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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 06 April, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. Let $A = \{x \in R : [x+3]+[x+4] \le 3\}$, $B = \left\{x \in R : 3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$, where [t] denotes greatest integer functions. Then

(1) A = B (2) $A \subset B$, , $A \ne B$ (3) $B \subset C$, , $A \ne B$ (4) $A \cap B = \phi$,

NTA Ans. (1)
Reso Ans. (1)

Sol. $A = \{x \in R : [x+3]+[x+4] \le 3\}$

$[x]+3+[x]+4 \le 3$

$[x] \le -2 \Rightarrow x \in (-\infty, -1)$

$B = \left\{x \in R : 3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$

$3^x\left(3\left(\frac{1}{10}+\frac{1}{10^2}+....\right)\right)^{x-3} < 3^{-3x}$

$\left(3\left(\frac{\frac{1}{10}}{1-\frac{1}{10}}\right)\right)^{x-3} < 3^{-4x}$

$3^{3-x} < 3^{-4x}$

3–x < -4x

$3x < -3 \Rightarrow x < -1$

A=B

2. $I(x) = \int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}dx$. If I(0)=, then $I\left(\frac{\pi}{4}\right)$ is equal to

(1) $\log_e \frac{(\pi+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$ (2) $\log_e \frac{(\pi+4)^2}{16} + \frac{\pi^2}{4(\pi+4)}$

(3) $\log_e \frac{(\pi+4)^2}{32} + \frac{\pi^2}{4(\pi+4)}$ (4) $\log_e \frac{(\pi+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$

NTA Ans. (1)
Reso. Ans. (1)

Sol. $I(x) = \int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}dx$

$\because \frac{d}{dx}(x\tan x + 1) = (x\sec^2 x + \tan x)$

$\therefore I(x) = x^2\left(\frac{-1}{x\tan x + 1}\right) + 2\int \frac{x}{x\tan x + 1}dx$

$$= \frac{-x^2}{x\tan x+1} + 2\int \frac{x\cos x}{(x\sin x+\cos x)}dx$$

$$= \frac{-x^2}{(x\tan x+1)} + 2\ln|(x\sin x+\cos x)| + c$$

$I(0) = 0 \Rightarrow C = 0$

$$I(\pi/4) = \ln\left(\frac{(\pi+4)^2}{32}\right) - \frac{\pi^2}{4(\pi+4)}$$

Correct option –(I)

3. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to

(1) 11 (2)12 (3) 10 (4) 9

NTA Ans. **(3)**

Reso. Ans. **(3)**

Sol. $\bar{x}_1 = 12$ $\bar{x}_2 = 14$ $\sigma_1^2 = 14$ $\sigma_2^2 = \sigma^2$

$n_1=15$, $n_2=15$

variance of 30 number$= \dfrac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \dfrac{n_1n_2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}$

$$13 = \frac{14+\sigma^2}{2} + \frac{(12-14)^2}{4}$$

$24 = 14 + \sigma^2 \Rightarrow \sigma^2 = 10$

4. Let $a_1, a_2, a_3, \ldots\ldots\ldots a_n$, be n positive consecutive terms of an arithmetic progression If $d > 0$ is its common difference, then $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \ldots\ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ is

(1) $\frac{1}{\sqrt{d}}$ (2) $\sqrt{d}$ (3) 0 (4) 1

NTA Ans. **(4)**

Reso. Ans. **(4)**

Sol. $$\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ----- + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

$$= \left(\frac{\sqrt{a_2}-\sqrt{a_1}}{a_2-a_1} + \frac{\sqrt{a_3}-\sqrt{a_2}}{a_3-a_2} + \frac{\sqrt{a_4}-\sqrt{a_3}}{a_4-a_3} + ----- + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-a_{n-1}}\right)$$

$$= \frac{1}{d}\left(\sqrt{a_n}-\sqrt{a_1}\right)$$

$$\therefore \underset{n\to\infty}{Lt}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ----- + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

$$= \lim_{n \to \infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_n} - \sqrt{a_1}}{d} \right)$$

$$= \lim_{n \to \infty} \left(\frac{\sqrt{a_1 + (n-1)d} - \sqrt{a_1}}{\sqrt{n}\sqrt{d}} \right) = \lim_{n \to \infty} \left(\sqrt{\frac{a_1}{nd} + \left(1 - \frac{1}{n}\right)} - \sqrt{\frac{a_1}{nd}} \right)$$

