JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Official Based Questions & Solutions

Date: 1 February, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 06.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

61. Let P(S) denote the power set of S = {1,2,3,…..,10}. Define the relation R_1 and R_2 on P(S) as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \phi$ and AR_2B if $A \cup B^c = B \cup A^c$, $\forall$ A,B $\in$ P(S). Then

(1) both R_1 and R_2 are not equivalence relations (2) only R_2 is an equivalence relation
(3) only R_1 is an equivalence relation (4) both R_1 and R_2 are equivalence relations

Ans. **(4)**

Sol. $AR_1B \Rightarrow (A \cap B') \cup (B \cap A') = \phi$

$\Rightarrow A \cap B' = \phi$ & $B \cap A' = \phi$

$\Rightarrow A = B$

$\therefore AR_1B$ is equivalence relation

$AR_2B \Rightarrow A \cup B' = B \cup A'$

$\Rightarrow A = B$

$\therefore AR_2B$ is equivalence relation

62. The sum $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$ is equal to

(1) $\frac{11e}{2} + \frac{7}{2e} - 4$ (2) $\frac{11e}{2} + \frac{7}{2e}$ (3) $\frac{13e}{4} + \frac{5}{4e} - 4$ (4) $\frac{13e}{4} + \frac{5}{4e}$

Ans. **(3)**

Sol.

Sol. $\sum_{n=1}^{\infty} \frac{2n^2+3n+4)}{2n!}$

$= \frac{1}{2}\sum_{n=1}^{\infty} \frac{2n(2n-1)+8n+8}{2n!}$

$= \frac{1}{2}\sum_{n=1}^{\infty} \left\{\frac{1}{(2n-2)!} + \frac{4}{(2n-1)!} + \frac{8}{2n!}\right\}$

$= \frac{1}{2}\left\{\left(\frac{e+e^{-1}}{2}\right) + 4\left(\frac{e-e^{-1}}{2}\right) + 8\left(\frac{e+e^{-1}}{2} - 1\right)\right\}$

$= \frac{1}{4}\left(13e + 5e^{-1} - 16\right)$

$= \frac{13}{4}e + \frac{5}{4e} - 4$

63. If $y(x) = x^x$, x > 0 then y"(2) – 2y′(2) is equal to

(1) $4(\log_e 2)^2 - 2$ (2) $8\log_e 2 - 2$ (3) $4(\log_e 2)^2 + 2$ (4) $4\log_e 2 + 2$

Ans. **(1)**

Sol. $y = x^x$

$\ell n\, y = x\, \ell n\, x$

$\frac{1}{y}.y' = 1 + \ell nx$

$y' = y(1 + \ell n\, x)$ ....(1)

$y' = x^x(1 + \ell n\, x)$

at x = 2 we have y = 4

So $y'(2) = 4(1 + \ell n\, 2)$ ....(2)

and $y'' = y'(1 + \ell n\, x) + \frac{y}{x}$

$y''(2) = y'(1 + \ell n\, 2) + 2$

$y''(2) - y'(2) = y'(\ell n\, 2) + 2$

$y''(2) - 2y'(2) = (\ell n\, 2 - 1)\, y'(2) + 2$

$= 4(\ell n\, 2 - 1)(\ell n\, 2 + 1) + 2$

$= 4(\ell n 2)^2 - 2$

64. Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$, then $|\vec{r}|$ is

(1) $\frac{11}{7}\sqrt{2}$ (2) $\frac{11}{7}$ (3) $\frac{\sqrt{914}}{7}$ (4) $\frac{11}{5}\sqrt{2}$

Ans. (1)

Sol. $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = 0$

$\Rightarrow (\vec{r} - \vec{c}) = \lambda\vec{a}$

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{a}$

$\Rightarrow \vec{r}.\vec{b} = \vec{c}.\vec{b} + \lambda\vec{a}.\vec{b}$

$\Rightarrow 0 = (1-3) + \lambda(2+5)$

$2 = 7\lambda \Rightarrow \lambda = \frac{2}{7}$

$\vec{r} = \vec{c} + \frac{2}{7}\vec{a}$

$= \hat{i} + 2\hat{j} - 3\hat{k} + \frac{2}{7}(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\vec{r} = \frac{11\hat{i} - 11\hat{k}}{7}$

$|\vec{r}| = \frac{\sqrt{121+121}}{7}$

$|\vec{r}| = \frac{11\sqrt{2}}{7}$

65. Let S = { $x \in R$, $0 < x < 1$ and $2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ } . If n(S) denotes the number of elements in S then

(1) n(S) = 2 and only one element in S is less than $\frac{1}{2}$.

