + \frac{V}{2}\frac{3}{5}$$

$$= \frac{8V}{15} + \frac{3V}{10}$$

$$= \frac{16V + 9V}{30}$$

$$= \frac{25V}{30} = \frac{5V}{6}$$

$$P_{C_m} = 3M\left(\frac{5V}{6}\right) = \frac{15mV}{6}$$

V_{C_m} along y-axis = 0

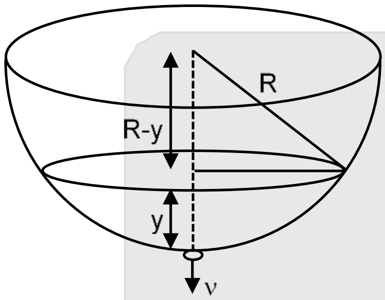
Ans. (b)

8. A large hemispherical water tank of radius R is filled with water initially upon a height $h = \frac{R}{2}$. The water starts dripping out through a small orifice of cross-section area 'a' at its spherical bottom. The time taken to get the tank completely empty (neglect viscosity) is

(a) $t = \frac{19\pi R^2}{60a}\sqrt{\frac{R}{g}}$ (b) $t = \frac{3\pi R^2}{10a}\sqrt{\frac{R}{g}}$ (c) $t = \frac{17\pi R^2}{60a}\sqrt{\frac{R}{g}}$ (d) $t = \frac{\pi R^2}{4a}\sqrt{\frac{R}{g}}$

Ans. **(c)**

Sol.



$$a\sqrt{2gy} = \pi(2Ry - y^2)V$$

$$V = \frac{a}{\pi}\sqrt{2g}\frac{\sqrt{y}}{(2Ry - y^2)}$$

$$-\frac{dy}{dt} = \frac{a}{\pi}\frac{\sqrt{2g}\sqrt{y}}{y(2R-y)}$$

$$\int_{R/2}^{0}\sqrt{y}(2R-y)\,dy = -\frac{a}{\pi}\sqrt{2g}\int_{0}^{t}dt$$

$$\int_{0}^{R/2}2R\sqrt{y}\,dy - \int_{0}^{R/2}y^{3/2}dy = \frac{a}{\pi}\sqrt{2g}(t)$$

$$\frac{2(2R)}{3}\left(\frac{R}{2}\right)^{3/2} - \frac{2}{5}\left(\frac{R}{2}\right)^{5/2} = \frac{a}{\pi}\sqrt{2g}\,t$$

$$\left(\frac{4}{3\times 2\sqrt{2}} - \frac{2}{5}4.\sqrt{2}\right)R^{5/2}\frac{\pi}{a\sqrt{2g}} = t$$

$$\left(\frac{2}{3\sqrt{2}} - \frac{1}{10\sqrt{2}}\right)\frac{R^2\pi}{a\sqrt{2}}\sqrt{\frac{R}{g}} = t$$

$$\left(\frac{2}{3} - \frac{1}{10}\right)\frac{R^2\pi}{2a}\sqrt{\frac{R}{g}} = t$$

$$\frac{17\pi R^2}{60a}\sqrt{\frac{R}{g}} = t$$

9. If Pascal (Pa), the unit of pressure volt(V), the unit of potential and meter (L), the unit of length are taken as fundamental units, the dimensional formula for the permittivity ε_0 of free space is expressed as

(a) $Pa^{-1} V^2 L^{-2}$ (b) $Pa^1 V^{-2} L^2$ (c) $Pa^1 V^2 L^{-2}$ (d) $Pa^{-1} V^{-2} L^2$

Ans. **(b)**

Sol. $\varepsilon_0 \propto (Pa)^a (V)^b (L)^c$

$$\varepsilon_0 = \frac{q^2}{Fr^2} = \frac{q^2}{ML^3T^{-2}}$$

$$q^2M^{-1}L^{-3}T^2 \propto (ML^{-1}T^{-2})^a \left(\frac{ML^2T^{-2}}{q}\right)^b L^c$$

$$q^2M^{-1}L^{-3}T^{+2} \propto M^{a+b} L^{-a+2b+c} T^{-2a-2b} q^{-b}$$

by comparison

$\therefore$ $-b = 2$

$b = -2$

$a + b = -1$

$a = 1$

$-q + 2b + c = -3$

$-(1) + 2(-2) + c = -3$

$C = 2$

$\varepsilon_0 \propto (Pa)^1 (V)^{-2} (L)^2$

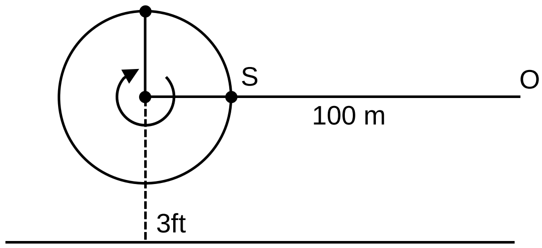
Ans. (b)

10. A cycle wheel of mass M and radius R fitted with a siren at a point on its circumference, is mounted with its plane vertical on horizontal axle at about 3 feet above the ground. An observer stands in the vertical plane of the wheel at 100m away from the axle of the wheel on a horizontal platform. The siren emits a sound of frequency 1000 Hz and the wheel rotates clockwise with uniform angular speed $\omega = \pi$ rad/sec. Initially at t = 0 sec. the siren is nearest to the observer and moves downwards. The observer records the highest pitch of sound for the first time after (speed of sound in air is 330 ms^{-1})

(a) 0.30 s (b) 1.8 s (c) 2.3 s (d) 9.8 s

Ans. **(b)**

Sol.