$= 1$

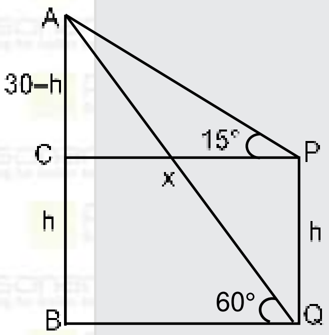
5. From the top A of a vertical wall AB of height 30m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60°respectively, B and Q are on the same horizontal level. If C is a point on AB such that CB = PQ, then the are (in m²) of the quadrilateral BCPQ is equal to

(1) $300(\sqrt{3}+1)$ (2) $200(3-\sqrt{3})$ (3) $600(\sqrt{3}-1)$ (4) $300(\sqrt{3}-1)$

NTA Ans. **(3)**

Reso. Ans. **(3)**

Sol.



$\tan 15° = \frac{30-h}{x}$

$\tan 60° = \frac{30}{x} \Rightarrow x = \frac{30}{\sqrt{3}}$

$\frac{2-\sqrt{3}}{\sqrt{3}} = \frac{30-h}{30} \quad \Rightarrow \quad 30\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) = 30 - h$

$h = 30 - 30 \times \frac{2}{\sqrt{3}} + 30 = 60 - \frac{60}{\sqrt{3}}$

Area $= \frac{30(60)(\sqrt{3}-1)}{3} = 600\ (\sqrt{3}-1)$

6. Statement $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is logically equivalent to :

(1) $(P \Rightarrow R) \wedge (Q \Rightarrow R)$ (2) $(P \Rightarrow R) \vee (Q \Rightarrow R)$

(3) $(P \wedge R) \Rightarrow Q$ (4) $(P \vee R) \Rightarrow Q$

NTA Ans. **(4)**

Reso. Ans. **(4)**

Sol. $(\sim P \vee Q) \wedge (\sim R \vee Q)$

$\equiv (\sim P \wedge \sim R) \vee Q$

$\equiv \sim (P \vee R) \vee Q$

$\equiv (P \vee R) \Rightarrow Q$

7. Sum of the first 20 terms of the series : 5 + 11 + 19 + 29 + 41+..........is

(1) 3520 (2) 3420 (3) 3450 (4) 3250

NTA Ans. (1)

Reso. Ans. (1)

Sol. 5, 11, 19, 29, 41

6 8 10 12

2 2 2

$T_n = an^2 + bn + c$

$T_1 = a + b + c = 5$ ...... (1)

$T_2 = 4a+2b+c=11$ ...... (2)

$T_3 = 9a + 3b+c=19$ .... (3)

eq. (2) –eq. (1)

$3a + b = 6$ .....(4)

eq. (3) –eq. (2)

$5a + b = 8$ .....(5)

eq (5) – eq(4)

$2a = 2 \Rightarrow a=1, b=3, c=1$

$T_n = n^2 + 3n + 1$

$$S_{20} = \sum_{n=1}^{20}\left(n^2+3n+1\right) = \frac{20.21.41}{6} + 3.\frac{20.21}{2} + 20 = 3520$$

8. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is :

(1) $11+\sqrt{3}$ (2) $9+\sqrt{3}$ (3) $9-\sqrt{3}$ (4) $11-\sqrt{3}$

NTA Ans. (2)

Reso. Ans. (2)

Sol. $\left|x^2 - 8x + 15\right| - 2x + 7 = 0$

$|(x-3)(x-5)| - 2x + 7 = 0$

Case-I : If $x \in (-\infty, 3] \cup [5, \infty)$

$x^2 - 8x + 15 - 2x + 7 = 0$

$x^2 - 10x + 22 = 0$

$x = \frac{10 \pm \sqrt{12}}{2} = 5 \pm \sqrt{3}$

$\therefore$ $x = 5 + \sqrt{3}$ is accepted

Case-II : If $x \in (3,5)$

$-x^2 + 8x - 15 - 2x + 7 = 0$

$x^2 - 6x + 8 = 0$

$x = 2, 4 \Rightarrow x = 4$ is only accepted

$\therefore$ Sum of roots $5 + \sqrt{3} + 4 = 9 + \sqrt{3}$

9. Let $A = [a_{ij}]_{2\times2}$, where $a_{ij} \neq 0$ for all I, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and b = |A|. Then $3a^2 + 4b^2$ is equal to

