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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 06 April, 2023 (SHIFT-2) | TIME : (03.00 p.m. to 06.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. Let P be a square matrix such that $P^2 = I - P$. For $\alpha,\beta,\gamma,\delta \in$ IN, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \gamma$ is equal to

(1) 40 (2) 24 (3) 18 (4) 22

NTA Ans. (2)

Reso Ans.(2)

Sol. $P^\alpha + P^\beta = \gamma I - 29 P$ .... (1)

$P^\alpha - P^\beta = \delta I - 13 P$ .... (2)

(1) + (2) $\Rightarrow P^\alpha = \frac{\gamma+\delta}{2} I - 21P$ .... (3)

(1) – (2) $\Rightarrow P^\beta = \frac{\gamma-\delta}{2} I - 8P$ ....(4)

$P^2 = I - P$

$P^3 = P - P^2 = P - (I - P) = 2P - I$

$P^4 = 2P^2 - P = 2(I - P) - P = 2I - 3P$

$P^5 = 2P - 3(I - P) = 5P - 3I$

$P^6 = 5(I - P) - 3P = 5I - 8P$ .... (5)

$P^7 = 5P - 8(I - P) = 13P - 8I$

$P^8 = 13(I - P) - 8P = 13I - 21P$ .... (6)

from (3) & f (6)

$\alpha = 8, \gamma + \delta = 26$

from (4) & (5)

$\beta = 6, \gamma - \delta = 10$

So $\alpha + \beta + \gamma - s$

= 14 + 10 = 24

2. Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{\lceil x \rceil - x}}$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. Then among the statements

(S1) : $A \cap B = (1, \infty) - N$ and

(S2) : $A \cup B = (1, \infty)$

(1) both (S1) and (S2) are true (2) only (S2) is true

(3) only (S1) is true (4) neither (S1) nor (S2) is true

NTA Ans. (3)

Reso Ans. (3)

Sol. $\lceil x \rceil - x > 0 \Rightarrow \lceil x \rceil > x$

$\Rightarrow x \notin I$ So A = R – I

Also $0 < \lceil x \rceil - x < 1$ when $x \in I$

So $f(x) \in (1, \infty) \Rightarrow B = (1, \infty)$

$A \cap B = (1, \infty) - N$

& $A \cup B = (1, \infty) \cup \{R - I\}$

So S_1 is true & S_2 is false

3. If the coefficient of x^7 in $\left(ax^2+\frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{3bx^2}\right)^{11}$ are equal. Then

(1) 729ab = 32 (2) 64ab = 243 (3) 32ab = 729 (4) 243ab = 64

NTA Ans. (1)
Reso Ans.(1)

Sol. $T_r = {}^{11}C_r\,(ax^2)^{11-r}\left(\frac{1}{2bx}\right)^r$

$22 - 2r - r = 7 \Rightarrow 3r = 15$

$\Rightarrow r = 5$

So coefficients of $x^7 = {}^{11}C_5\,\frac{a^6}{2^5.b^5}$ …… (1)

$T_r = {}^{11}C_r\,(ax)^{11-r}\left(-\frac{1}{3bx^2}\right)^r$

$11 - r - 2r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

So coefficients of $x^{-7} \Rightarrow {}^{11}C_6\,a^5\,\frac{1}{3^6.b^6}$ …… (2)

from (1) & (2)

$${}^{11}C_5.\frac{a^6}{2^5.b^5} = {}^{11}C_6\,\frac{a^5}{3^6.b^6}$$

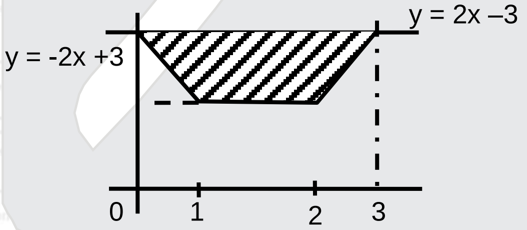
$$\frac{a}{32} = \frac{1}{729b}$$

729ab = 32

4. The area bounded by the curves $y = |x-1| + |x-2|$ and y = 3 is equal to

(1) 3 (2) 5 (3) 4 (4) 6

NTA Ans. (3)
Reso Ans.(3)



area = $\frac{1}{2}\times(1+3)\times 2 = 4$

5. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial number. The serial number of the word PUBLIC is