$\theta = \frac{3\pi}{2} = \omega t$

$= \pi t$

t = 3/2 m = 1.5 m

time to receives sound signal = $\frac{100}{330} = 0.3\,\text{sec}$

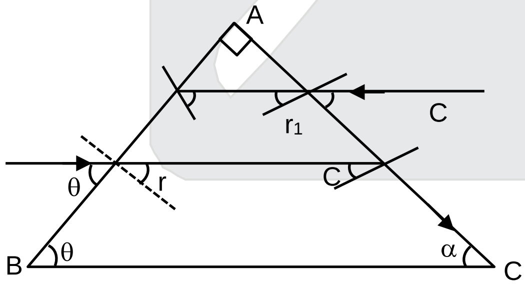
total time = 1.5 + 0.3 = 1.8 sec

11. On a right angled transparent triangular prism ABC, when a ray of light is incident on face AB, parallel to the hypotenuse BC, it emerges out of the prism grazing along the surface AC. If instead the rays is made incident on face AC, parallel to the hypotenuse CB it gets totally reflected on face AB. The refractive index μ of the material of the prism is :

(a) $\mu > \sqrt{2}$ (b) $\sqrt{2} > \mu > \sqrt{\frac{3}{2}}$ (c) $\sqrt{3} > \mu > \sqrt{2}$ (d) $\mu < \sqrt{\frac{3}{2}}$

Ans. **(b)**

Sol.



$1 \sin(90 - \theta) = \mu \sin r = \mu \sin(90 - C)$

$\cos\theta = \mu \cos C$ ...(i)

$1 \sin(90 - \alpha) = \mu \sin r_1$

$\sin r_1 = \frac{\cos\alpha}{\mu} < \sin(90 - C)$

$\cos\alpha < \mu \cos C$ ...(ii)

from (i)

$\mu \cos C < 1$

$\mu\sqrt{1-\frac{1}{\mu}} < 1$

$\mu^2 - 1 < 1$

$\mu^2 < 2$

$\mu < \sqrt{2}$

From (i) & (ii)

$\cos^2\theta + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - \cos^2\theta < \mu^2\cos^2C$

$1 - \mu^2\cos^2C < \mu^2\cos^2C$

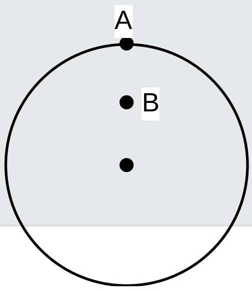
$2\mu^2\cos^2C > 1$

$2\mu^2\left(1-\frac{1}{\mu^2}\right) > 1$

$2\mu^2 - 2 > 1$

$\mu^2 > \frac{3}{2}$

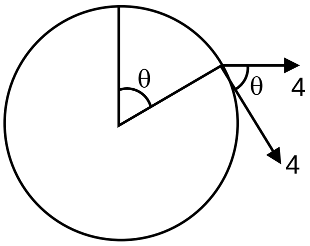
12. A circular disc of radius R = 10 cm is uniformly rolling on a horizontal surface with a velocity v = 4 ms^{-1} of centre of mass without slipping, the time taking by the disc to have the speed of point A (which lies on the circumference) equal to the present speed of point B (point B lies midway between centre and the point A) is



(a) t = 0.025 s (b) t = 0.0036 s (c) t = 0.046 s (d) t = 0.064 s

Ans. **(b)**

Sol.



ω = V/R = 40 rad/m

$V_B = r\omega + v$ = 6 m/sec

$V_A = 2 \times 4 \times \cos\theta/2$

$$\cos\frac{\theta}{2} = \frac{3}{4}$$

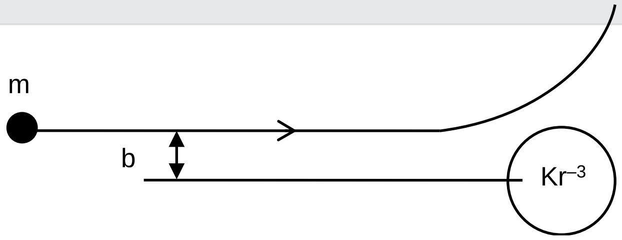
$$\frac{\theta}{2} = 41.5^\circ$$

$$\theta = 83^\circ = 83 \times \frac{\pi}{180} = 1.45\,\text{rad}$$

$$t = \frac{1.45}{40} = 0.036\ \text{sec}$$

ans (b)

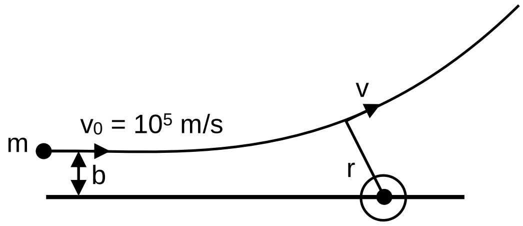
13. As shown in the figure, a particle of mass m = 10^{-10} kg, moving with velocity $v_0 = 10^5$ m/s approaches a stationary fixed target with impact parameter b from a large distance. If the fixed rigid target has a core with repulsive central force $F(r) = \frac{K}{r^3}$ where constant K > 0 and the particle scatters elastically. The closest distance of approach (if numerically K = b^2) is



(a) b (b) $b\sqrt{2}$ (c) $b\sqrt{3}$ (d) 2b

Ans. **(b)**

Sol.



$mv_0b = mvr$

$vr = v_0b$ .....(1)

$F = \frac{K}{r^3}$

$U = \frac{K}{2r^2}$, $y = 0$

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{k}{2r^2}$

$\frac{1}{2}\times10^{-10}\times10^{10} = \frac{1}{2}\times10^{-10}\times\frac{V_0^2b^2}{r^2} + \frac{k}{2r^2}$

$0.5 = 0.5\frac{b^2}{r^2} + \frac{k}{2r^2} = \frac{k}{r^2}$ as $k = b^2$

$\frac{k}{r^2} = 0.5$

$r^2 = 2k$

$r = \sqrt{2k} = \sqrt{2}b$

14. If the specific activity of C^{14} nuclide in a certain ancient wooden toy is known to be $\frac{3}{5}$ of that in a recently fallen tree of the same class, the age of the ancient wooden toy is (The half-life of C^{14} is 5570 years)

