JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Official Based Questions & Solutions

Date: 1 February, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

61. Let $f(x) = 2x + \tan^{-1}x$ and $g(x) = \log_e\left(x+\sqrt{1+x^2}\right)$, $x \in [0, 3]$. Then

(1) min $f'(x) = 1 +$ max $g'(x)$

(2) there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x)$, $\forall x \in (x_1,x_2)$

(3) max $f(x) >$ max $g(x)$

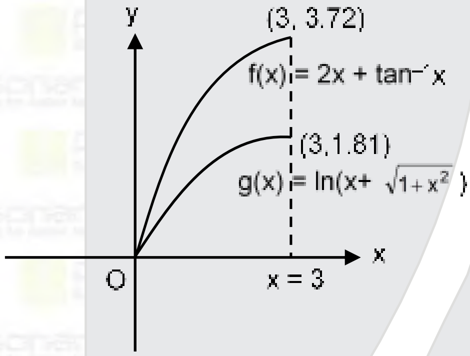
(4) there exists $x \in [0, 3]$ such that $f'(x) < g'(x)$

Ans. **(3)**

Sol. $f'(x) = 2+\dfrac{1}{1+x^2}$, $g'(x) = \dfrac{1}{\sqrt{1+x^2}}$

both are strictly increasing on [0, 3]

$f(0) = g(0) = 0$



$f(3) > g(3)$

so, max $f(x) >$ max $g(x)$

62. Let R be a relation on IR, given by R = {(a, b) : $3a - 3b + \sqrt{7}$ is an irrational number}. Then R is

(1) reflexive and symmetric but not transitive
(2) reflexive and transitive but not symmetric
(3) reflexive but neither symmetric nor transitive
(4) an equivalence relation

Ans. **(3)**

Sol. **(1)** $\forall a \in R$ $\quad 3a-3a+\sqrt{7} = \sqrt{7} \in Q^c$

so R is reflexive

(2) $\left(\dfrac{\sqrt{7}}{3},0\right) \in R \Rightarrow 3\times\dfrac{\sqrt{7}}{3} - 0 + \sqrt{7} = 2\sqrt{7} \in Q^c$

But for $\left(0,\dfrac{\sqrt{7}}{3}\right)$, $0-\sqrt{7}+\sqrt{7} = 0 \notin Q^c$

so $\left(0,\dfrac{\sqrt{7}}{3}\right) \notin R$

So R is not symmetric

(3) $\left(0,\sqrt{3}\right) \in R$ and $\left(\sqrt{3},\dfrac{\sqrt{7}}{3}\right) \in R$

But $\left(0,\dfrac{\sqrt{7}}{3}\right) \notin R$

So R is not transitive.

63. The shortest distance between the lines

$\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ is

(1) $7\sqrt{3}$ (2) $6\sqrt{3}$ (3) $4\sqrt{3}$ (4) $5\sqrt{3}$

Ans. (2)

Sol. The shortest distance between lines

$\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$

$\vec{r_1}=(5i+2j+4k)+\lambda_1(i+2j-3k)=\vec{a_1}+\lambda_1\vec{b_1}$

$\vec{r_2}=(-3i-5j+k)+\lambda_2(i+4j-5k)=\vec{a_2}+\lambda_2\vec{b_2}$

Shortest distance between $\vec{r_1}$ and $\vec{r_2}$ be

$$d=\left|\frac{\left(\vec{b_1}\times\vec{b_2}\right).\left(\vec{a_2}-\vec{a_1}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}\right|$$

Where , $\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-3\\1&4&-5\end{vmatrix}=2\hat{i}+2\hat{j}+2\hat{k}$

and $\left|\vec{b_1}\times\vec{b_2}\right|=2\sqrt{3}$

$$d=\left|\frac{(2\hat{i}+2\hat{j}+2\hat{k}).(-8\hat{i}-7\hat{j}-3\hat{k})}{2\sqrt{3}}\right|$$

$$d=\left|\frac{-16-14-6}{2\sqrt{3}}\right|$$

$$d=\frac{36}{2\sqrt{3}}=6\sqrt{3}$$

64. The sum to 10 terms of the series

$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+.......$is

(1) $\frac{56}{111}$ (2) $\frac{58}{111}$ (3) $\frac{55}{111}$ (4) $\frac{59}{111}$

Ans. (3)