(2) n(S) = 1 and the element in S is less than $\frac{1}{2}$.

(3) n(S) = 1 and the element in S is more than $\frac{1}{2}$.

(4) n(S) = 0

Ans. **(2)**

Sol. Put x = tanθ $\theta \in \left(0, \frac{\pi}{4}\right)$

$$2\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$2\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right] = \cos^{-1}[\cos(2\theta)]$$

$$\Rightarrow 2\left(\frac{\pi}{4}-\theta\right) = 2\theta \Rightarrow \theta = \frac{\pi}{8}$$

$$\Rightarrow x = \tan\frac{\pi}{8} = \sqrt{2}-1 < \frac{1}{2}$$

66. For the system of linear equations αx + y + z = 1, x + αy + z = 1, x + y + αz = β which one of the following statements is NOT correct ?

(1) It has infinitely many solutions if α = 2 and β = – 1

(2) x + y + z = $\frac{3}{4}$ if α = 2 and β = 1

(3) It has infinitely many solutions if α = 1 and β = 1

(4) It has no solution if α = –2 and β = 1

Ans. **(1)**

Sol. $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = \alpha(\alpha^2 - 1) - (\alpha - 1) + (1 - \alpha) = \alpha^3 - 3\alpha + 2 = (\alpha - 1)(\alpha^2 + \alpha - 2)$

(α – 1) (α – 1) (α + 2)

for α =1 system of equations is

x + y + z = 1

x + y + z =1

x + y + z = β ⇒ β = 1 then infinitely equation

θ.f α= 2, β = – 1 then

θx + y + z = 1

x + 2y + z = 1

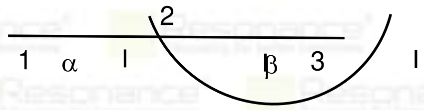
x + y + 2z = –1 No solution

67. The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1, 2) and its other root lies in the interval (2, 3) is

(1) 2 (2) 0 (3) 1 (4) 3

Ans. **(3)**

Sol. Let $f(x) = 2x^2 - 8x + k = 0$



$f(1) > 0$

$f(2) < 0$

$f(3) > 0$

$f(1) > 0 \Rightarrow 2 - 8 + k > 0 \quad \Rightarrow k > 6 \quad (1)$

$f(2) < 0 \Rightarrow 8 - 16 + k < 0 \quad \Rightarrow k < 8 \quad (2)$

$f(3) > 0 \Rightarrow 18 - 24 + k > 0 \Rightarrow k > 6 \quad (3)$

$k \in (6, 8) \quad \therefore$ integral k = 7

68. If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then

(1) $A^{30} - A^{25} = 2I$ (2) $A^{30} = A^{25}$ (3) $A^{30} + A^{25} - A = I$ (4) $A^{30} + A^{25} + A = I$

Ans. **(3)**

Sol. $A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$ ....(1)

$\because A(\alpha) \cdot A(\beta) = A(\alpha + \beta)$

So by above.

$$A^{30}\left(\frac{\pi}{3}\right) = \begin{bmatrix} \cos 30\left(\frac{\pi}{3}\right) & \sin 30\left(\frac{\pi}{3}\right) \\ -\sin 30\left(\frac{\pi}{3}\right) & \cos 30\left(\frac{\pi}{3}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{25}\left(\frac{\pi}{3}\right) = \begin{bmatrix} \cos 25\left(\frac{\pi}{3}\right) & \sin 25\left(\frac{\pi}{3}\right) \\ -\sin 25\left(\frac{\pi}{3}\right) & \cos 25\left(\frac{\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & +\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = A$$

$A^{30} + A^{25} - A = I$

69. Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE ?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

Ans. **(BONUS)**

Sol. $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k},\ \vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$

(1) Projection of $\vec{a}$ on $\vec{b} = \vec{a}.\hat{b} = \frac{(5-3-15)}{\sqrt{1+9+25}} = \frac{-13}{\sqrt{35}}$