(a) 5570 years (b) 4105 years (c) 3342 years (d) 2785 years

Ans. **(b)**

Sol. $A = \frac{3}{5}A_0$

$e^{-\lambda t} = \frac{3}{5}$

$t = \frac{1}{\lambda}\ell n\frac{5}{3}$

$= \frac{T_H.\ell n\frac{5}{3}}{\ell n2}$

$= \frac{0.510}{0.693}\times5570$

= 4105 year

In questions 15 and 16 mark your answer as

(a) If statement-I is true and statement-II is true and also if the statement-II is a correct explanation of statement-I

(b) If statement-I is true and statement-II is true but the statement-II is a not a correct explanation of statement-I

(c) If statement-I is true but the statement-II is false

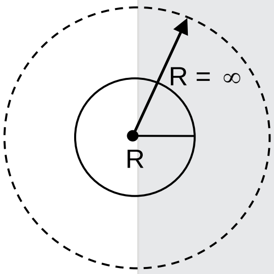
(d) If statement-I is false but statement-II is true

15. Statement-I: Work done in bringing a charge q from infinity to the center of a uniformly charged non-conducting solid sphere of radius R (with a total charge Q) is zero.

Statement-II: The potential difference between the centre and the surface of the uniformly charged non–conducting solid sphere of radius R (with a total charge Q) is $\frac{1}{4\pi\varepsilon_0}\times\frac{Q}{2R}$.

Ans. **(d)**

Sol.



$u_1 = \frac{1}{2}\theta v$ → For radius R

$= \frac{1}{2}\theta \times \frac{\theta}{4\pi\varepsilon R}$

$u_1 = \frac{\theta^2}{8\pi\varepsilon R}$ → For radius R → ∞

$u = u_1 + u_2$

$= \frac{\theta^2}{\theta\pi\varepsilon R} + 0 = \frac{\theta^2}{8\pi\varepsilon R}$

16. Statement-I: The current flowing through a p-n junction is more in forward bias then that in the reverse bias.

Statement-II: The diffusion current, dominant in forward bias, is more than the drift current, dominant in the reverse bias.

Ans. **(a)**

Sol. Holes diffused from the t\p-side to n-side while electron diffuse from n-side to the p-side this is due to concentration gradient across p and n side p. Which giving ripe to a diffusion current across the function.

17. Consider the process of the melting of a spherical ball of ice originally at 0°C. Assuming that the heat is being absorbed uniformly through the surface and the rate of absorption is proportional to the instantaneous surface area. Which of the following is true for the radius (r) of the ice ball at any instant of time? Assume that the radius of the ice ball at t = 0 is r = R_0 and that the shape of the ball always remains spherical during melting. Also assume that L and ρ are respectively the latent heat and density of ice at 0°C.

(a) radius decreases exponentially with time as $r = R_0 e^{-\frac{kt}{\rho L}}$. Here k is constant

(b) radius decrease exponentially with time as $r = R_0 e^{-\frac{k\rho t}{2L}}$

(c) radius of the ice ball decreases with time linearly with a slope $-\frac{k}{\rho L}$

(d) radius of the ice ball decreases with time linearly with a slope $-\frac{k}{2L}$

Ans. **(c)**

Sol. $\left(\frac{dm}{dt}\right)L = -KA$

$\frac{dm}{4\pi r^2}L = Kdt$

$\frac{\rho 4\pi r^2 dr}{4\pi r^2}L = -Kdt$

$-\frac{\rho L}{k}\int dr = \int dt$

$-\frac{\rho L}{k}(R_0 - R_0) = t$

$\frac{\rho L}{k}(R_0 - R) = t$

$R_0 - R = \frac{kt}{\rho L}$

$R = R_0 - \frac{kt}{\rho L}$

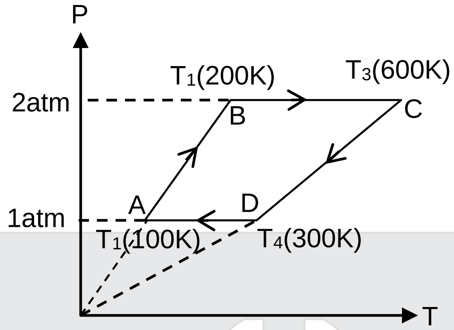
$R = -\frac{k}{\rho L}t + R_0$

Slope = $-\frac{k}{\rho L}$

18. The work done by the three moles of an ideal gas in the cyclic process ABCD shown in the diagram is approximately. Given that

T_1 = 100 K, T_2 = 200 K and

T_3 = 600 K, T_4 = 300 K



(a)7.5 kJ (b) 5.0 kJ (c) 2.5 kJ (d) zero

Ans. **(b)**

Sol. $\mu = 3$,

cyclic process

T_1 = 100 K

T_2 = 200 K

T_3 = 6000 K

T_4 = 300 K

AB

$P \propto T$ C $\because$ V $\rightarrow$ Constant)

Similarly W.D. AB and CD is zero.

W.D = W_{BC} + W_{DA}

$\downarrow$ $\downarrow$

Isobaric Isobaric

= μR (600 – 200) + μR (100 – 300)

= μR [400 – 200]

= (μR × 200) i = 5 kJ

19. The molar specific heat capacity of a certain gas is expressed as $C = C_V + \alpha\frac{P}{T}$. The equation of state for the process can be written as (α & A are constant)

(a) PV = RT (b) $V = aT^2$ (c) $V^2 = \alpha \ell nT$ (d) $T = Ae^{\frac{V}{\alpha}}$