Sol. $t_n=\frac{n}{1+n^2+n^4}=\frac{n}{(n^2+n+1)(n^2-n+1)}$

$=\frac{1}{2}\left(\frac{1}{(n^2-n+1)}-\frac{1}{(n^2+n+1)}\right)$

$t_1=\frac{1}{2}\left(1-\frac{1}{3}\right)$

$t_2 = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{7}\right)$

$t_3 = \frac{1}{2}\left(\frac{1}{7} - \frac{1}{13}\right)$

|
|
|

$t_{10} = \frac{1}{2}\left(\frac{1}{91} - \frac{1}{111}\right)$

$\therefore S_{10} = t_1 + t_2 + t_3 + \ldots + t_{10}$

$= \frac{1}{2}\left(1 - \frac{1}{111}\right) = \frac{55}{111}$

65. Let S = $\left\{x : x \in \mathbb{R} \text{ and } \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10\right\}$. Then n(S) is equal to

(1) 2 (2) 4 (3) 0 (4) 6

Ans. **(2)**

Sol. $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10$

$\left(\left(\sqrt{3}+\sqrt{2}\right)^2\right)^{\frac{x^2-4}{2}} + \left(\left(\sqrt{3}-\sqrt{2}\right)^2\right)^{\frac{x^2-4}{2}} = 10$

$\Rightarrow \left(5+2\sqrt{6}\right)^{\frac{x^2-4}{2}} + \left(5-2\sqrt{6}\right)^{\frac{x^2-4}{2}} = 10$

Now $\left(5+2\sqrt{6}\right)\left(5-2\sqrt{6}\right) = 25 - 24 = 1$

& $\left(5+2\sqrt{6}\right) + \left(5-2\sqrt{6}\right) = 10$

$\therefore \frac{x^2-4}{2} = \pm 1 \quad \Rightarrow x^2 = 4 \pm 2$ → 6, 2

$\Rightarrow x = \pm\sqrt{6}, \pm\sqrt{2}$

$\therefore$ n (S) = 4

66. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is

(1) 1216 (2) 1456 (3) 1072 (4) 1792

Ans. **(3)**

Sol. Let remaining two observations are α, β.

Mean = $\frac{1+3+5+\alpha+\beta}{5} = 5$

$\alpha + \beta = 16$ ____(1)

$\sigma^2 = \frac{1+9+25+\alpha^2+\beta^2}{5} - 5^2$

$8 = \frac{35+\alpha^2+\beta^2}{5} - 25$

$33 \times 5 - 35 = \alpha^2 + \beta^2$

$\alpha^2 + \beta^2 = 130$ _____(2)

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$256 - 130 = 2\alpha\beta \Rightarrow \alpha\beta = 63$

$\alpha^3 + \beta^3 = (\alpha+ \beta)^3 - 3\alpha\beta\ (\alpha + \beta)$

$= 4096 - 3024$

$= 1072$

67. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi,\ x \in \left[-\frac{1}{2},\frac{1}{2}\right]$. Then $\sum_{x\in S} 2\sin^{-1}(x^2-1)$ is equal to

(1) 0 (2) $\frac{-2\pi}{3}$ (3) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (4) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Ans. **(2)**

Sol. Given, $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi,\ x \in \left[-\frac{1}{2},\frac{1}{2}\right]$

$\therefore -2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi - \cos^{-1}(2x)$

by using $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$

$\Rightarrow -\cos^{-1}\left(2\left(\sqrt{1-x^2}\right)^2 - 1\right) = \pi - \cos^{-1}(2x)$

$\Rightarrow -\cos^{-1}(2 - 2x^2 - 1) = \pi - \cos^{-1}(2x)$

$\Rightarrow \cos^{-1}(1 - 2x^2) = -\pi + \cos^{-1}(2x)$

$\Rightarrow 1 - 2x^2 = \cos(-\pi + \cos^{-1}(2x))$

$\Rightarrow 1 - 2x^2 = -2x$

$\Rightarrow 2x^2 - 2x - 1 = 0$

$\Rightarrow x = \frac{-(-2)\pm\sqrt{4+4.2}}{4}$

$\Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$

$\therefore x = \frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}$

So, $x = \frac{1-\sqrt{3}}{2} \in \left[-\frac{1}{2},\frac{1}{2}\right]$

$\therefore S = \{x : x = \frac{1-\sqrt{3}}{2}\}$

$\therefore \sum_{x\in S} 2\sin^{-1}(x^2-1) = 2\sin^{-1}\left(\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)^2 - 1\right) = 2\sin^{-1}\left\{\frac{1}{4}+\frac{3}{4}-\frac{\sqrt{3}}{2}-1\right\}$

$= 2\sin^{-1}\left\{-\frac{\sqrt{3}}{2}\right\} = 2 \times \left(-\frac{\pi}{3}\right) = \frac{-2\pi}{3}$