(1) 578 (2) 576 (3) 582 (4) 580

NTA Ans. (3)
Reso Ans.(3)

Sol. 'PUBLIC '
561432

4!5 + 4!4 + 0 + 2!2 + 0 + 2!2 + 1!1 + 1

= 480 + 96 + 4 1 + 1 =582

6. Let f(x) be a function satisfying $f(x) + f(\pi - x) = \pi^2$, $\forall x \in IR$. Then $\int_0^{\pi} f(x)\sin x\,dx$ is equal to

(1) $\frac{\pi^2}{2}$ (2) π^2 (3) $\frac{\pi^2}{4}$ (4) $2\pi^2$

NTA Ans. (2)
Reso Ans.(2)

Sol. Let $I = \int_0^{\pi} f(x)\sin(x)dx$ .... (1)

$I = \int_0^{\pi} f(\pi - x)\sin(x)dx$ .... (2)

(1) + (2) $\Rightarrow 2I = \int_0^{\pi} \pi^2 \sin(x)dx = 2\pi^2 \Rightarrow I = \pi^2$

7. Among the statements:
(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8
(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in IN$
(1) only (S2) is correct (2) only (S1) is correct
(3) both (S1) and (S2) are incorrect (4) both (S1) and (S2) are correct

NTA Ans. (4)
Reso Ans.(4)

Sol. Statement – 1 : $x^{2022} - y^{2022}$ is divisible by $(x - y)$
Hence $(2023)^{2022} - (1999)^{2022}$ is divisible by 2023 – 1999 = 24
$\Rightarrow$ statement : 1 is correct
Statement – 2 : $13(13)^n - 11n - 13$
$= 13(12 + 1)^n - 11n - 13$
$= 13\,[12^n + n(12)^{n-1} + \frac{n(n-1)}{2}(12)^{n-2} + ... + {}^nC_n] - 11n - 13$
$= 13\,[12^n + n(12)^{n-1} + {}^nC_2(12)^{n-2} + ... + {}^nC_{n-2}(12)^2] - 11n - 13$

$= 13\,[12^n + n(12)^{n-1} + {}^nC_2(12)^{n-2} + ... + {}^nC_{n-2}(12) + 1] + 156n - 11n$
$= 13\,[12^n + n.12^{n-1} + {}^nC_2(12)^{n-2} + ... + {}^nC_{n-2}(12)^2] + 145n$
Which is not divisible by 144 for all $n \in N$ but divisible by 144 for infinitely many values of n if n = 144m where $m \in N$.

8. $\lim_{n\to\infty}\left\{\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right)......\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)\right\}$ is equal to

(1) 0 (2) $\frac{1}{\sqrt{2}}$ (3) $\sqrt{2}$ (4) 1

NTA Ans. (1)
Reso Ans.(1)

Sol. $\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n < \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)......\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n-1}}\right) < \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)^n$

$\Rightarrow \lim_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)^n < L < \lim_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n-1}}\right)^n$

$\Rightarrow L = 0$

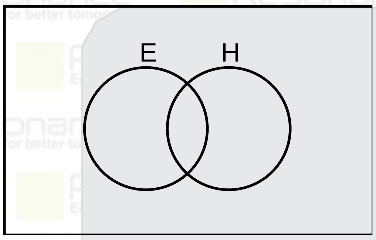
9. In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If number of persons, who speak only English is α and the number of personas who speak only Hindi is β, then the eccentricity of the ellipse $25(\beta^2x^2 + \alpha^2y^2) = \alpha^2\beta^2$ is

(1) $\frac{\sqrt{117}}{12}$ (2) $\frac{\sqrt{129}}{12}$ (3) $\frac{\sqrt{119}}{12}$ (4) $\frac{3\sqrt{15}}{12}$

NTA Ans. (3)