(1) 4 (2) 14 (3) 7 (4) 3

NTA Ans. **(1)**

Reso. Ans. **(1)**

Sol. Let $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ & $A^2 = I \Rightarrow |A|^2 = 1 \Rightarrow b^2 = 1$

$A^2=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$=\begin{bmatrix} a_{11}^2 + a_{12}a_{21} & a_{11}a_{12} + a_{12}a_{22} \\ a_{21}a_{11} + a_{22}a_{21} & a_{21}a_{12} + a_{22}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a_{12}(a_{11} + a_{22})=0 \quad \Rightarrow \quad a_{11} + a_{22} = a = 0$

$\therefore\ 3a^2+4b^2 = 4$

10. One vertex of a rectangular parallelepiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. . Let P be the vertex (3, 4, 5) . Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is :

(1) $12\sqrt{5}$ (2) $\frac{12}{5\sqrt{5}}$ (3) $\frac{12}{5}$ (4) $\frac{12}{\sqrt{5}}$

NTA Ans. **(3)**

Reso. Ans. **(3)**

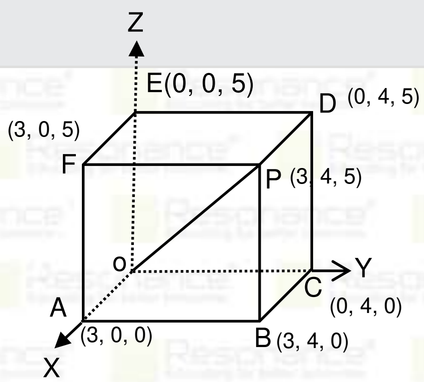
Sol. Eq. of line CD is

$\vec{r} = 4\hat{j} + \lambda 5\hat{k}$

Eq of line OP is

$\vec{r} = \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$

$$\text{S.D.}=\left|\frac{(\vec{a}_1 - \vec{a}_2).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \frac{\begin{vmatrix} 0 & 4 & 0 \\ 0 & 0 & 5 \\ 3 & 4 & 5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 5 \\ 3 & 4 & 5 \end{vmatrix}}$$



S.D. = $\frac{12}{5}$

11. If the system of equations

x + y + az = b

2x + 5y + 2z = 6

X + 2y + 3z = 3

Has infinitely many solutions, then 2a + 3b is equal to

(1) 25 (2) 28 (3) 23 (4) 20

NTA Ans. **(3)**

Reso. Ans. **(3)**

Sol. $\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 1(15-4) + 1(2-6) + a(4-5) = 7-a$

For Infinite solution $\Delta = 0 \Rightarrow a = 7$

$\Delta_x = \begin{vmatrix} b & 1 & 7 \\ 6 & 5 & 2 \\ 3 & 2 & 3 \end{vmatrix} = b(15-4) + 1(6-18) + 7(12-15) = 11b-12-21 = 11b-33 = 0$

$\Rightarrow b = 3$

$\Delta_y = \begin{vmatrix} 1 & b & 7 \\ 2 & 6 & 2 \\ 1 & 3 & 3 \end{vmatrix} = 1(18-6) + b(2-6) + 7(6-6)$

$= 12-4b = 0 \Rightarrow b = 3$

$\Delta_z = \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 1(15-12) + 1(6-6) + b(4-5)$

$= 3-b = 0 \quad \Rightarrow \quad b = 3$

$\therefore \quad 2a + 3b = 14 + 9 = 23$

12. If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$, then a + b + c is equal to

(1) 14 (2) 15 (3) 13 (4) 12

NTA Ans. **(1)**

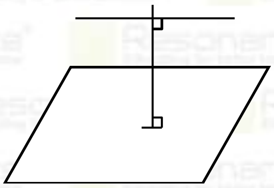
Reso. Ans. **(1)**

Sol. Equation of family of plane passing through the line of intersection of given planes is

$(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$

$(2 + 4\lambda)x - (1 + 3\lambda)y + (1 + 5\lambda)z + (9\lambda - 3) = 0$

parallel to $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$



$\therefore -2(2 + 4\lambda) + 4(-1 - 3\lambda) + 5(1 + 5\lambda) = 0$

$-4 - 8x - 4 - 12\lambda + 5 + 25\lambda = 0$

$5\lambda - 3 = 0 \Rightarrow \lambda = \frac{3}{5}$

So, plane is

$5(2x - y + z - 3) + 3(4x - 3y + 5z + 9) = 0$

$22x - 14y + 20z + 12 = 0$

$11x - 7y + 10z + 6 = 0$

$a = 11.\ b = -7,\ c = 10$

$a + b + c = 11 - 7 + 10 = 14$

correct option (1)