Ans. **(d)**

Sol. $C = C_V + \alpha \dfrac{P}{T}$

$$\frac{d\theta}{ndT} = C_V + \alpha \frac{P}{T}$$

$$\frac{dn}{ndT} + \frac{PdV}{ndt} = C_V + \frac{\alpha P}{T}$$

$$C_V + \frac{PdV}{dt} = C_V + \frac{\alpha P}{T}$$

$$\frac{PdV}{dt} = \frac{\alpha P}{T}$$

$$dV = \alpha \frac{dT}{T}$$

$$V = \alpha(\ln T - \ln A)$$

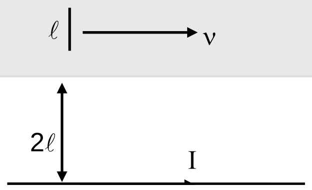
$$V = \alpha \ln \frac{T}{A}$$

$$\frac{V}{\alpha} = \ln \frac{T}{A}$$

$$\frac{T}{A} = e^{V/\alpha}$$

$$T = Ae^{V/\alpha}$$

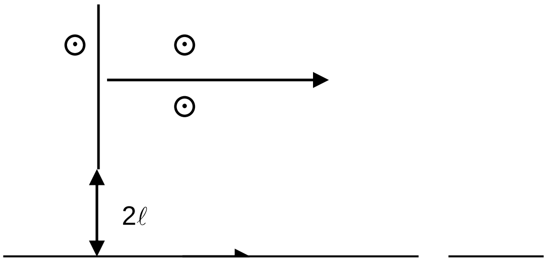
20. A metal bar of length ℓ moves with a velocity v parallel to an infinitely long straight wire carrying a current I as shown in the figure. If the nearest end of the perpendicular bar always remains at a distance 2ℓ from the current carrying wire, the potential difference (in volt) between two ends of the moving bar is :



(a) $\dfrac{\mu_0 I v}{2\pi}$ (b) $\dfrac{\mu_0 I v}{6\pi}$ (c) $\dfrac{\mu_0 I v}{2\pi}\ell n2$ (d) $\dfrac{\mu_0 I v}{2\pi}\ell n1.5$

Ans. **(d)**

Sol.



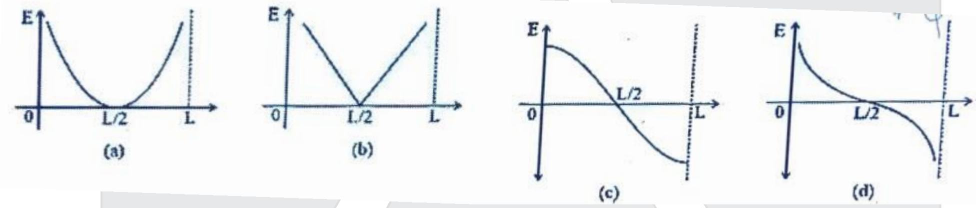
consider a element of length dy at a distance y from the wire

$$de = \frac{\mu_0}{2\pi y} idy.V$$

$$de = \int_{\ell}^{2\ell+\ell} \frac{\mu_0 iv}{2\pi}\frac{dy}{y} = \frac{\mu_0 iv}{2\pi}[\log y]_{2\ell}^{3\ell}$$

$$= \frac{\mu_0 iv}{2\pi}\log\frac{3\ell}{2\ell} = \frac{\mu_0 iv}{2\pi}\log(1.5)$$

21. Two point charges +Q each are located at (0, 0) and (L, 0) at a distance L apart on the X-axis. The electric field (E) in the region 0 < x < L is best represented by



(a) figure a (b) figure b (c) figure c (d) figure d

Ans. **(d)**

Sol.

Let
$Q_1 = Q$

Let A, B and C are three points

At A

$$E_{net} = \frac{kQ_1}{0^2} + \frac{KQ_2}{L^2} = \infty + \frac{KQ^2}{L^2} \rightarrow \infty$$

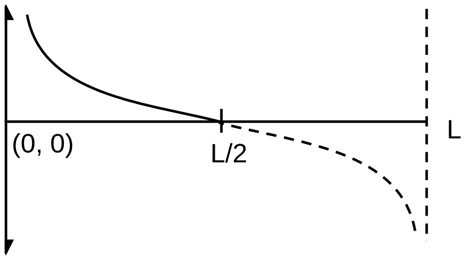
at B

$$E_{net} = E_1(\hat{i}) + E_2(-\hat{i})$$

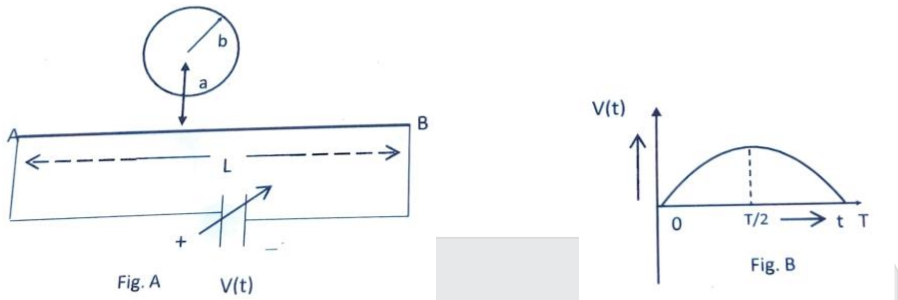
But $|E_1| = |E_2| = 0$

Between B and C

$E_1 < E_2 \rightarrow$ resultant electric field is in $(-\hat{i})$



22. A long straight wire AB of length L(L >> a, L >> b) and resistance R is connected to a time varying source of emf V(t). The variation of applied emf V(t) with time is shown in Figure B. A circular metallic loop of radius r = b is placed coplanar with the current carrying wire with its centre at a distance a from the axis of the wire as shown. The induced current in the loop is