Reso Ans.(3)

Sol.



n (E∪H) = 100

n (E) + n (H) – n (E∩H) = 100

75 + 40 – n (E∩H) = 100

n (E∩H) = 15

n (only English) = 75 – 15 = 60 = α

n (only Hindi) = 40 – 15 = 25 = β

Now ellipse $25(25^2x^2 + 60^2 y^2) = (60)^2 \times (25)^2$

$$\frac{x^2}{\frac{(60)^2}{25}} + \frac{y^2}{\frac{(25)^2}{25}} = 1$$

$$\text{eccentricity} = e = \sqrt{1 - \frac{(25)^2}{(60)^2}} = \sqrt{\frac{(85)(35)}{(60)^2}} = \frac{5}{60}\sqrt{17 \times 7} = \sqrt{\frac{119}{12}}$$

10. The Sum of all values of α for which the points whose position vectors are $\hat{i} - 2\hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{j} + 4\hat{k}$, $(\alpha + 1)\ \hat{i} + 2\hat{k}$ and $9\hat{i} + (\alpha - 8)\ \hat{j} + 6\hat{k}$ are coplanar, is equal to

(1) 6 (2) -2 (3) 4 (4) 2

NTA Ans. (4)

Reso Ans.(4)

Sol. Let A $(\hat{i} + 2\hat{j} + 3\hat{k})$, B $(2\hat{i} - 3\hat{j} + 4\hat{k})$, C $(\alpha + 1\hat{i} + 2\hat{k})$ and D $(9\hat{i} + (\alpha - \delta)\hat{j} + 6\hat{k})$

$\Rightarrow$ Now A,B,C,D are coplanar iff

$[\vec{AB}\ \vec{AC}\ \vec{AD}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ \delta & \alpha - 6 & 3 \end{vmatrix} = 0$$

$\Rightarrow$ 1 (6 + α – 6) + 1 (3α + 8) + 1 (α^2 – 6α-16) =0

$\Rightarrow$ α + 3α + 8 + α^2 -6α-16 = 0

$\Rightarrow$ α^2 – 2 α - 8 = 0

$\Rightarrow$ (α-4) (α+2) = 0

$\Rightarrow$ α = 4, –2

11. If gcd (m, n) = 1 and

$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (2021)^2 - (2022)^2 + (2023)^2 = 1012\, m^2 n$ the $m^2 - n$ is equal to

(1) 200 (2) 180 (3) 240 (4) 220

NTA Ans. (3)

Reso Ans.(3)

Sol. $1^2 + (3-2)(3+2) + (5-4)(5+4) + \ldots\ldots(2023-2022)(2023+2022)$

$= 1 + 2 + 3 + 4 + 5 + \ldots + 2022 + 2023$

$\frac{2023\times(2023+1)}{2} = \frac{2024\times 2023}{2}$

$= 1012 \times 2023 = 1012 \times (17)^2 \times 7$

M = 17, n = 7

$m^2 - n^2 = (m+n)(m-n) = (17+7)(17-7) = 24 \times 10 = 240$

12. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ represent three conterminous edges of a parallelepiped of volume V. Then the volume of the parallelepiped, whose coterminous edges are represented by $\vec{a}, \vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to:

(1) 2V (2) 6V (3) 3V (4) V

NTA Ans. (4)

Reso Ans.(4)

Sol. Since $V = [\vec{a}\vec{b}\vec{c}]$

Now new volume is $= [\vec{a} \quad \vec{b}+\vec{c} \quad \vec{a}+2\vec{b}+3\vec{c}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}] = V$

13. If the solution curve f(x,y) = 0 of the differential equation $(1+\log_e x)\frac{dx}{dy} - x\log_e x = e^y, x > 0$, passes through the points (1, 0) and (α , 2), then α^α is equal to

(1) $e^{2e^{\sqrt{2}}}$ (2) e^{2e^2} (3) e^{e^2} (4) $e^{\sqrt{2}e^2}$

NTA Ans. (2)

Reso Ans.(2)