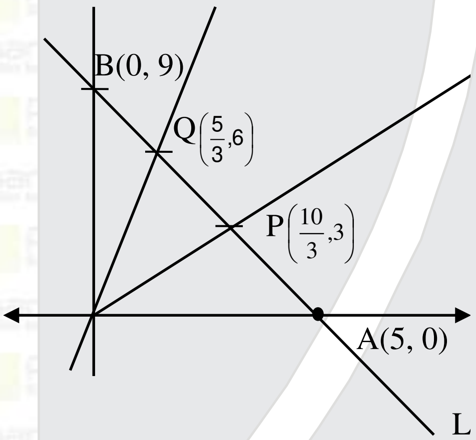
13. The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line L : $9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2 then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on

(1) $y-2x = 15$ (2) $6x-y = 15$ (3) $6x+y = 10$ (4) $y - x = 5$

NTA Ans. **(4)**

Reso. Ans. **(4)**

Sol.



$m_1 = \dfrac{3}{(10/3)} = \dfrac{9}{10}$

$m_2 = \dfrac{6}{(5/3)} = \dfrac{18}{5}$

$m_1 + m_2 = \dfrac{9+36}{10} = \dfrac{45}{10} = \dfrac{9}{2}$

So, $y = (m_1 + m_2)x$ is $y = \dfrac{9}{2}x$

Now solving $y = \dfrac{9}{2}x$ and $9x + 5y = 45$

$\Rightarrow 2y + 5y = 45 \Rightarrow y = \dfrac{45}{7}$, $x = \dfrac{10}{7}$

$y - x = \dfrac{35}{7} = 5$

$\therefore$ correct option (4)

14. If ${}^{2n}C_3 : {}^{n}C_3 = 10 : 1$, then the ratio $(n^2 + 3n) : (n^2 - 3n + 4)$ is

(1) 2:1 (2) 35:16 (3) 27:11 (4) 65: 37

NTA Ans. (1)

Reso. Ans. (1)

Sol. $\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{10}{1}$

$\frac{4(2n-1)}{n-2} = \frac{10}{1}$

$8n - 4 = 10n - 20 \Rightarrow n = 8$

$\frac{n^2+3n}{n^2-3n+4} = \frac{64+24}{64-24+4} = \frac{88}{44} = 2:1$

15. Let $\vec{a} + 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$ if $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a}.\vec{d} = 18$, then $|\vec{a} \times \vec{d}|^2$ is equal to

(1) 720 (2) 680 (3) 760 (4) 640

NTA Ans. (1)

Reso. Ans. (1)

Sol. As $\bar{d}$ is perpendicular to both $\bar{b}$ and $\bar{c} \Rightarrow \bar{d} = \lambda\,(\bar{b} \times \bar{c}), \lambda \in R$

$\bar{d} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = \lambda\,(\hat{i}(2) - \hat{j}(1) + \hat{k}(2))$

$= \lambda\,(2\hat{i} - \hat{j} + 2\hat{k})$

$\bar{a}.\bar{d} = 18 \Rightarrow |\lambda| = 2$

$(\bar{a} \times \bar{d})^2 = \lambda^2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 2 & -1 & 2 \end{vmatrix} = 4\,(10\hat{i} + 4\hat{j} - 8\hat{k})^2$

$= 4\,(100 + 16 + 64) = 4\,(180) = 720$

16. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to

(1) 123 (2) 164 (3) 82 (4) 75

NTA Ans. (1)

Reso. Ans. (1)

Sol. Let P = probability of getting sum '5' in single throw = $\frac{4}{36} = \frac{1}{9}$

$q = \frac{8}{9}.$

$P(x \geq 4) = P(x = 4) + P(x = 5)$

$= {}^5C_4 P^4 \cdot q + {}^5C_5 \cdot P^5$

$= P^4 \cdot \left(5 \cdot \frac{8}{9} + \frac{1}{9}\right)$

$= \left(\frac{1}{9}\right)^4 \cdot \frac{41}{9} = \frac{41}{3^{10}} = \frac{123}{3^{11}}$