(a) clockwise from 0 to T/2 and anticlockwise from T/2 to T

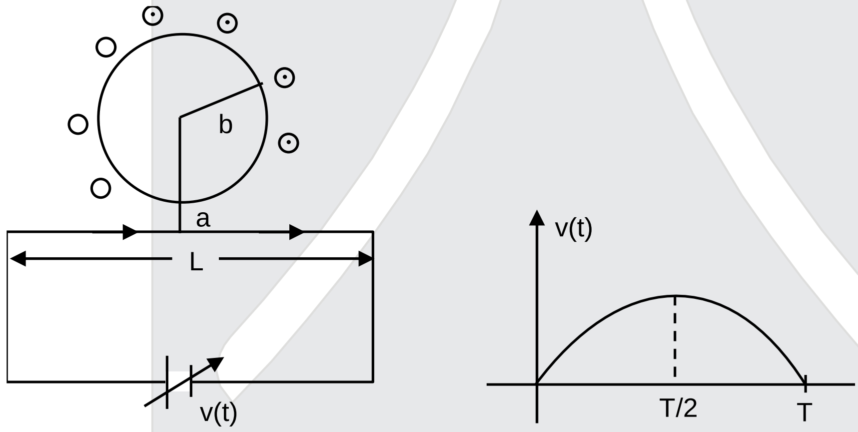
(b) anticlockwise from 0 to T/2 and clockwise from T/2 to T

(c) clock wise from 0 to T

(d) anticlockwise from 0 to T

Ans. **(a)**

Sol.



Slope of V–t

$\frac{dv(t)}{dt} = (+) \rightarrow$ from $\left(0 - \frac{T}{2}\right)$

$\frac{dv(t)}{dt} = (-)$ for $\left(\frac{T}{2} \rightarrow T\right)$

$e = -\frac{\mu_0 b^2}{2a} \frac{1}{R} \cdot \frac{dv(t)}{dt}$

$e \propto -\frac{dv(t)}{dt} \rightarrow$ according to lenz law

$\rightarrow$ 0 to T/2 $\left(\frac{dv}{dt}(t)\right) \rightarrow$ induced current will be C.W.

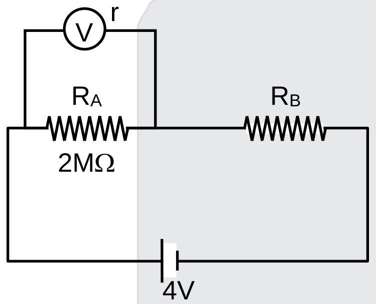
$\rightarrow$ T/2 to T $\left(\frac{dv}{dt}(t)\right) \rightarrow$ induced current will be A.C.W.

23. A simple circuit consists of a known resistance R_A = 2MΩ and an unknown resistance R_B both in series with a battery of 9 volt and negligible internal resistance. When the voltmeter is connected across the resistance R_A, it measures 3 volt but when the same voltmeter is connected across R_B it reads 4.5 volt. The voltmeter measures 9V across the battery. Considering that the voltmeter has a finite resistance r, the correct option is

(a) R_B = 3MΩ and r = 6.0 MΩ
(b) R_B = 2.5 MΩ and r = 6.0 MΩ
(c) R_B = 4 MΩ and r = 12MΩ
(d) R_B = 2.5MΩ and r= 6.0 MΩ

Ans. **(a)**

Sol.



1st situation

$iR_B = 6V$

$$i\frac{rR_A}{r+R_A} = 3V$$

$$\therefore R_B = \frac{2rR_A}{r+R_A}$$

$$\frac{2}{R_B} = \frac{1}{r} + \frac{1}{R_A} \quad .....(i)$$

2nd situation

$iR_A = 4.5$ V $\qquad i\frac{rR_B}{r+R_B} = 4.5$

$$R_A = \frac{rR_B}{r+R_B}$$

$$\frac{1}{R_A} = \frac{1}{r} + \frac{1}{R_B} \quad .....(ii)$$

from (ii) in (i)

$$\frac{2}{R_A} - \frac{2}{r} = \frac{1}{r} = \frac{1}{R_A} \;;\; \frac{1}{R_A} = \frac{3}{r}$$

r = 3 R_A = 6 MΩ

$\therefore R_B$ = 3 MΩ

24. The optical powers of the objective and the eyepiece of a compound microscope are 100 D and 20 D respectively. The microscope magnification being equal to 50 when the final image is formed at d = 25 cm i.e., the least distance of vision. If the separation between the objective and the eyepiece is increased by 2 cm, the magnification of the microscope will be

(a) 62 (b) 50 (c) 38 (d) 25

Ans. **(a)**

Sol. f_0 = 1cm, fe = 5 cm

$\frac{\ell}{f_0}\left(\frac{d}{fe}+1\right)=50$ .....(i)

$\frac{\ell+2}{f_0}\left(\frac{D}{f_e}+1\right)=M$ ....(ii)

(ii) .... (ii) M – 50 = $\frac{2}{f_0}\left(\frac{D}{f_e}+1\right)$

= $\frac{2}{1}\left(\frac{25}{5}+1\right)$; M = 62

A-2

ANY NUMBER OF OPTIONS 4, 3, 2 or 1 MAY BE CORRECT

MARKS WILL BE AWARDED ONLY IF ALL CORRECT OPTIONS ARE BUBBLED AND NO WRONG OPTION

25. The electric field component of an electromagnetic wave is expressed as $E=(3j+bk)\times10^{-3}\sin[10^7(x+2y+3z-\beta t)$ in SI units. Taking c = 3 × 10^8 ms^{-1} as the speed of electromagnetic wave in vacuum, choose the correct option(s)

(a) The value of constant beta is β = 3 × 10^8 × $\sqrt{14}$

(b) The value of constant b is b = 2.

(c) The average energy density of the em wave is U = 6.5 × 10^{-6} ε_0 in SI units.