Sol. $(1+\ell nx)\frac{dx}{dy} - x\,\ell nx = e^y$

Let $x\ell nx = t$

$\left(x\frac{1}{x} + \ell nx\right)\frac{dx}{dy} = \frac{dt}{dy}$

$(1+\ell nx)\frac{dx}{dy} = \frac{dt}{dy}$

$\frac{dt}{dy} - t = e^y$

I.F. = $e^{\int -dy} = e^{-y}$

Complete solution is

t. $e^{-y} = \int e^{-y}.e^{y}dy$

$x\ell nx\ e^{-y} = y + c$

$x\ell nx = e^{y}(y + c)$

(1, 0)

x = 1, y = 0

0 = 1. (0 + c) ⇒ c = 0

(α , 2)

x = α , y= 2

$\alpha \ln \alpha = e^2(2+0)$

$\ln \alpha^{\alpha} = 2e^2$

$\alpha^{\alpha} = e^{2e^2}$

14. For the system of equations

x + y + z = 6

x + 2y + αz = 10

x + 3y + 5z = β, which one the following is Not true?

(1) System has a unique solution for α = – 3, β = 14.

(2) System has infinitely many solutions for α = 3, β = 14.

(3) System has a unique solution for α = 3, β ≠ 14.

(4) System has no solution for α = 3, β = 24.

NTA Ans. (3)

Reso Ans.(3)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & \alpha-1 \\ 1 & 2 & 4 \end{vmatrix} = 4 - 2(\alpha - 1) = 6 - 2\alpha$

$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & \alpha \\ \beta & 3 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 10-12 & 1 & \alpha-2 \\ \beta-18 & 3 & 2 \end{vmatrix} = -1.(-4 - (\beta-18)(\alpha-2)) = (\alpha-2)(\beta-18) + 4$

$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & \alpha \\ 1 & \beta & 5 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 1 \\ 0 & 4 & \alpha-1 \\ 0 & \beta-6 & 4 \end{vmatrix} = 16 - (\alpha-1)(\beta-6)$

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 3 & \beta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & 4 \\ 0 & 1 & \beta-10 \end{vmatrix} = \beta - 10 - 4 = \beta = 14$

Option 1 : If $\alpha = -3 \Rightarrow D = 6 + 6 = 12 \neq 0$ So unique solution

Option 2 : If $\alpha = 3$ & $\beta = 14 \Rightarrow D = 0, D_x = 1 + (-4) + 4 = 0, D_y = 16 - 2 \times 8 = 0, D_z = 0$

Eqⁿ : x + y + z = 6

x + 2y + 3z = 10   y + 2z = 4

x + 3y + 5z = 14   y + 2z = 4   So infinite no of solutions

Option 3 : $\alpha = 3, \beta \neq 14 \Rightarrow D = 0, D_x \neq 0$ So no solution

Option 4 : $\alpha = 3, \beta = 24 \Rightarrow D = 0, D_x \neq 0$ So no solution

15. Among the statements

(S1) : $(p \Rightarrow q) \vee ((\sim p) \wedge q)$ is a tautology

(S2) : $(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q)$ is a contradiction

(1) only (S2) is True
(2) neither (S1) and (S2) is True
(3) Both (S1) and (S2) are True
(4) only (S1) is True

NTA Ans. (2)

Reso Ans.(2)

Sol. S1 : $(p \to q) \vee ((\sim p) \wedge q$

$\equiv (\sim p \vee q) \vee (\sim p \wedge q)$

$= \sim p \vee (q \vee (\sim p \wedge q))$

$= \sim p \vee q \neq t$

S2 : $(q \to p) \to (\sim p \wedge q)$

$= (q \to p) \to \sim(p \vee \sim q)$

$= (q \to p) \to \sim(q \to p)$ (Let $q \to p = s$)

$= s \to \sim s$

$= (\sim s) \vee (\sim s) = \sim s = \sim (q \to p) = \sim (\sim q \vee p)$

$= q \wedge \sim p \neq c$

16. Let the line L pass through the point (0, 1, 2), intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point P(1, –9 ,2) from the line L is.