70. Which of the following statements is a tautology

(1) $p\vee(p\wedge q)$
(2) $(p\wedge(p\to q))\to \sim q$
(3) $(p\wedge q)\to(\sim(p)\to q)$
(4) $p\to(p\wedge(p\to q))$

Ans. **(3)**

Sol. **(1)** $p\vee(p\wedge q) \neq t$

(2) $p\wedge(\sim p\vee q)\to\sim q$

$\equiv ((p\wedge\sim p)\vee(p\wedge q))\to \sim q$

$\equiv (p\wedge q)\to\sim q$

$\equiv \sim(p\wedge q)\vee\sim q$

$\equiv \sim p\vee\sim q\vee\sim q$

$\equiv \sim p\vee\sim q \neq t$

(3) $(p\wedge q)\to((\sim p)\to q)$

$(p\wedge q)\to(p\vee q)$

$\sim(p\wedge q)\vee(p\vee q)$

$(\sim p\vee\sim q)\vee(p\vee q) \equiv t$

(4) $\sim p\vee(p\wedge(\sim p\vee q))$

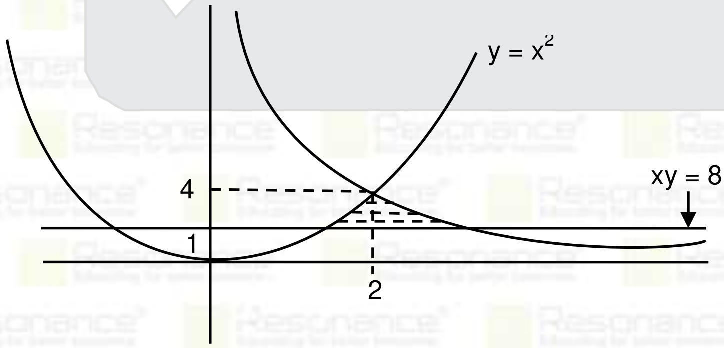
$\equiv \sim p\vee(p\wedge q) \equiv t\wedge(\sim p\vee q) \equiv \sim p\vee q \neq t$

71. The area of the region given by $\{(x, y) : xy \le 8,\ 1 \le y \le x^2\}$ is

(1) $16\log_e 2 - \frac{14}{3}$ (2) $8\log_e 2 - \frac{13}{3}$ (3) $8\log_e 2 + \frac{7}{6}$ (4) $16\log_e 2 + \frac{7}{3}$

Ans. **(1)**

Sol. $A = \int_1^4 \left(\frac{8}{y} - \sqrt{y}\right) dt$



$= \left[8\ln y - \frac{y^{3/2}}{3/2}\right]_1^4 = 8\,(\ln 4 - \ln 1) - \frac{2}{3}(4^{3/2} - 1)$

$= 8\ln 4 - \frac{16}{3} + \frac{2}{3} = 8\ln 4 - \frac{14}{3}$ Ans. C

72. Let $\alpha x = \exp(x^{\beta}y^{\gamma})$ be the solution of differential equation $2x^2y\,dy - (1 - xy^2)\,dx = 0$, $x > 0$, $y = (2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals

(1) 0 (2) –1 (3) 1 (4) 3

Ans. **(3)**

Sol. $2x^2y\,dy = (1 - xy^2)\,dx$

$2x^2y\frac{dy}{dx} = 1 - xy^2$

$2y\frac{dy}{dx} + \frac{1}{x}y^2 = \frac{1}{x^2}$

Let $y^2 = t$ $\therefore 2y\frac{dy}{dx} = \frac{dt}{dx}$ $\therefore \frac{dt}{dx} + \frac{1}{x}t = \frac{1}{x^2}$ Linear in t

$\therefore$ I.F solution is $= e^{\int \frac{dx}{x}} = e^{\ln x} = x$

$t \cdot x = \int \frac{1}{x^2}x\,dx + c$

$y^2x = \ln x + c$

$\because y(2) = \sqrt{\ln 2}$ $\therefore \ln 2 . 2 = \ln 2 + c$ $c = \ln 2$

$\therefore$ solution is $xy^2 = \ln x + \ln 2$

$xy^2 = \ln 2x \Rightarrow 2x = e^{xy^2}$

given solution is $\alpha x = e^{x^{\beta}y^{\gamma}}$

$\therefore \alpha = 2,\ \beta = 1, \gamma = 2$

$\therefore \alpha + \beta - y = 1$

73. Let a, b be two real numbers such that $ab < 0$. If the complex number $\frac{1+ai}{b+i}$ is unit modulus and $a + ib$ lies on the circle $|z - 1| = |2z|$, then a possible value of $\frac{1+[a]}{4b}$, where [t] is greatest integer function, is