∴ Correct option (1)

17. Let the position vectors of the points A, B, C and D be $5\hat{i}+5\hat{j}+2\lambda\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, -2\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-\hat{i}+5\hat{j}+6\hat{k}$. Let the set S = {λ∈R : the points A, B, C and D are coplanar }. Then $\sum_{\lambda\in x}(\lambda+2)^2$ is equal to

(1) 41 (2) $\frac{37}{2}$ (3) 13 (4) 25

NTA Ans. **(1)**
Reso. Ans. **(1)**

Sol $\vec{a} = 5\hat{i}+5\hat{j}+2\lambda\hat{k}$

$\hat{b} = \hat{i}+2\hat{j}+3\hat{k}$

$\vec{c} = -2\hat{i}+\lambda\hat{j}+4\hat{k}$

$\hat{d} = -\hat{i}+5\hat{j}+6\hat{k}$

$\overrightarrow{BA}\cdot(\overrightarrow{BC}\times\overrightarrow{BD}) = 0$

$\begin{vmatrix} 4 & 3 & 2\lambda-3 \\ -3 & \lambda-2 & 1 \\ -2 & 3 & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 2, 3$

$\Sigma(\lambda + 2)^2 = 16 + 25 = 41$

Correct option (1)

18. Let $5f(x)+4f\left(\frac{1}{x}\right) = \frac{1}{x}+3, x>0$. Then $18\int_1^2 f(x)dx$ is equal to :

(1) 5 $\log_e 2 - 3$ (2) 5 $\log_e 2 + 3$ (3) 10 $\log_e 2 + 6$ (4) 10 $\log_e 2 - 6$

NTA Ans. **(4)**
Reso. Ans. **(4)**

Sol. ∵ $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$

$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3$

$9f(x) = \frac{5}{x} + 15 - 4x - 12$

$18f(x) = \frac{10}{x} - 8x + 6$

$\int_1^2 18f(x)dx = \left(10\ln x - 4x^2 + 6x\right)_1^2$

$= 10 \ln 2 - 16 + 12 + 4 - 6$

$= 10 \ln 2 - 6$

19. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to :

(1) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$ (2) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$ (3) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$ (4) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$

NTA Ans. **(2)**
Reso. Ans. **(2)**

Sol. Differentiating $2 \cdot x^y\left(\frac{dy}{dx}\ln x + y\frac{1}{x}\right) + 3 \cdot y^x\left(1.\ln y + x\frac{y'}{y}\right) = 0$

Put $x = 2,\quad y = 2$

$2 \cdot 4\left(\frac{dy}{dx}\ln 2 + 1\right) + 3 \cdot 4\left(\ln 2 + \frac{dy}{dx}\right) = 0$

$(2 \cdot \ln 2 + 3)\frac{dy}{dx} + (2 + 3\ln 2) = 0$

$\frac{dy}{dx} = -\frac{(2+3\ln 2)}{(3+2\ln 2)} = -\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

20. If the ratio of the fifth term from the beginning to the fifth term form the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6} : 1$, then the third term from the beginning is :

(1) $30\sqrt{2}$ (2) $60\sqrt{3}$ (3) $60\sqrt{2}$ (4) $30\sqrt{3}$

NTA Ans. **(2)**
Reso. Ans. **(2)**

Sol. given $\dfrac{{}^nC_4\left(2^{\frac{1}{4}}\right)^{n-4} \cdot \left(3^{\frac{-1}{4}}\right)^4}{{}^nC_4\left(3^{\frac{-1}{4}}\right)^{n-4} \cdot \left(2^{\frac{1}{4}}\right)^4} = \sqrt{6}$

$\Rightarrow 2^{\frac{n-8}{4}} \cdot 3^{\frac{n-8}{4}} = \sqrt{6}$

$\Rightarrow 6^{n-8} = 6^2$

$\Rightarrow n = 10$

So $T_3 = {}^{10}C_2\left(2^{\frac{1}{4}}\right)^8 \cdot \left(3^{\frac{-1}{4}}\right)^2 = \frac{15 \times 3 \times 4}{\sqrt{3}} = 60\sqrt{3}$

21. Let $a \in Z$ and $[t]$ be the greatest integer $\leq t$. Then the number of points, where the function $f(x) = [a+13\sin x]$, $x \in (0, \pi)$ is not differentiable, is ________.