(d) The amplitude of magnetic field is B = 1.20 × 10^{-11} Tesla

Ans. **(a,c,d)**

Sol. $\vec{E}=(3\hat{j}+b\hat{k})\times10^{-3}\sin[10^7(x+2y+3z-\beta t]$

$\vec{k}=(\hat{i}+2\hat{j}+3\hat{k})\times10^7$

K = $\sqrt{1+4+9}\times10^7$ = $\sqrt{14}$ × 10^7

C = $\frac{\omega}{k}$

ω = ck

$\beta \times 10^7 = 3 \times 10^8 \times \sqrt{14} \times 10^7$

$\beta = 3\sqrt{14} \times 10^8$ .....(a)

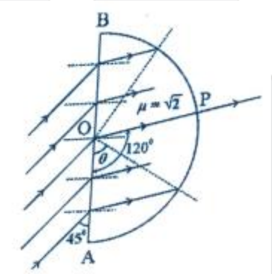
$\vec{E}.\vec{k} = 0$

6 + 3b = 0, b = – 2

$u = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2} \times \varepsilon_0 \times 13 \times 10^{-6} = 6.5\ \varepsilon_0 \times 10^{-6}$

$B_0 = \frac{E_0}{c} = \frac{\sqrt{14} \times 10^{-3}}{3 \times 10^8} = 1.2 \times 10^{-11}$ T

26. A parallel beam of light is made incident (as shown on the flat diametric plane of transparent semi-circular thin sheet of thickness t (t << R) of refractive index $\mu = \sqrt{2}$ at an angle of 45º. As a result of refraction, the light enters the semi-circular sheet and comes out at its curved surface.



(a) Light rays comes out at the curved surface for values of θ in the range $75º \le \theta \le 165º$.

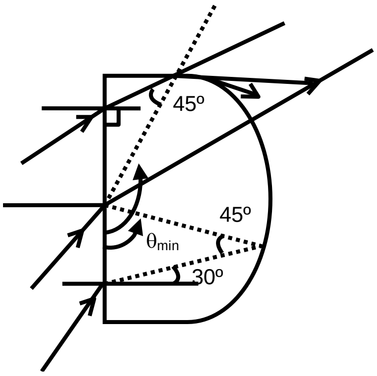
(b) The range of angle θ is independent of the angle of incidence.

(c) The range of angle θ depends on the refractive index of the material

(d) All the emergent rays of light shall cross the line OP which is a refracted ray at θ = 120º.

Here θ is the angle between the vertical diameter AB and the concerned radius of the semicircular sheet of radius R.

Ans. **(a,c,d)**



Sol. $1 \sin 45 = \sqrt{2} \sin r$

$\frac{1}{\sqrt{2}} = \sqrt{2} \sin r = 30º$

for $\mu = \sqrt{2}$, c = 45

$\theta_{max} = 120 + 45 = 165º$

$\theta_{min} = 180 - 60 - 45 = 75º$

27. A certain of uniform area of cross section A (a = 1.0 cm^2) with its length = 2 m is thermally insulated on its lateral surface. The thermal conductivity (K) of the material of the rod varies with temperature T as $K = \frac{a}{T}$ where α is a constant. The two ends of the rod are maintained at temperature of T_1 = 90ºC and T_2 = 10ºC. The correct option(s) is/are

(a) The temperature at 50 cm from the colder end is 17.32ºC

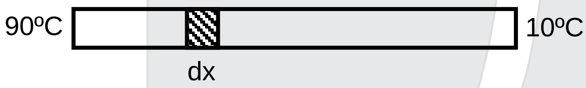
(b) The temperature at 50 cm from the hotter end is 51.96ºC

(c) The rate of heat flow per unit area of cross section of the rod is 1.1 α in SI units.

(d) The temperature gradient is numerically higher near the hot end compare to the near the cold end.

Ans. **(a,b,c,d)**

Sol. $k = \frac{\alpha}{T}$



$$q = -KA\frac{dT}{dx} = -\frac{\alpha}{T}A\frac{dT}{dx}$$

$$q\int_0^{\ell} dx = -\alpha A\int_{T_1}^{T_2}\frac{dT}{T}$$

$$q = \frac{\alpha A\ell n(T_1/T_2)}{\ell}$$

(a) $$\frac{\alpha A\ell n\left(\frac{90}{10}\right)}{4} = \frac{\alpha A\ell n\frac{T}{10}}{0.5}$$

$$\frac{T}{10} = 1.733$$

T_2 = 17.33

(b) $$\frac{\alpha A\ell n\frac{90}{10}}{4} = \frac{\alpha A\ell n\frac{90}{7}}{0.5}$$

$$T = \frac{90}{1.73} = 53^{o}\,C$$

(c) $$q/A = \frac{\alpha\ell n}{\ell}\frac{90}{10} = \frac{\alpha}{2}\times 2.2 = 1.1\,\alpha$$

(d) $$\frac{dT}{dx} = -\frac{q}{ka} = \frac{q.T}{\alpha A}$$

28. Positronium is a short-lived $(\approx 10^{-9}\,s)$ bound state of an electron and positron (a positively charged particle with mass and charge equal (in magnitude) to an electron) revolving round their common centre of mass. If E_0. v_0 and a_0 are respectively the ground state energy, the orbital speed of electron in first orbit and the radius of the first (n = 1) Bohr orbit for Hydrogen atom, the corresponding quantities E, v and a for the positronium are

(a) $E = \frac{E_0}{2}$ (b) $a = a_0$ (c) $a = 2a_0$ (d) $E = E_0$, $v = v_0$, $a = a_0$

Ans. **(a,b)**

Sol. H–atom

$a_0 = \frac{h^2 \epsilon_0}{\pi m e^2}$ $v_0 = \frac{e^2}{h \epsilon_0}$ $E_0 = -\frac{m e^4}{8n^2h^2 \epsilon_0}$

In positronium

$2 \times mvr = \frac{nh}{2\pi} = \frac{h}{2\pi}$

$vr = \frac{h}{4\pi m}$ ...(i)

$\frac{1}{4\pi \epsilon_0} \cdot \frac{e^2}{4r^2} = \frac{mv^2}{r}$

$v^2 r = \frac{e^2}{16\pi \epsilon_0 m}$ ...(ii)

(ii) / (i)

$v = \frac{e^2}{16\pi \epsilon_0 m} \times \frac{4\pi m}{h} = \frac{e^2}{4h \epsilon_0} = \frac{v_0}{4}$

$r = \frac{h}{4\pi m} \times \frac{4h \epsilon_0}{e^2} = \frac{4h^2 \epsilon_0}{4\pi m e^2} = a_0$ ...(b)

$E = \frac{1}{2} m v^2 \times L + \frac{e \in e}{4\pi \epsilon_0 2r}$

$= m \times \frac{e^4}{16h^2 \epsilon_0^2} - \frac{e^2}{4\pi \epsilon_0 .2} \times \frac{\pi m e^2}{h^2 \epsilon_0}$

$= -\frac{m e^4}{16h^2 \epsilon_0^2} = \frac{E_0}{2}$

29. A thin double convex lens of radii of curvature $R_1 = 20$ cm and $R_2 = 60$ cm is made-up of a transparent material refractive index $\mu = 1.5$. Choose the correct option(s).