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) 1 (4) –1

NTA. **(Bouns)**

Reso. **(Bouns)**

Sol. $ab < 0$, $\left|\frac{1+ai}{b+i}\right| = 1 \Rightarrow a = \pm b$ ___(1)

$a + ib$ lies on $|z-1| = |2z|$

$\Rightarrow |a + ib - 1| = |2a + 2ib|$

$\Rightarrow a^2 - 2a + 1 + b^2 = 4a^2 + 4b^2$

$\Rightarrow 3a^2 + 3b^2 + 2a - 1 = 0$ ____ (2)

by (1) & (2) $6a^2 + 2a - 1 = 0$

$a = \frac{-1 \pm \sqrt{7}}{6} \left\langle \begin{array}{l} \frac{-1+\sqrt{7}}{6} \\ \frac{-1-\sqrt{7}}{6} \end{array} \right.$

$[a] = 0$ or -1

$b = -a$

$\frac{1+[a]}{4b} = 0$ or $\frac{+1}{4b}$ No option is correct

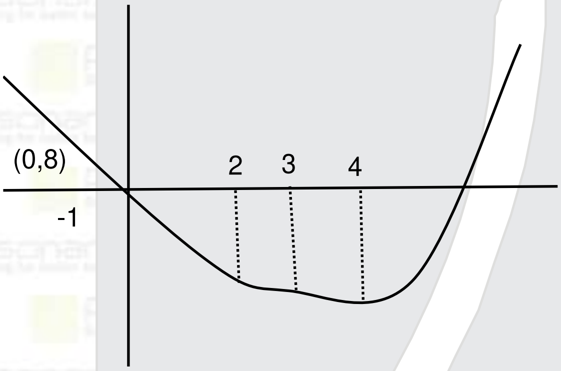
74. The sum of the absolute maximum and minimum values of the function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in the interval $[-1, 3]$ is equal to

(1) 12 (2) 10 (3) 24 (4) 13

Ans. **(2)**

Sol. $f(x) = \begin{cases} x^2 - 5x + 6 - 3x + 2 & , \quad x \in (-\infty, 2) \cup [3, \infty) \\ -(x^2 - 5x + 6) - 3x + 2 & , \quad x \in [2, 3] \end{cases}$

$\Rightarrow f(x) = \begin{cases} x^2 - 8x + 8 & , \quad x \in (-\infty, 2) \cup [3, \infty) \\ -x^2 + 2x - 4 & , \quad x \in [2, 3] \end{cases}$



absolute maximum $= f(-1) = (-1)^2 - 8(-1) + 8 = 17$

absolute minimum $= f(3) = -7$

sum = 17 – 7 = 10

75. Let $9 = x_1 < x_2 < \ldots\ldots < x_7$ in an A.P. with common difference d. If the standard deviation of $x_1, x_2 \ldots\ldots x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to

(1) $18\left(1 + \frac{1}{\sqrt{3}}\right)$ (2) $2\left(9 + \frac{8}{\sqrt{7}}\right)$ (3) 34 (4) 25

Ans. **(3)**

Sol. Mean $\Rightarrow \bar{x} = \frac{\sum_{i=1}^{7} x_i}{7} = \frac{\frac{7}{2}[2a + 6d]}{7} = a + 3d = x_4$

Variance $= \frac{\sum_{i=1}^{7} (x_i - \bar{x})^2}{7} = (4)^2 \Rightarrow \frac{\sum_{i=1}^{7} (x_i - x_4)^2}{7} = 16$

$\Rightarrow \frac{(3d)^2 + (2d)^2 + d^2 + 0 + d^2 + (2d)^2 + (3d)^2}{7} = 16$

$= 4d^2 = 16 \quad \Rightarrow \quad d = 2$

$\Rightarrow \bar{x} = 9 + 3(2) = 15$

& $x_6 = a + 5d = 9 + 5(2) = 19 \Rightarrow \bar{x} + x_6 = 34$

76. The value of the integral $\int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$ is

(1) $\frac{\pi^2}{12\sqrt{3}}$ (2) $\frac{\pi^2}{6}$ (3) $\frac{\pi^2}{3\sqrt{3}}$ (4) $\frac{\pi^2}{6\sqrt{3}}$