NTA Ans. **(25)**
Reso. Ans. **(25)**

Sol. $f(x) = a + [13\sin x]$ $\quad\because\ a \in I$ and $x \in (0, \pi)$

$\because$ total number of points of non-differentiability of $[p\sin x] = 2p - 1$ here $p = 13$

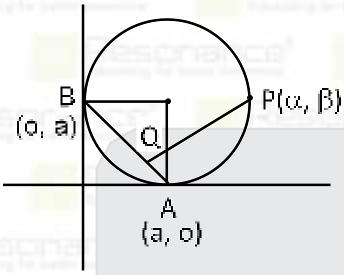
$\therefore$ total number of points of non-differentiability of $[13\sin x] = 25$

22. A circle passing through the point $P(\alpha,\beta)$ in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then the value of $\alpha\beta$ is ___________.

NTA Ans. (121)

Reso. Ans. (121)

Sol.



Equation of line AB

$x + y = a$ ....(i)

$\alpha^2 + \beta^2 - 2a\alpha - 2a\beta + a^2 = 0$ ....(i)

$PQ = \dfrac{|\alpha+\beta-a|}{\sqrt{2}} = 11$

$\alpha^2 + \beta^2 + 2\alpha\beta - 2ax - 2\alpha\beta + a^2 = 121 \times 2$

$0 + 2\alpha\beta = 2 \times 121$

$\alpha\beta = 121$

23. Let A = {1,2,3, 4,.......10} and B = {0, 1,2,3,4} . The number of elements in the relation $R = \{(a,b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$ is ________.

NTA Ans. (18)

Reso. Ans. (18)

Sol $(a-b)\,(2(a-b) + 3) \in B$

If $a - b = 0$ $\quad 0 \in B$

Number of order pair (a, b) = 10

If $a - b = -2 \Rightarrow 2 \in B$

Number of order pair (a – b) = 8

$\therefore$ Number of elements = 18

24. Let the image of the point P(1,2,3) in the plane $2x - y + z = 9$ be Q. If the coordinates of the point R are (6, 10, 7) , then the square of the area of the triangle PQR is _______.

NTA Ans. (594)

Reso. Ans. (594)

Sol. Image of P(1, 2, 3)

$\dfrac{x-1}{2} = \dfrac{y-2}{-1} = \dfrac{z-3}{1} = \dfrac{-2(2-2+3-9)}{4+1+1}$

$Q \equiv (5, 0, 5)$

and $\equiv (6, 10, 7)$

$\overrightarrow{PQ} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$\overrightarrow{PR} = 5\hat{i} - 8\hat{j} + 4\hat{k}$

$$\Delta = \frac{1}{2}\left\|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 2 \\ 5 & 8 & 4 \end{vmatrix}\right\|$$

$= 3\ |4\hat{i} + \hat{j} - 7\hat{k}| = 3\sqrt{66}$

$\Delta^2 = 594$

25. Let the point (p, p + 1) lie inside the region $\left\{E = \{x, y\} : 3 - x \le y \le \sqrt{9 - x^2}, 0 \le x \le 3\right\}$. If the set of sall values of p is the interval (a, b), then $b^2 + b - a^2$ is equal to ________.

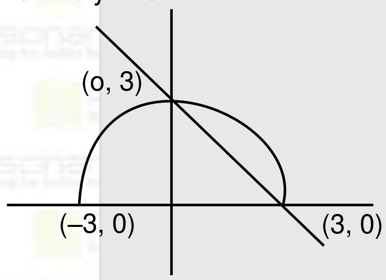
NTA Ans. **(3)**

Reso. Ans. **(3)**

Sol. $3 - x \le y \Rightarrow x + y - 3 \ge 0$....... (i)

$y \le \sqrt{9 - x^2}$

$\Rightarrow x^2 + y^2 \le 9$



$P + P + 1 - 3 \ge 0$

$\Rightarrow\ P \ge 1$

and $P^2 + (P + 1)^2 \le 9$

$2P^2 + 2P - 8 \le 0$

$P^2 + P - 4 \le 0$

$\Rightarrow \quad P \in \left[-\frac{(1+\sqrt{17})}{2}, \frac{\sqrt{17}-1}{2}\right]$

$\therefore \quad P \in \left[1, \frac{\sqrt{17}-1}{2}\right]$

$a = 1, b = \frac{\sqrt{17}}{2} - \frac{1}{2}$

$b^2 + b - a^2 = 3$

26. If the area of the region $S = \{(x, y) : 2y - y^2 \le x^2 \le 2y, x \ge y\}$ is equal to $\frac{n+2}{n+1} - \frac{\pi}{n-1}$ then the natural number n is equal to _________.