(a) The focal length of the lens is f = 30 cm when in air.

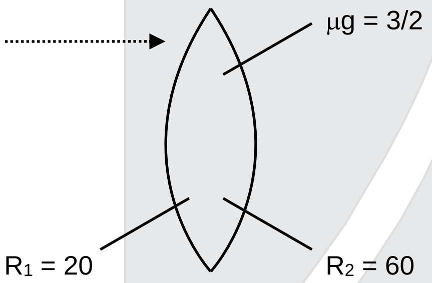
(b) The lens behaves as a concave mirror of focal length $f_M = 10$ cm when silvered on the surface of radius $R_2 = 60$ cm

(c) The lens behave as a concave lens (diverging lens) if the image space beyond R_2 60 cm radius surface is filled with a transparent liquid of refractive index $\mu = \frac{5}{3}$. The object space prior to the surface of radius $R_1 = 20$ cm is air.

(d) A beam of rays incident parallel to principal axis focuses at 48 cm behind the lens if water $\left(\mu = \frac{4}{3}\right)$ fills the entire space behind the surface of radius $R_2 = 60$ cm. The object space prior to the e surface of radius $R_1 = 20$ cm is air.

Ans. **(a,b,d)**

Sol. **(a)**



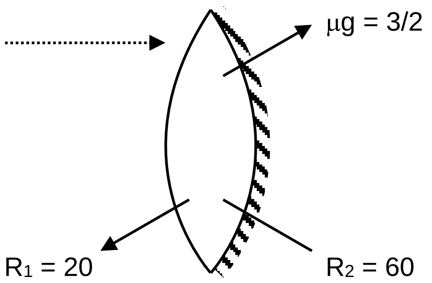
When is

$$\frac{1}{f} = \left(\frac{3/2}{1} - 1\right)\left\{\frac{1}{20} - \frac{1}{-60}\right\} = \left(\frac{3-2}{2}\right) \times \left\{\frac{3+1}{60}\right\}$$

$$= \frac{1}{2} \times \frac{1}{15}$$

f = 30 cm

(b)



$R_1 = 20$

$$\frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f\ell}$$

$$= \frac{1}{R/2} - \frac{2}{30}$$

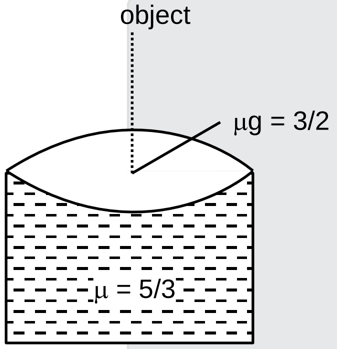
$$= \frac{1}{30} - \frac{1}{15}$$

$$= \frac{1-2}{30}$$

$$= -\frac{1}{30}$$

$F_{eq} = -30$ cm

(c)



$$\frac{\mu_t}{v} - \frac{\mu_i}{4} = \frac{\left(\frac{3}{2}-1\right)}{20} + \frac{\left(\frac{4}{3}-\frac{3}{2}\right)}{-60}$$

$$\frac{4}{3v} - \frac{1}{\infty} = \frac{3-2}{40} + \frac{\left(\frac{\theta-9}{6}\right)}{-60}$$

$$\frac{4}{3v} = \frac{1}{40} + \frac{1}{6 \times 60}$$

$$= \frac{1}{10}\left[\frac{1}{4} + \frac{1}{36}\right] = \frac{1}{10}\left[\frac{9+1}{36}\right] = \frac{1}{36}$$

$$\frac{4}{3v} = \frac{1}{36}$$

$$v = \frac{4 \times 36}{3}$$

= 48

30. A thick hollow cylinder of height h and inner and outer radii a and b (b > a) made up of a poorly conducting material of resistivity ρ lies conducting material of resistivity ρ lies coaxially inside a long solenoid at its middle. The radius of the solenoid is larger than b. Throughout the interior of the solenoid, a uniform time varying magnetic field $B = \beta t$ is produced parallel to solenoid axis. Here β is a constant. In this time varying magnetic field.

(a) the emf induced at a certain radius r (a < r < b) in the hollow cylinder is $\pi r^2 \beta$

(b) the induced current circulating in the thick hollow cylinder between radii a and b is

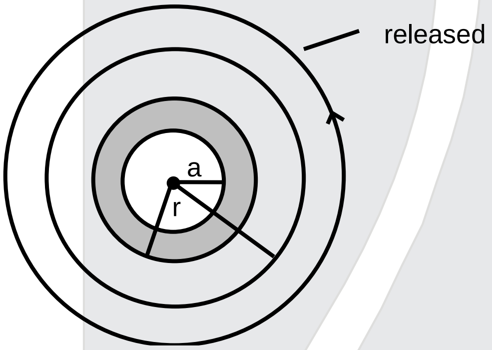
$$i = \frac{\beta h}{4\rho}\left(b^2 - a^2\right)$$

(c) the resistance offered to the circulation of current by the thick hollow cylinder is $R = \dfrac{2\pi\rho}{h \times \ell n \dfrac{b}{a}}$

(d) no electric field is detectable outside the solenoid.

Ans. **(a,b,c)**

Sol.