Ans. (4)

Sol. $I = \int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$ ....(1)

replacing x by – x

$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \frac{-x+\frac{\pi}{4}}{2-\cos 2x} dx$ ....(2)

(1) + (2) $\Rightarrow$ $2I = \int_{-\pi/4}^{\pi/4} \frac{\frac{\pi}{2}}{2-\cos 2x} dx$

$I = \frac{\pi}{4} \cdot \int_{-\pi/4}^{\pi/4} \frac{dx}{2-\cos 2x}$

$= \frac{\pi}{4} \cdot 2\int_{0}^{\pi/4} \frac{dx}{2-\cos 2x} = \frac{\pi}{2} \cdot \int_{0}^{\pi/4} \frac{dx}{2-\frac{1-\tan^2 x}{1+\tan^2 x}}$

$I = \frac{\pi}{2} \cdot \int_{0}^{\pi/4} \frac{\sec^2 x}{1-3\tan^2 x} dx$ Let tan x = t

Þ $\sec^2 x\, dx = dt$

$= \frac{\pi}{2} \int_{0}^{1} \frac{dt}{1+3t^2}$

$= \frac{\pi}{2} \cdot \frac{1}{\sqrt{3}} \left(\tan^{-1} \sqrt{3}t\right)_0^1 = \frac{\pi}{2\sqrt{3}} \left(\tan^{-1}\sqrt{3} - \tan^{-1} 0\right)$

$I = \frac{\pi^2}{6\sqrt{3}}$

77. Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$ which is nearest to line $3x + 2y = 1$. Then $\sqrt{2}(y_0 - x_0)$ is equal to

(1) –9 (2) 3 (3) 9 (4) –3

Ans. (1)

Sol. Equation of tangent at (x_0, y_0) is T = 0

$\Rightarrow 3xx_0 - 4yy_0 = -36$

it should be parallel to 3x + 2y = 1

$\frac{3x_0}{4y_0} = \frac{-3}{2} \Rightarrow \quad x_0 = -\ 2y_0$

$\because$ (x_0, y_0) lies on curve $\Rightarrow 3x_0^2 - 4y_0^2 = 36$

$\Rightarrow 3(-2y_0)^2 - 4y_0^2 = 36$

$\Rightarrow y_0^2 = \frac{9}{2} \Rightarrow y_0 = \pm\frac{3}{\sqrt{2}} \Rightarrow x_0 = \mp 3\sqrt{2}$

Point $\Rightarrow \left(3\sqrt{2}, -\frac{3}{\sqrt{2}}\right)$ or $\left(-3\sqrt{2}, \frac{3}{\sqrt{2}}\right)$

Nearest point is $\left(3\sqrt{2}, -\frac{3}{\sqrt{2}}\right)$

$\Rightarrow \sqrt{2}(y_0 - x_0) = \sqrt{2}\left(\left(-\frac{3}{\sqrt{2}}\right) - 3\sqrt{2}\right) = -9$

78. Two dice are thrown independently. Let A be the event that the number appeared on the first die is less than the number appeared on the second die, B be the event that the number appeared on first die is even and that on the second die is odd and C be the event that the number appeared on the first die is odd and that on second is even. Then

(1) A and B are mutually exclusive

(2) the number of favourable cases of the events A,B and C are 15, 6 and 6 respectively

(3) B and C are independent

(4) the number of favourable cases of the event $(A \cup B) \cap C$ is 6

Ans. **(4)**

Sol.

A (I < II)	B (EO)	C(OE)
$n(A) = 15$	$n(B) = 9$	$n(C) = 9$
$n(A \cap B) = 3$	$n(A \cap C) = 6$	$n(B \cap C) = 0$

$n(A \cap B \cap C) = 0$

$n((A \cup B) \cap C) = n(A \cap C) + n(B \cap C) - n(A \cap B \cap C) = 6$

79. Let $f : R - \{0, 1\} \to R$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. then f(2) is equal to

(1) $\frac{7}{3}$ (2) $\frac{9}{2}$ (3) $\frac{9}{4}$ (4) $\frac{7}{4}$

Ans. **(3)**

Sol. , $x = 2 \Rightarrow f(2) + f(-1) = 3$ ....(1)