(1) $\sqrt{74}$
(2) $\sqrt{69}$
(3) $\sqrt{54}$
(4) 9

NTA Ans. (1)

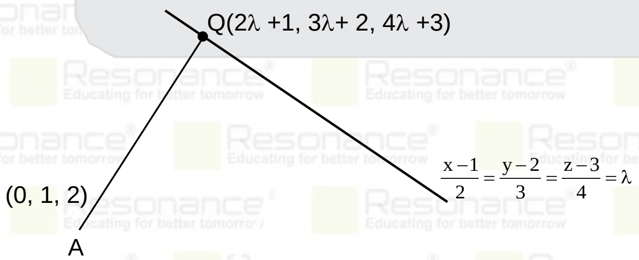
Reso Ans.(1)

Sol. AQ is parallel to $2x + y - 3z = 4$

so $\overrightarrow{AQ}.(2\hat{i} + \hat{j} - 3\hat{k}) = 0$

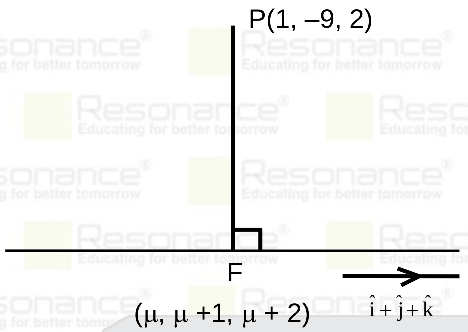
$\Rightarrow 2(2\lambda + 1) + (3\lambda + 1) - 3(4\lambda + 1) = 0$

$\Rightarrow \lambda = 0$



so $\overrightarrow{AQ} = \hat{i} + \hat{j} + \hat{k}$

Line L is $\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{1} = \mu$



$\overrightarrow{PF}.(\hat{i}+\hat{j}+\hat{k})=0$

$\Rightarrow \mu$ -1 + μ + 10 +μ = 0 $\Rightarrow$ = – 3

F (– 3, – 2, –1)

So PF = $\sqrt{16+49+9}=\sqrt{74}$

17. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to

(1) 4 (2) 2 (3) 3 (4) 1

NTA Ans. (1)

Reso Ans.(1)

Sol. Total number of outcomes = $6 \times 6 \times 6$

Total number of favourable out comes = $6 \times 5 \times 4$

So probability = $\frac{5}{9}=\frac{p}{q}$

18. A plane P contains the line of intersection of planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k})=-5$.

If P passes through the point (0,2,-2), then square of distance of point (12,12,18) from the plane P is

(1) 310 (2) 1240 (3) 620 (4) 155

NTA Ans. (3)

Reso Ans.(3)

Sol. Given planes

P_1 : $x+y+z-6=0$

P_2 : 2x + 3y + 4z +5 =0

Equation of plane passing through line of intersection of

$P_1 = 0$ & $P_2 = 0$ is $P_1 + \lambda P_2 = 0$

(1 +2 λ) x+ (1+3 λ) y+ (1+4λ)z +(5 λ -6)=0

It passes through (0, 2,–2)

2(1+3λ) + 6(1+4 λ) (–2) + 5 λ–6 = 0

2 + 6λ –2-8λ +5λ–6 = 0

3 λ-6 =0, λ = 2

So plane : $5x + 7y + 9z + 4 = 0$

distance of (12,12,18)= $\dfrac{5\times12+7\times12+9\times18+4}{\sqrt{25+49+81}}$

$d = \dfrac{310}{\sqrt{155}} = 2\sqrt{155}$

$d^2 = 4 \times 155 = 620$

19. Let $a \neq b$ be two non-zero real numbers. Then the number of elements in the set $X = \{z \in C : \text{Re}(az^2 + bz) = a \text{ and } \text{Re}(bz^2 + az) = b\}$ is equal to

(1) 2 (2) 0 (3) 1 (4) 3

NTA Ans. (2)

Reso Ans.(Infinite)