68. The value of $\frac{1}{1!\,50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+......+\frac{1}{49!2!}+\frac{1}{51!\,1!}$ is

(1) $\frac{2^{51}}{50!}$ (2) $\frac{2^{50}}{50!}$ (3) $\frac{2^{51}}{51!}$ (4) $\frac{2^{50}}{51!}$

Ans. **(4)**

Sol. $\frac{1}{1!\,50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+......+\frac{1}{49!2!}+\frac{1}{51!\,1!}$

$\frac{1}{51!}\left(\frac{51!}{1!50!}+\frac{51!}{3!48!}+.........+\frac{51!}{49!2!}+\frac{51!}{51!0!}\right)$

$\frac{1}{51!}\left({}^{51}C_1+{}^{51}C_3+.........+{}^{51}C_{51}\right)$

$\frac{1}{51!}\left(\frac{2^{51}}{2}\right)=\frac{2^{50}}{51!}$

69. $\lim_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+.....+\frac{1}{2n}\right)$ is equal to

(1) $\log_e 2$ (2) $\log_e\left(\frac{2}{3}\right)$ (3) 0 (4) $\log_e\left(\frac{3}{2}\right)$

Ans. **(1)**

Sol. $\lim_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+.....+\frac{1}{2n}\right)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r+n}$

$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\left(\frac{r}{n}+1\right)}\cdot\frac{1}{n}$

$\int_0^1\frac{dx}{1+x}=\left[\ell n(1+x)\right]_0^1$

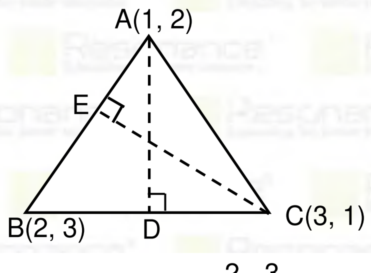
$\ell n2-\ell n1=\ell n2$

70. If the orthocentre of the triangle, whose vertices are (1, 2), (2, 3) and (3, 1) is (α, β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is

(1) $x^2 - 19x + 90 = 0$ (2) $x^2 - 20x + 99 = 0$ (3) $x^2 - 18x + 80 = 0$ (4) $x^2 - 22x + 120 = 0$

Ans. **(2)**

Sol.



Slope (AB) = $m_{AB}=\frac{2-3}{1-2}=1$

$\therefore$ Slope (CE) = – 1

$\therefore$ Equation of CE is: y – 1 = –1(x – 3)

$\Rightarrow$ y – 1 = –x + 3

$\Rightarrow$ x + y = 4 ……(i)

Similarly $m_{BC} = \frac{2}{-1} = -2$

Equation of AD is: $y - 2 = \frac{1}{2}(x - 1)$

2y – 4 = x – 1

$\therefore$ x – 2y + 3 = 0 …….(ii)

Solving equation (i) &(ii)

We have $x = \frac{5}{3}$, $y = \frac{7}{3}$

$\therefore \alpha = \frac{5}{3}, \beta = \frac{7}{3}$

$\therefore \alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$

$4\alpha + \beta = 4 \times \frac{5}{3} + \frac{7}{3} = \frac{27}{3} = 9$

$\therefore$ Quadratic Equation is,

x^2 – (sum of roots)x + (product of roots) = 0

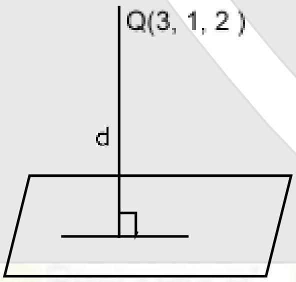
$x^2 - 20x + 99 = 0$

71. Let the image of the point P(2,–1,3) in the plane x + 2y – z = 0 be Q. Then the distance of the plane 3x + 2y + z + 29 = 0 from the point Q is

(1) $2\sqrt{14}$ (2) $\frac{22\sqrt{2}}{7}$ (3) $3\sqrt{14}$ (4) $\frac{24\sqrt{2}}{7}$

Ans. **(3)**

Sol. Image of point P(2, –1, 3) in the plane x + 2y –z = 0 be



$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-1} = \frac{-2(2-2-3)}{(1)^2+(2)^2+(-1)^2}$$

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-1} = \frac{6}{6} = 1$$

so , x – 2 = 1 , y + 1 = 2 , z – 3 = –1

$\therefore$ x = 3 , y = 1 , z = 2

So : Q (3, 1 , 2)

$$\therefore\ d = \left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+2^2+1^2}}\right|$$

$$d = \left|\frac{9+4+29}{\sqrt{14}}\right|$$

$$d = \frac{42}{\sqrt{14}} = 3\sqrt{14}$$

72. Let S denote the set of all real values of λ such that the system of equations

$\lambda x + y + z = 1$
$x + \lambda y + z = 1$
$x + y + \lambda z = 1$

is inconsistent, then $\sum_{\lambda \in S}\left(|\lambda|^2 + |\lambda|\right)$ is equal to