, $x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ ....(2)

, $x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0$ ....(3)

(2) …….. (3) $\Rightarrow f(2) - f(-1) = \frac{3}{2}$ ....(4)

(1) + (4) $\Rightarrow 2f(2) = \frac{9}{2} \quad \Rightarrow f(2) = \frac{9}{4}$

80. Let the plane P pass through the intersection of the planes 2x + 3y – z = 2 and x + 2y + 3z = 6, and be perpendicular to the plane 2x + y – z + 1 = 0. If d is the distance of P from the point (–7,1,1), then d^2 is equal to

(1) $\frac{25}{83}$ (2) $\frac{250}{83}$ (3) $\frac{15}{53}$ (4) $\frac{250}{82}$

Ans. **(2)**

Sol. A plane through intersection of 2x + 3y – z = 2 & x + 2y + 3z = 6 is

$2x + 3y - z - 2 + \lambda (x + 2y + 3z - 6) = 0$

$\Rightarrow (2 + \lambda)x + (3 + 2\lambda)y + (3\lambda - 1)z - (2 + 6\lambda) = 0$

Plane is $\perp$ to 2x + y – z + 1 = 0

$\therefore 2(2 + \lambda) + 1(3 + 2\lambda) - (3\lambda - 1) = 0$

$4 + 2\lambda + 3 + 2\lambda - 3\lambda + 1 = 0$

$\lambda = -8$

$\therefore$ plane is 6x + 13y + 25z – 46 = 0

$\therefore d = \frac{|-42+13+25-46|}{\sqrt{36+169+625}} = \frac{50}{\sqrt{830}}$

$\therefore d^2 = \frac{50\times 50}{830} = \frac{250}{83}$

81. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is

Ans. **(81)**

Sol. Unit digit must be 4 since number should be divisible by 2.

Four out of remaining five places, each has 3 options and remaining one place will have only one option so total number of six digit numbers = 3.3.3.3.1 = 81

82. Number of integral solutions to the equation x + y + z = 21 where $x \geq 1, y \geq 3, z \geq 4$, is equal to

Ans. **(105)**

Sol. $x \geq 1, y \geq 3, z \geq 4 \Rightarrow x - 1 \geq 0, y - 3 \geq 0, z - 4 \geq 0$

Let x – 1 = X, y – 3 = Y, z – 4 = Z

So x + y + z = 21

X + 1 + Y + 3 + Z + 4 = 21

$X + Y + Z = 13 \quad X \geq 0, Y \geq 0, Z \geq 0$

No. of integral solution = ${}^{13+3-1}C_2 = {}^{15}C_2 = \frac{15\times 14}{2} = 105$

83. Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}} + \sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is

Ans. **(4)**

Sol. $t_1 = {}^mC_1,\ t_3 = {}^mC_2,\ t_5 = {}^mC_3$

$\Rightarrow 2\ {}^mC_2 = {}^mC_1 + {}^mC_3$

$\Rightarrow 2 = \frac{{}^mC_1}{{}^mC_2} + \frac{{}^mC_3}{{}^mC_2}$

$\Rightarrow 2 = \dfrac{1}{\dfrac{m-2+1}{2}} + \dfrac{m-3+1}{3}$

$\Rightarrow 2 = \dfrac{2}{m-1} + \dfrac{m-2}{3}$

$\Rightarrow 6m - 6 = 6 + m^2 - 3m + 2$

$\Rightarrow m^2 - 9m + 14 = 0$

$\Rightarrow (m - 2)(m - 7) = 0$

$\Rightarrow m = 2$ & $m = 7$

$\therefore m = 7 \quad (m \neq 2)$

Now $\left(\sqrt{10-3^x} + 5\sqrt{3^{(x-2)}}\right)^7$

$t_6 = {}^7C_5 . \left(\sqrt{(10-3^x)}\right)^2 . 3^{x-2} = 21$

$(10 - 3^x) . \dfrac{3^x}{9} = 1$

$10.\ 3^x - (3^x)^2 = 9$

$t^2 - 10t + 9 = 0$

$(t - 1)(t - 9) = 0$

$t = 1, \quad t = 9$

$3^x = 1, \quad 3^X = 3^2$

$X = 0, \quad X = 2$

$\therefore$ sum of squares $= 0^2 + 2^2 = 4$

84. If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \dfrac{\alpha}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is equal to