Sol. Let $z = x + iy$

So $\text{Re}(az^2 + bz) = a$

$\Rightarrow a(x^2 - y^2) + bx = a$

$\Rightarrow x^2 - y^2 + \dfrac{b}{a}x = 1$.......(1)

& $\text{Re}(bz^2 + az) = b$

$\Rightarrow b(x^2 - y^2) + ax = b$

$\Rightarrow x^2 - y^2 + \dfrac{a}{b}x = 1$ .... (2)

(1) – (2) $\Rightarrow \left(\dfrac{b}{a} - \dfrac{a}{b}\right)x = 0 \Rightarrow x = 0$ or $a = -b$ (as $a \neq b$)

C– I. If $x = 0$

From (1) $\Rightarrow x^2 - y^2 + \dfrac{b}{a}x = 1 \Rightarrow -y^2 = \Rightarrow y^2 = -1 \Rightarrow y \in \phi$

so $z \in \phi$.

Case–II. if $a = -b$

then from (1) $\Rightarrow x^2 - y^2 - x = 1$

has infinite solutions

20. If the tangents at the points P and Q on the circle $x^2 + y^2 - 2x + y = 5$ meet at the point $R\left(\dfrac{9}{4}, 2\right)$, then the area of the triangle PQR is:

(1) $\dfrac{5}{4}$ (2) $\dfrac{5}{8}$ (3) $\dfrac{13}{8}$ (4) $\dfrac{13}{4}$

NTA Ans. (2)

Reso Ans.(2)

Sol. Length of tangent from R on circle $= \sqrt{\dfrac{81}{16} + 4 - \dfrac{9}{2} + 2 - 5}$

$L = \sqrt{\dfrac{81-72+16}{16}} = \left(\dfrac{5}{4}\right)$

Radius of circle is $\Rightarrow r = \sqrt{1 + \dfrac{1}{4} + 5}$

$= \sqrt{\dfrac{25}{4}} = \dfrac{5}{2}$

Hence area of Δ PQR is = $\dfrac{L^3 r}{L^2+r^2} = \dfrac{\frac{125}{64}\cdot\frac{52}{2}}{\frac{25}{16}+\frac{25}{4}} = \dfrac{625}{128}\times\dfrac{16}{125} = \dfrac{5}{8}$

21. Let f(x) = $\dfrac{x}{\left(1+x^2\right)^{\frac{1}{n}}}, x \in IR - \{-1\}, n \in N, n > 2.$

If f^n (x) = (fofof…. upto n times) (x), then $\lim\limits_{n\to\infty}\int_0^1 x^{n-2}\left(f^n(x)\right)dx$ is equal to ______ .

NTA Ans. (0)
Reso Ans.(0)

Sol. $f^n(x) = \dfrac{x}{\left(1+nx^n\right)^{\frac{1}{n}}}$

$\lim\limits_{n\to\infty}\int_0^1 x^{n-2}\left(f^n(x)\right)dx = \lim\limits_{n\to\infty}\int_0^1 \dfrac{x^{n-1}}{\left(1+nx^n\right)^{\frac{1}{n}}}dx$ (1+nx^n + t$^n \Rightarrow$ n^2. x^{n-1} dx = nt^{n-1}dt)

$= \int_1^{(n+1)^{\frac{1}{n}}} \dfrac{t^{n-1}}{n.t}dt = \left[\dfrac{t^{n-1}}{n(n-1)}\right]_1^{(n+1)^{\frac{1}{n}}}$ $= \lim\limits_{n\to\infty}\dfrac{(n+1)^{\frac{n-1}{n}}-1}{n(n-1)}$

$\lim\limits_{n\to\infty}\dfrac{\dfrac{(n+1)}{(n+1)^{\frac{1}{n}}}-1}{n(n-1)} = 0$

22. If the mean and variance of the frequency distribution

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is ______ .