(1) 12 (2) 4 (3) 2 (4) 6

Ans. (4)

Sol. $D = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda-1)^2(\lambda+2)$ If $D = 0 \Rightarrow \lambda = 1, -2$

$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda-1)^2$ When $\lambda = -2$, $D \neq 0$

when $\lambda = 1$, all equations are identical so number of solutions are infinite.
So for inconsistent system , λ can be equal to – 2 only.

$\therefore \sum_{\lambda \to 5}\left(|\lambda|^2 + |\lambda|\right) = |-2|^2 + |-2| = 4 + 2 = 6$

73. Let f(x) = $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then

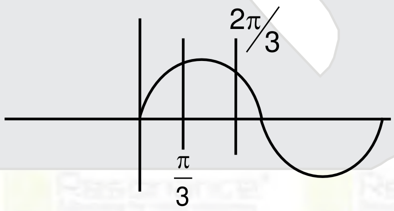
(1) $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$ (2) $\alpha^2 + \beta^2 = \frac{9}{2}$ (3) $\alpha^2 - \beta^2 = 4\sqrt{3}$ (4) $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$

Ans. (4)

Sol. $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin(2x) \\ \sin^2 x & \cos^2 x & 1+\sin(2x) \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

$R_1 \to R_1 - R_2$
$R_2 \to R_2 - R_3$

$f(x) = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \sin^2 x & \cos^2 x & 1+\sin(2x) \end{vmatrix}$



$= 1\,(1+\sin(2x)) + \cos^2 x + \sin^2 x$

$f(x) = 2 + \sin 2x$ $\left[\begin{matrix} x \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right) \\ 2x \in \left(\frac{\pi}{3}, \frac{2\pi}{3}\right) \end{matrix}\right]$

$\max(f(x)) = 3 = \alpha$

$\text{Min}(f(x)) = 2 + \frac{\sqrt{3}}{2} = \beta$

$\therefore \beta^2 = \left(2 + \frac{\sqrt{3}}{2}\right)^2 = 4 + \frac{3}{4} + 2\sqrt{3} = \frac{19}{4} + 2\sqrt{3}$ $\therefore \beta^2 - 2\sqrt{\alpha} = \frac{19}{4} + 2\sqrt{3} - 2\sqrt{3} = \frac{19}{4}$

74. The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)\,(a'x + b'y + c') = 0$. The equation of the angle bisectors of the lines represented by the equation $2x^2 + xy - 3y^2 = 0$ is

(1) $3x^2 + 5xy + 2y^2 = 0$ (2) $x^2 - y^2 - 10xy = 0$ (3) $3x^2 + xy - 2y^2 = 0$ (4) $x^2 - y^2 + 10xy = 0$

Ans. **(2)**

Sol. Equation of angle bisectors is $\dfrac{x^2 - y^2}{2+3} = \dfrac{xy}{\frac{1}{2}}$

$\Rightarrow \dfrac{x^2 - y^2}{5} = 2xy$

$\Rightarrow x^2 - y^2 = 10xy$

$\Rightarrow x^2 - y^2 - 10xy = 0$

75. For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct ?

(1) area of ΔABC is $\dfrac{27\sqrt{3}}{2}$ (2) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$

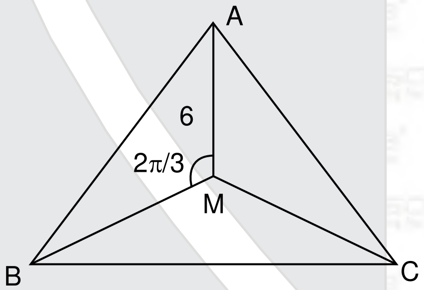
(3) perimeter of ΔABC is $18\sqrt{3}$ (4) $\overrightarrow{MA}.\overrightarrow{MB} = -18$

Ans. **(1)**

Sol. $\cos 2A + \cos 2B + \cos 2C$ is least

when $A = B = C = 60^\circ$

So M will be coincident with circumcentre



$r = \dfrac{\Delta}{s} = 3 \Rightarrow \dfrac{\frac{\sqrt{3}}{4}a^2}{\frac{3a}{2}} = 3$