$B = \beta t$

(a) ϕ 2 $\pi r^2 B$

$$|e| = \left|\frac{d\phi}{dt}\right| 2\ \pi r^2\ \frac{dB}{dt} = \pi r^2 \beta$$

$$\text{(b) } \pi r^2\ \beta = e = di \times \rho.\ \frac{2\pi r}{drh}$$

$$di = \pi r^2 \beta\ \frac{drh}{2\pi\rho r}$$

$$i = \frac{\pi\beta h}{2\pi\rho}\int_a^b r dr = \frac{\beta h}{2\rho}\left(\frac{b^2 - a^2}{2}\right) = \frac{\beta h}{4\rho}(b^2 - a^2)$$

$$\text{(c) } d\left(\frac{1}{R}\right) = \frac{drh}{\rho 2\pi r}$$

$$\frac{1}{R} = \frac{h}{2\pi\rho}\int_a^b \frac{dr}{r} = \frac{h}{2\pi\rho}\ell n(b/a)$$

$$R = \frac{2\pi\rho}{h\ell n(b/a)}$$

31. A simple pendulum consisting of a small bob of mass m attached to a massless inextensible string of length ℓ = 2m, hanging vertically from the ceiling, is oscillating in a vertical plane with an angular amplitude θ_m such that the maximum tension in its string is three times the minimum tension in the string i.e., $T_{max} = 3T_{min}$. The correct option(s) is/are

(a) The maximum tension in the string is $T_{max} = mg(3 - 2\cos\theta_m)$

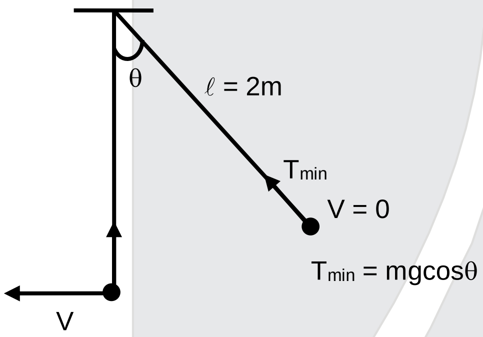
(b) The maximum tension in the string is $T_{max} = \frac{9}{5}mg$

(c) The maximum velocity of the bob on its way is $v_{max} = 3.96\ ms^{-1}$

(d) The angular amplitude θ_m lies in the range $\frac{\pi}{4} < \theta_m < \frac{\pi}{3}$

Ans. **(a,b,c,d)**

Sol.



$$\frac{1}{2}mv^2 = mg\ell(1-\cos\theta_m)$$

$$v = \sqrt{2g\ell(1-\cos\theta_m)}$$

option (A) $T_{max} = mg + \frac{mv^2}{\ell} = mg + \frac{m}{\ell}2g\ell(1-\cos\theta_m) = mg(3 - 2\cos\theta_m)$

$\frac{T_{max}}{T_{min}} = 3$ (given)

$mg + m2g(1 - \cos\theta_m) = 3\,mg\cos\theta_m$

$1 + 2 - 2\cos\theta = 3\cos\theta_m$

$3 = 5\cos\theta_m$

$\cos\theta_m = \frac{3}{5}$ $\qquad\theta_m = 53^o$ (D) option

option (B) $t_{max} = mg\left(3 - 2 \times \frac{3}{5}\right) = mg\left(\frac{9}{5}\right)$

option (C) $v_m = \sqrt{2g(2)\left(1-\frac{3}{5}\right)} = \sqrt{\frac{8g}{5}} = 3.96$ m/s

32. Two small masses m and M lie on a large horizontal frictionless circular track of radius R. The two masses are free to slide on the track but constrained to move along a circle. Initially the two masses are tied by a thread with a compressed spring between them (spring of negligible length being attached with none of the two masses). The compressed spring stores a potential energy U_0. At a certain time t = 0 the thread is burnt and the two masses are released to run opposite to each other leaving the spring behind. The total mechanical energy remaining conserved. On the circular track the two masses make a head on perfectly elastic collision. Take M = 2m for all calculation. Which of the following option(s) is /are correct ?

(a) The angle turned by mass m before the collision is $\theta = 4\frac{\pi}{3}$

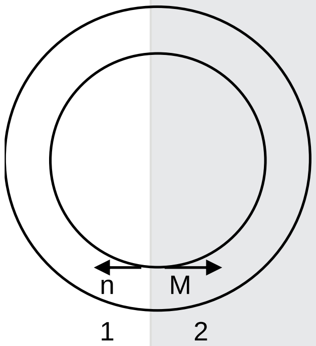
(b) The velocity of mass m on the track is $u = \sqrt{\frac{4U_0}{3m}}$

(c) The time taken to collide for the first time is $t_1 = 2\pi R\sqrt{\frac{m}{3U_0}}$

(d) The time taken for the second collision is $t_2 = 2\pi R\sqrt{\frac{2m}{3U_0}}$

Ans. **(a,b,c)**

Sol.



P = mv = mv ...(i)

$v = \frac{P}{m}$ $\qquad V = \frac{P}{M}$

as $\quad m = \frac{M}{2}$ $\qquad \therefore v = 2V$

a. $\quad Vt + vt = 2\pi R$

$vt + \frac{vt}{2} = 2\pi R$; $vt = \frac{4\pi R}{3}$; $\theta = \frac{4\pi}{3}$

b. $\quad \frac{K_1}{K_2} = \frac{M}{m} = \frac{2}{1}$ $\qquad K_1 + K_2 = U_0$

$\therefore \quad K_1 = \frac{2}{3}U_0 = \frac{1}{2}mv^2$; $\quad v = \sqrt{\frac{4}{3}\frac{U_0}{m}}$

c. $\quad t = \frac{4\pi R}{3v} = \frac{4\pi R}{3}\sqrt{\frac{3m}{4U_0}} = 2\pi R\sqrt{\frac{m}{3U_0}}$

d. $\quad$ as collision is elastic relative speed will be same after 1^{st} collision.