NTA Ans. (25)
Reso Ans.(25)

Sol. Subtract 9 in all obs.

x_i	–7	–5	–3	–1	1	3	5	7
f_i	4	4	α	15	8	β	4	5

has $\bar{x} = 0$ & $\sigma^2 = 15.08$

$\vec{x} = 0 \Rightarrow -28-20-3\alpha-15+8+3\beta+20+35 = 0 \Rightarrow \alpha = \beta$

Now $\sigma^2 = 15.08 \Rightarrow \dfrac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2 = 15.08$

$\Rightarrow \dfrac{4\times 49+4\times 25\times+\alpha.9++15\times 1+8\times 1+\beta\times 9+4\times 25+5\times 49}{40+2\alpha} = 15.08$

$\Rightarrow \alpha = 5 \Rightarrow \alpha^2 = 25$

Now $\alpha^2 + \beta^2 - \alpha\beta = \alpha^2 + \alpha^2 - \alpha^2 = \alpha^2 = 25$

23. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots + 20(21)^{19} = k(20)^{19}$, the k is equal to _______ .

NTA Ans. (400)

Reso Ans.(400)

Sol. $k = 1 + 2\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \ldots\ldots\ldots + 20\left(\frac{21}{20}\right)^{19}$

$k = 1 + 2r + 3r^2 + 4r^3 + \ldots\ldots\ldots 20r^{19}$ where $r = \frac{21}{20}$ ………… (1)

$kr = r + 2r^2 + 3r^3 + \ldots\ldots + 20r^{20}$ …………..(2)

by (1) – (2)

$(1 - r)k = 1 + r + r^2 + \ldots\ldots + r^{19} - 20r^{20}$

$$-\frac{1}{20}k = 1\frac{(r^{20}-1)}{r-1} - 20r^{20}$$

$$-\frac{1}{20}k = \frac{\left(\frac{21}{20}\right)^{20} - 1}{\left(\frac{21}{20} - 1\right)} - 20\left(\frac{21}{20}\right)^{20}$$

$$-\frac{1}{20}k = 20\left(\frac{21}{20}\right)^{20} - 20 - 20\left(\frac{21}{20}\right)^{20}$$

$k = 400$

24. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is…….

NTA Ans. (2)

Reso Ans.(2)

Sol. Let e_1 be eccentricity of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Let e_2 be eccentricity of hyperbola $2x^2 - 2y^2 = 1$

$e_2 = \sqrt{2} \Rightarrow e_1 = \frac{1}{\sqrt{2}}$

focus of ellipse $S_1 \equiv (ae_1, 0)$

focus of hyperbola $S_2 = \left(\frac{1}{\sqrt{2}}.\sqrt{2}, 0\right) \equiv (1, 0)$

since both are orthogonal so S_1 & S_2 are coincident

$ae_1 = 1 \Rightarrow a = \sqrt{2}$

$b^2 = a^2\ a^2\left(1 - \frac{1}{2}\right) = 2 \times \frac{1}{2} = 1$

LL'= length of latus rectum $= \frac{2b^2}{a} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$(LL')^2 = 2$

25. For α.β, z ∈ C and λ > 1, if $\sqrt{\lambda-1}$ is the radius of the circle $|z-\alpha|^2+|z-\beta|^2=2\lambda$, then $|\alpha-\beta|$ is equal to_____ .

NTA Ans. (2)
Reso Ans.(2)

Ans. $|z|^2+|\alpha|^2-2\text{Re}(z\bar{\alpha})+|z|^2+|\beta|^2-2\text{Re}(z\bar{\beta})=2\lambda$

$\Rightarrow |z|^2-\frac{\alpha+\beta}{2}\bar{z}-\frac{\overline{\alpha+\beta}}{2}z+\frac{|\alpha|^2+|\beta|^2}{2}-\lambda=0$

radius = $\sqrt{\left|\frac{\alpha+\beta}{2}\right|^2-\left(\frac{|\alpha|^2+|\beta|^2}{2}-\lambda\right)}=\sqrt{\lambda-1}$

$\Rightarrow\left|\frac{\alpha-\beta}{2}\right|^2=1\Rightarrow|\alpha-\beta|^2=4$

$\Rightarrow|\alpha-\beta|=2$

26. If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is _____ .

NTA Ans. (18)
Reso Ans.(18)