$\dfrac{a}{2\sqrt{3}} = 3 \Rightarrow a = 6\sqrt{3}$

Perimeter = $3a = 18\sqrt{3}$

Area of $\Delta ABC = \dfrac{\sqrt{3}}{4}a^2 = \dfrac{\sqrt{3}}{4} \times 36 \times 3 = 27\sqrt{3}$

$\sin 2A + \sin 2B + \sin 2c = \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{2}$

$\sin A + \sin B + \sin C = \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{2}$

$\overrightarrow{MA}.\overrightarrow{MB} = |\overrightarrow{MA}||\overrightarrow{MB}| \cos\dfrac{2\pi}{3}$

$|\overrightarrow{MA}| = |\overrightarrow{MB}| = R = \dfrac{a^3}{4.\frac{\sqrt{3}}{4}a^2} = \dfrac{a}{\sqrt{3}} = 6$

So, $\overrightarrow{MA}.\overrightarrow{MB} = 6 \times 6 \times \left(-\dfrac{1}{2}\right) = -18$

76. If y = y(x) is the solution curve of the differential equation $\frac{dy}{dx} + y\tan x = x\sec x$, $0 \le x \le \frac{\pi}{3}$, y(0) = 1, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$ (2) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(3) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ (4) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

Ans. (1)

Sol. $\frac{dy}{dx} + y\tan x = x\sec x$ and $0 \le x \le \frac{\pi}{3}$

it is LDE in y

I. F.= $e^{\int \tan x dx} = e^{\ell n \sec x}$ =secx

∴ solution is

y. sec x= $\int x\sec x.\sec x dx + c$

ysecx = $\int x\sec^2 x dx + c$

$y\sec x = x\tan x - \int \tan x dx + c$

$y\sec x = x\tan x - \ell n\sec x + c$

y(0) = 1 ∴ 1 = 0 – 0 + c

c = 1

∴ $y\sec x = x\tan x - \ell n\sec x + 1$

now $y\left(\frac{\pi}{6}\right) = \frac{\frac{\pi}{6}.\frac{1}{\sqrt{3}} - \ell n\frac{2}{\sqrt{3}} + 1}{\frac{2}{\sqrt{3}}} = \frac{\frac{\pi}{6\sqrt{3}} - \ell n\frac{2}{\sqrt{3}e}}{\frac{2}{\sqrt{3}}} = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\frac{2}{\sqrt{3}e}$

77. If the centre and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α,β) and γ, then 3(α + β + γ) is equal to

(1) 12 (2) 11 (3) 10 (4) 9

Ans. (1)

Sol. $\left|\frac{z-2}{z-3}\right| = 2$

(z–2) $(\bar{z}-2) = 4(\bar{z}-3)(z-3)$

$\Rightarrow z\bar{z} - 2z - 2\bar{z} + 4 = 4(z\bar{z} - 3z - 3\bar{z} + 9)$

$3z\bar{z} - 10z - 10\bar{z} + 32 = 0$

$z\bar{z} - \frac{10}{3}z - \frac{10}{3}\bar{z} + \frac{32}{3} = 0$

$\alpha = -\frac{10}{3}$ $c = \frac{32}{3}$

centre $\left(\frac{10}{3},0\right)$ $\gamma = \sqrt{|\alpha|^2 - c} = \sqrt{\frac{100}{9} - \frac{32}{3}} = \frac{2}{3}$

$\alpha = \frac{10}{3}, \beta = 0, \gamma = \frac{2}{3}$

∴ $\alpha+\beta+\gamma = \frac{10}{3} + \frac{2}{3} = \frac{12}{3}$ ∴ 3(α+β+γ) = 12

78. In a binomial distribution B(n, p), the sum and the product of the mean and the variance are 5 and 6 respectively, then 6(n + p – q) is equal to

(1) 50 (2) 53 (3) 52 (4) 51

Ans. **(3)**

Sol. Mean = np = x

variance = npq = y

x + y = 5

xy = 6

So x = 2, y = 3 or x = 3, y = 2

np = 2 | np = 3

npq = 3 | npq = 2

$q = \frac{3}{2}$ | $q = \frac{2}{3}$ ...(1)

not possible | $p = \frac{1}{3}$

n = 9

now $6 (n + p - q) = 6 (9 + \frac{1}{3} - \frac{2}{3})$

$= 6 (9 - \frac{1}{3})$

= 2 (27 – 1)

= 52

79. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$, y(1) = 0 is 4π. Let P and Q be the points of intersection of the curve C and the y – axis. If normals at P and Q on the curve C intersect x –axis at points R and S respectively, then length of the line segment RS is