Ans. **(1)**

Sol. $T_{r+1} = {}^{22}C_r . \left(\dfrac{\alpha}{x^3}\right)^r . \left(x^{\frac{2}{3}}\right)^{22-r}$

$= {}^{22}C_r . \alpha^r . x^{\frac{44-11r}{3}}$

for $x^0 \Rightarrow r = 4$

coefficient of $x^0 = {}^{22}C_4 . \alpha^4 = 7315$

$\Rightarrow \dfrac{22.21.20.19}{4.3.2} \alpha^4 = 7315$

$\Rightarrow (11 \times 7 \times 5 \times 19)\alpha^4 = 7315$

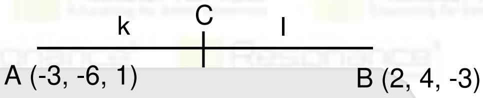
$\Rightarrow \alpha^4 = 1$

$\Rightarrow |\alpha| = 1$

85. The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the points A(–3, –6, 1) and B(2,4,–3) divides the line segment AB internally in the ration k : 1. If a,b,c (|a|, |b|, |c|) are co-prime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$ then |a + b + c| is equal to

Ans. **(10)**

Sol. $C\left(\frac{2k-3}{k+1},\frac{4k-6}{k+1},\frac{-3k+1}{k+1}\right)$



C lies on $8x + y + 2z = 0$

$\therefore 8(2k-3) + (4k-6) + 2(-3k+1) = 0$

$16k - 24 + 4k - 6 - 6k + 2 = 0$

$14k = 28 \Rightarrow k = 2$

$\therefore$ C is $\left(\frac{1}{3},\frac{2}{3},\frac{-5}{3}\right)$

Any point on line $\frac{x-1}{-1}=\frac{y+4}{2}=\frac{z+2}{2}$ is D $(1-\lambda, -4+2\lambda, -2+3\lambda)$

Drs of CD = $1-\lambda-\frac{1}{3}, -4+2\lambda-\frac{2}{3}, -2+3\lambda+\frac{5}{3}$

$= \frac{2-3\lambda}{3}, \frac{-14+6\lambda}{3}, \frac{-1+9\lambda}{3}$

CD $\perp$ to line : $-1(2-3\lambda) + 2(6\lambda-14) + 3(9\lambda-1) = 0$

$\Rightarrow -2 + 3\lambda + 12\lambda - 28 + 27\lambda - 3 = 0$

$42\lambda = 33 \Rightarrow \lambda = \frac{33}{42} = \frac{11}{14}$

$\therefore$ drs of CD = $\frac{2-\frac{33}{14}}{3}, \frac{-14+\frac{66}{14}}{3}, \frac{-1+\frac{99}{14}}{3}$

$= -5, -130, 85$

$= -1, -26, 17$

$\therefore (-1 - 26 + 17) = 10$

86. The sum of common terms of the following three arithmetic progressions.
3,7,11,15,………,399
2,5,8,11,………,359 and
2,7,12,17,………,197
is equal to

Ans. **(321)**

Sol. S_1 = {2,5,8,11,14, _ _ _ _ _ ,359} S_2
S_2 = {3,7,11,15, _ _ _ _ _ ,239} S_1
S_3 = {7,12,17,22, _ _ _ _ _ 197} S_3
common AP

S = {47,107,167}

Hence sum of common AP = 321

87. If $\int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx}{1+5^{\cos x}} = \frac{k\pi}{16}$, then k is equal to

Ans. (13)

Sol. $I = \int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx}{1+5^{\cos x}}$

$= 2\int_0^{\pi/2} \left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx$

$= \int_0^{\pi/2} 2+\cos 4x+\cos 2x+1+\cos 2x+2\cos 3x\left(\frac{\cos 3x+3\cos x}{4}\right)dx$

$= \int_0^{\pi/2} 3dx+0+0+0+\int_0^{\pi/2}\frac{1}{2}(\cos^2 3x+3\cos x\ \cos 3x)dx$

$= \frac{3\pi}{2}+\frac{1}{4}\int_0^{\pi/2}(1+\cos 6x+3(\cos 4x+\cos 2x))dx$

$= \frac{3\pi}{4}+\frac{1}{4}.\frac{\pi}{2} = \frac{13\pi}{8} = \frac{k\pi}{16}$

$\Rightarrow k = 26$

88. If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to

Ans. (16)