NTA Ans. **(5)**

Reso. Ans. **(5)**

Sol. $2y - y^2 \le x^2 \Rightarrow x^2 + (y - 1)^2 \ge 1$ ....(i)

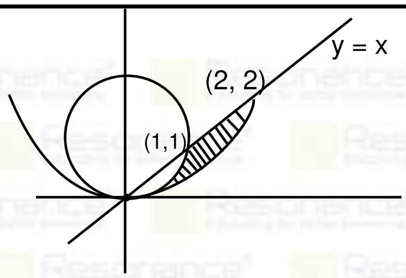
$x^2 \le 2y$ ....(ii)

$x \ge y$ ....(iii)

Solving (i) and (ii)

$2y + y^2 - 2y + 1 = 1$

$y = 0$



$\therefore \quad A = \int_0^2 \left(x - \frac{x^2}{2}\right)dx - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \left(\frac{x^2}{2} - \frac{x^3}{6}\right)_0^2 - \left(\frac{\pi - 2}{4}\right)$

$= 2 - \frac{4}{3} - \frac{\pi}{4} + \frac{1}{2} = \frac{7}{6} - \frac{\pi}{4}$

n = 5

27. The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is _________.

NTA Ans. **(5005)**
Reso. Ans. **(5005)**

Sol. $T_{r+1} = {}^{15}C_r (x^4)^{15-r} (-\frac{1}{x^3})^r$

$60 - 4r - 3r = 18$

$7r = 42$

$r = 6$

Coff.= ${}^{15}C_6 = 5005$

28. Let y = y(x) be a solution of the differential equation (xcosx)dy + (xysinx+ycosx–1)dx = 0, $0<x<\frac{\pi}{2}$. If $\frac{\pi}{3} y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to __________.

NTA Ans. **(2)**
Reso. Ans. **(2)**
Sol. xcosx dy + [xysinx + ycosx – 1]dx = 0

cos x (xdy + ydx) + (xy) sinxdx – dx = 0

$\frac{d(xy)}{dx} + (\tan x) \cdot xy = \sec x$

I.F = $e^{\int \tan x \cdot dx} = e^{\ell n \sec x} = \sec x$

complete solution

$(xy) \sec x = \int \sec^2 x dx$

(xy) secx = tan x + c

$x = \frac{\pi}{3}, \ y = \frac{3\sqrt{\pi}}{\pi}$

$\sqrt{3} \cdot 2 = \sqrt{3} + C$

$C = \sqrt{3}$

$xy = \sin x + \sqrt{3}\cos x$

$xy' + y = \cos x - \sqrt{3}\sin x$

$xy'' + 2y' = -\sin x - \sqrt{3}\cos x$

$\left|\frac{\pi}{6}\cdot y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right| = \left|-\frac{1}{2} - \frac{3}{2}\right| = 2$

29. Let the tangent to curve $x^2 + 2x - 4y + 9x = 0$ at the point P(1,3) on it meet the y-axis at A. Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B. If B lies on the line $2x - 3y = 8$, then $(AB)^2$ is equal to ______________.

NTA Ans. **(292)**

Reso. Ans. **(292)**

Sol. Equation of tangent at P(1, 3)

$x + (x + 1) - 2(y + 3) = 9 = 0$

$2x - 2y + 4 = 0$

$A \equiv (0, 2)$

Equation of line through P(1, 3) parallel to

$x - 3y = 6$ is

$x - 3y + 8 = 0$…….(i)

$\therefore B \equiv (16, 8)$

$(AB)^2 = 16^2 + 36 = 292$

30 The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is _________

NTA Ans. **(171)**

Reso. Ans. **(171)**

Sol. $x_1 + x_2 + x_3 = 20$

let $x_1 = 1 + t_1$, $x_2 = 1 + t_2$, $x_3 = 1 + t_3$

when $t_1, t_2, t_3 \in \{0, 1, 2, \ldots\ldots.17\}$

$t_1 + t_2 + t_3 = 17$--------(1)

So number of such distribution is equal to number of non-negative integral solutions of equation (1)

So required solution = ${}^{17+3-1}C_{3-1} = {}^{19}C_2$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