(1) $\frac{4\sqrt{3}}{3}$ (2) $\frac{2\sqrt{3}}{3}$ (3) 2 (4) $2\sqrt{3}$

Ans. **(1)**

Sol. (y – 2)dy + (x + a) dx = 0

$\frac{x^2}{2} + ax + \frac{y^2}{2} - 2y = c \Rightarrow y(1) = 0 \Rightarrow c = \frac{1}{2} + a$

$x^2 + y^2 + 2ax - 4y - 1 - 2a = 0$

area = 4π

$\Rightarrow \pi (a^2 + 4 + 1 + 2a) = 4\pi$

$\Rightarrow (a + 1)^2 = 0 \Rightarrow a = -1$

Now curve is

$x^2 + y^2 - 2x - 4y + 1 = 0$

Intersection points with

y – axis are P $(0, 2 - \sqrt{3})$ and Q $(0, 2 + \sqrt{3})$

Normal at P and Q are $y - 2 = \sqrt{3}(x - 1)$

and $y - 2 = -\sqrt{3}(x - 1)$

So R $(1 - \frac{2}{\sqrt{3}}, 0)$ and S $(1 + \frac{2}{\sqrt{3}}, 0)$

so Distance RS $= \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

80. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

(1) $p \wedge (\sim q)$ (2) $(\sim p) \wedge (\sim q)$ (3) $(\sim p) \vee (\sim q)$ (4) $(\sim p) \vee q$

Ans. **(2)**

Sol. Negation of given statement is

$\sim (q \vee (\sim q) \wedge p))$

$\equiv \sim q \wedge \sim ((\sim q) \wedge p)$

$\equiv \sim q \wedge (q \vee \sim p)$

$\equiv (\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$\equiv \sim q \wedge \sim p$

$\equiv \sim p \wedge \sim q$

81. If $\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7} dx = \frac{1}{\ell}(11)^{m/n}$ where ℓ, m, n $\in$ N, m and n are co-prime then ℓ + m + n is equal to

Ans. **(63)**

Sol. $\int_0^1 (x^{20} + x^{13} + x^6)(2x^{21} + 3x^{14} + 6x^7)^{1/7} dx$

$2x^{21} + 3x^{14} + 6x^7 = t$

$42(x^{20} + x^{13} + x^6)\, dx = dt$

$$\frac{1}{42}\int_0^{11} t^{\frac{1}{7}} dt = \left(\frac{t^{\frac{8}{7}}}{\frac{8}{7}} \times \frac{1}{42}\right)_0^{11} = \frac{1}{48}\left(t^{\frac{8}{7}}\right)_0^{11} = \frac{1}{48}(11)^{8/7}$$

$\ell = 48$, m = 8, n = 7

ℓ + m + n = 63

82. Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is $-\alpha\sqrt{3401}$ and $|\vec{u}.\hat{i}|^2 = \frac{m}{n}$ where m and n are co-prime natural numbers then m + n is equal to

Ans. **(3501)**

Sol. $[\vec{u}\vec{v}\vec{w}] = \vec{u}.(\vec{v} \times \vec{w})$

$= |\vec{u}||\vec{v} \times \vec{w}| \cos\theta$

$= \alpha |\vec{v} \times \vec{w}| \cos\theta$

when $\theta = \pi$

$[\vec{u}\vec{v}\vec{w}]_{mm} = -\alpha|\vec{v} \times \vec{w}| = -\alpha\sqrt{3401}$

$\Rightarrow |\vec{v} \times \vec{w}| = \sqrt{3401}$

$$\Rightarrow \left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{matrix}\right\| = \sqrt{3401}$$

$\Rightarrow |\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}| = \sqrt{3401}$

$\Rightarrow 1 + 25\alpha^2 + 9\alpha^2 = 3401$

$\Rightarrow 34\alpha^2 = 3400$

$\Rightarrow \alpha^2 = 100$

$\Rightarrow \alpha = 10$

Here $\vec{v} \times \vec{w} = \hat{i} - 50\hat{j} - 30\hat{k}$

$$\vec{u} = 10\left(-\frac{\vec{v} \times \vec{w}}{|\vec{v} \times \vec{w}|}\right) = -10\left(\frac{\hat{i} - 50\hat{j} - 30\hat{k}}{\sqrt{1 + 2500 + 900}}\right)$$

$$\vec{u} = \frac{-10}{\sqrt{3401}}\left(\hat{i} - 50\hat{j} - 30\hat{k}\right)$$

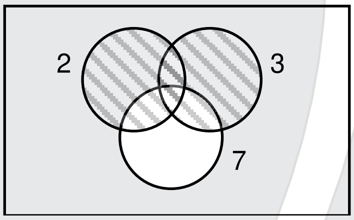
$$\left|\vec{u}.\hat{i}\right|^2 = \frac{100}{3401} = \frac{m}{n}$$

so m + n = 3501.