Sol. $y^2 = 8x + 4y + 4$

$y^2 - 4y = 8x + 4$

$(y-2)^2 = 8\ (x + 1)$

a = 2,

focus (a, 0)

$x + 1 = 2,\ y - 2 = 0$

$x = 1,\ y = 2\ \therefore$ focus is (1 , 2)

$\therefore$ Eq. of focal chand is $y - 2 = m\ (x - 1)$

But y = 0 $\quad -2 = m\ (3 - 1) \quad \Rightarrow m = -1$

$\therefore$ Eq. of focal chand is $y - 2 = -\ (x - 1)$

L. of focal chand = $4a\ \text{cosec}^2\,\alpha$

$= 4.2\ \cos a^2\ (135°)$

$= 8.\ (\sqrt{2})^2 = 16$

89. Let $\alpha x + \beta y + \gamma z = 1$ be the equation of a plane passing through the point (3, –2, 5) and perpendicular to the line joining the points (1,2,3) and (–2,3,5). Then the value of $\alpha\beta\gamma$ is equal to

Ans. **(6)**

Sol. $\alpha x + \beta y + \gamma z = 1$

Drs of line A (1, 2, 3) & B (-2, 3, 5) is 3, -1, -2

$\therefore \frac{\alpha}{3} = \frac{\beta}{-1} = \frac{\gamma}{-2} = \lambda$

$\Rightarrow \alpha = 3\lambda,\ \beta = -\lambda,\ \gamma = -2\lambda$

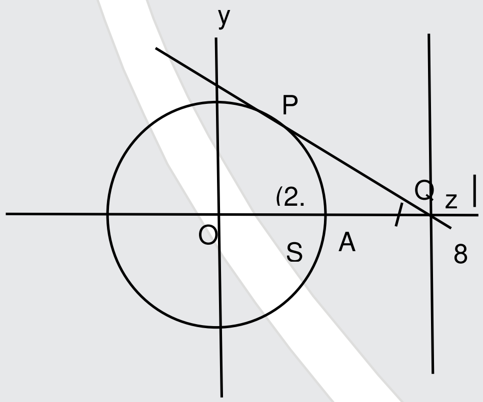
& $3\alpha - 2\beta + 5y = 1$

$\therefore 9\lambda + 2\lambda - 10\lambda = 1 \quad \therefore \lambda = 1$

$\therefore \alpha = 3,\ \beta = -1,\ y = -2 \quad \therefore \alpha.\beta.y = 6$

90. The line x = 8 is the directrix of the ellipse E : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus (2, 0). If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the x-axis at Q, then $(3PQ)^2$ is equal to

Ans. **(39)**



Sol. E : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

ae = 2

$\frac{a}{e} = 8$

$e = \frac{1}{2} \quad a^2 = 16 \quad a = 4$

$b^2 = a^2 (1 - e^2) = 16\left(1 - \frac{1}{4}\right) = 16 \times \frac{3}{4} = 12$

$\therefore$ Ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$

Tangent at P is $\frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$

Passing through $(0,\ 4\sqrt{3})$

$\therefore 0 + \frac{4\sqrt{3}\sin\theta}{2\sqrt{3}} = 1$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^0$

$\therefore$ Tangent is $\frac{x}{4} \cdot \frac{\sqrt{13}}{2} + \frac{y}{2\sqrt{3}} \cdot \frac{1}{2} = 1$

$$\frac{\sqrt{3}x}{8} + \frac{y}{4\sqrt{3}} = 1$$

$$Q\left(\frac{8}{\sqrt{3}}, 0\right) \quad P\,(4\cos\theta,\ 2\sqrt{3}\ \sin\theta)$$

$$P\,(2\sqrt{3}, \sqrt{3})$$

$$PQ = \sqrt{\left(\frac{8}{\sqrt{3}} - 2\sqrt{3}\right)^2 + 3} = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 + 3} = \sqrt{\frac{4}{3} + 3}$$

$$= \sqrt{\frac{13}{3}}$$

$$P\,Q^2 = \frac{13}{3} \quad \Rightarrow (3PQ)^2 = \frac{13}{3} \times 9 = 39$$

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