83. The number of 3-digit numbers, that are divisible by 2 or 3 but not divisible by 7, is

Ans. **(514)**

Sol.



Divisible by 2 100 , 102 , 104, …… , 998

$t_n = a + (n - 1)\, d \Rightarrow 998 = 100 + (n - 1)\, 2$

$\Rightarrow \frac{898}{2} = n - 1$

$\Rightarrow n = 449 + 1 = 450$

Divisible by 3 102, 105, 108, ….. , 999

$t_n = a + (n - 1)\, d \Rightarrow 999 - 102 = (n - 1)\, 3$

$\Rightarrow \frac{897}{3} = n - 1$

$\Rightarrow n = 299 + 1 = 300$

Divisible by 2 & 3 both 102, 108, 114, ….. , 996

$996 - 102 = (n - 1)\, 6$

$n = 1 + \frac{894}{6} = 1 + 149 = 150$

$\therefore$ No. divisible by 2 or 3 = 450 + 300 – 150 = 600

No. divisible by 2 and 7 both 112, 126, 140, 154, ………… 994

$994 - 112 = (n - 1)\, 14$

$882 = (n - 1)\, 14$

$\Rightarrow n = 63 + 1 = 64$

No. divisible by 3 and 7 both 105, 126, 147, …… , 987

$987 - 105 = (n - 1)\, 21$

$\Rightarrow n = 1 + 42 = 43$

No. divisible by 2, 3 and 7 126, 168, 252, 294, …… 966

$966 - 126 = (\, n - 1\ 42)$

$\Rightarrow 840 = (n - 1)\, 42$

$\Rightarrow n = 1 + 20 = 21$

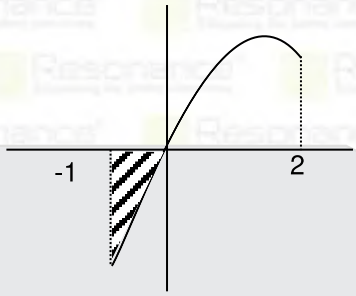
$\therefore$ Total required numbers = 600 – (64 + 43 – 21) = 600 +21 – 107

= 621 – 107 = 514

84. Let A be the area bounded by the curve y = x|x – 3|, the x-axis and the ordinates x = –1 and x = 2. Then 12A is equal to

Ans. **(62)**

Sol. $A = -\int_{-1}^{0}(3x - x^2)dx + \int_{0}^{2}(3x - x^2)dx$



$= -\left(\frac{3x^2}{2} - \frac{x^3}{3}\right)_{-1}^{0} + \left(\frac{3x^2}{2} - \frac{x^3}{3}\right)_{0}^{2}$

$= -\left(-\frac{3}{2} - \frac{1}{3}\right) + \left(6 - \frac{8}{3}\right)$

$= \frac{3}{2} + \frac{1}{3} + 6 - \frac{8}{3} = \frac{31}{6}$

$\Rightarrow 12A = 62$

85. The number of words, with or without meaning, that can be formed using all the letters of the word "ASSASSINATION" so that the vowels occur together, is

Ans. **(50400)**

Sol. **ASSASSINATION**

Vowels → A,A,A,I,I,O

all vowels are together A,A,A,I,I,O , S,S,S,S,N,N,T

$\therefore$ Number of words = $\frac{8!}{4!2!} \times \frac{6!}{3!2!} = \frac{8 \times 7 \times 6 \times 5}{2} \times \frac{6 \times 5 \times 4}{2}$

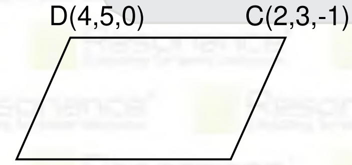
= 56 × 30× 30

= 56× 900 = 50400

86. A(2, 6, 2), B(–4, 0, λ), C(2, 3, –1) and D(4, 5, 0), |λ| ≤ 5 are the vertices of a quadrilateral ABCD. If its area is 18 square units, then 5 – 6λ is equal to

Ans. **(11)**

Sol.



$\overrightarrow{AC} = -3\hat{j} - 3\hat{k}$

$\overrightarrow{BD} = 8\hat{i} + 5\hat{j} - \lambda\hat{k}$

$\overrightarrow{AC} \times \overrightarrow{BD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -3 \\ 8 & 5 & -\lambda \end{vmatrix}$

$= \hat{i}(3\lambda+15)-\hat{j}(0+24)+\hat{k}(0+24)$

$= 3\left[(\lambda+5)\hat{i}-8\hat{j}+8\hat{k}\right]$

Area of quadrilateral ABCD = $\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$

$= \frac{1}{2}.3\sqrt{(\lambda+5)^2+64+64}$

$= \frac{3}{2}\sqrt{\lambda^2+10\lambda+153}$

Area = 18

$\frac{3}{2}\sqrt{\lambda^2+10\lambda+153}=18$

$\lambda^2+10\lambda+153=144$

$\lambda^2+10\lambda+9=0$

$(\lambda+1)(\lambda+9)=0$ (here $|\lambda| \le 5$)

So λ = -1

$\therefore$ 5 – 6 λ = 5 + 6 = 11

87. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to

Ans. (14)

Sol. $f(x) = x^2 + xg'(1) + g''(2)$ ......(1)

$\therefore f'(x) = 2x + g'(1)$

$f''(x) = 2$

$\therefore g(x) = f(1)x^2 + x(2x + g'(1)) + 2 = f(1)x^2 + 2x^2 + xg'(1) + 2$

$g'(x) = 2f(1)x + 4x + g'(1)$ ......(2)

$g''(x) = 2(1) + 4$ ......(3)

from (2)put x = 1

$g'(1) = 2f(1) + 4 + g'(1)$

$\Rightarrow 2f(1) + 4 = 0$

$\Rightarrow f(1) = -2 \Rightarrow g''(2) = -4 + 4 = 0$ from (3)

from (1)

$f(1) = 1 + g'(1) + 0$

$\Rightarrow g'(1) = -2 - 1 = -3$

$\because f'(x) = 2x - 3$

$f(x) = x^2 - 3x + c$

put x = 1 , f(1) = – 2 $\therefore$ c = 0

$\therefore f(x) = x^2 - 3x$

$\therefore g(x) = -3x + 2$

$\Rightarrow f(4) - g(4) = 4 - (-10) = 14$

88. Let f : IR $\rightarrow$ IR be a differential function such that $f'(x) + f(x) = \int_0^2 f(t)\,dt$. If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to

Ans. (1)

Sol. Given $f'(x) + f(x) = \int_0^2 f(t)\,dt$

Let $\int_0^2 f(t)\,dt = a$

$\therefore f'(x) + f(x) = a$

$f'(x) = a - f(x)$

$\dfrac{f'(x)}{a - f(x)} = 1$

$\therefore -\ell n\,(a - f(x)) = x + c$

Put x = 0

$-\ell n\,(a - e^{-2}) = c$

$\therefore -\ell n\,(a - f(x)) = x - \ell n\,(a - e^{-2})$

$\ell n \dfrac{a - e^{-2}}{a - f(x)} = x$

$\Rightarrow \dfrac{a - e^{-2}}{a - f(x)} = e^{x}$

$a - f(x) = (a - e^{-2})\, e^{-x}$

$f(x) = a - (a - e^{-2})e^{-x}$

Now $\int\limits_0^2 f(t)\,dt = a$

$\Rightarrow \int\limits_0^2 (a - (a - e^{-2})e^{-t})\,dt = a$

$\Rightarrow \left[at + (a - e^{-2})e^{-t}\right]_0^2 = a$

$\Rightarrow 2a + (a - e^{-2})\, e^{-2} - (a - e^{-2}) = a$

$\Rightarrow a + ae^{-2} - e^{-4} - a + e^{-2} = 0$

$a = \dfrac{e^{-4} - e^{-2}}{e^{-2}} = e^{-2} - 1$

Now 2f(0) – f(2)

$= 2e^{-2} - a + (a - e^{-2})\, e^{-2}$

$= 2e^{-2} - e^{-2} + 1 - e^{-2}$

= 1

89. If $a_1 = 8$, a_2 , a_3.......a_n, be an A.P. If the sum of first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is

Ans. **(754)**

Sol. $S_4 = 50$

$2(16 + 3d) = 50$

$d = 3$

$4a + d(4n - 10) = 170$

$32 + 3(4n - 10) = 170$

$4n - 10 = 46$

$n = 14$

Middle terms are T_7, T_8

$T_7\, T_8 = (8 + 6 \times 3)\,(8 + 7 \times 3) = 26 \times 29$

= 754

90. The remainder, when $19^{200} + 23^{200}$ is divided by 49, is

Ans. **(29)**

Sol. $(21 - 2)^{200} + (21 + 2)^{200} = 49\lambda + 2^{201}$

$2^{201} = 8^{67} = (7 + 1)^{67} = 49\lambda + 7\times 67 + 1$

$= 49\lambda + 470$

$= 49(\lambda + 9) + 29$

So, remainder $= 29$

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