Sol. Let $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}=\lambda$

$\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}=\mu$

So (1+2λ, 2+3λ, 3 +αλ) = (4+5μ, 1+2μ,βμ)

1 +2λ = 4 +5 μ ⇒ $\left(\frac{3+5\mu}{2}=\frac{2\mu-1}{3}\Rightarrow 9+15\mu=4\mu-2\Rightarrow\mu=-1\right)$

2 +3λ = 1 +2 μ ⇒ (2+3λ = 1 – 2 ⇒ λ = –1)

3 + αλ = βμ

3 – α = – β

⇒ β = α – 3

Now 8αβ = 8α(α –3) = 8(α² – 3α)

$\geq 8\left(-\frac{D}{4a}\right)=8\times\left(-\frac{9}{4}\right)=-18$ Ans. 18

27. The number of points, where the curve y = x⁵ – 20x³ + 50x +2 crosses the x–axis, is ______ .

NTA Ans. (5)
Reso Ans.(5)

Sol. $y = x^5-20x^3+50x+2$

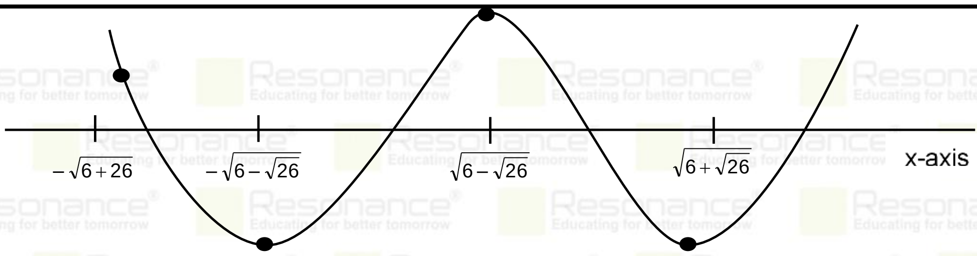
$Y' = 5x^4-60x^2+50$

$= 5(x^4-12x^2+10)=0$

$= 5((x^2-6)^2-26)=0$

$= 5(x^2-6-\sqrt{26})(x^2-6+\sqrt{26})$

$= 5(x-\sqrt{6+\sqrt{26}})(x+\sqrt{6+\sqrt{26}})(x-\sqrt{6-\sqrt{26}})(x+\sqrt{6-\sqrt{26}})$



For values of x where y' = 0 i.e. $x^4 - 12x^2 + 10 = 0$

$y = x(x^4 - 20x^2 + 50) + 2$

$= x(-8x^2 + 40) + 2$

$= -8x(x^2 - 5) + 2$

For $x^2 = 6 + \sqrt{26}$

$Y = -8x(1 + \sqrt{26}) + 2$

For $x = -\sqrt{6+\sqrt{26}}$ y is positive

& for $x = \sqrt{6+\sqrt{26}}$ y is negative

$x^2 - 6 = \pm\sqrt{26}$

for $x^2 = 6 - \sqrt{26}$

$y = -8x(1 - \sqrt{26})$

for $x = -\sqrt{6-\sqrt{26}}$ y is negative

for $x = \sqrt{6-\sqrt{26}}$ y is positive

28. The value of $\tan 9^o - \tan 27^o - \tan 63^o + \tan 81^o$ is____________ .

NTA Ans. (4)

Reso Ans.(4)

Sol. $\tan 9^o - \tan 27^o - \cot 27^o + \cot 9^o$

$(\tan 9^o + \cot 9^o) - (\tan 27^o + \cot 27^o)$

$$\frac{\sin^2 9^o + \cos^2 9^o}{\sin 9^o \cos 9^o} - \frac{\sin^2 27^o + \cos^2 27^o}{\cos 27^o \sin 27^o}$$

$$\frac{2}{\sin 18^o} - \frac{2}{\sin 54^o} = \frac{8}{\sqrt{5}-1} - \frac{8}{\sqrt{5}+1} = \frac{8(\sqrt{5}+1-\sqrt{5}+1)}{5-1} = \frac{16}{4} = 4$$

29. Let a curve y = f(x), x ∈ (0, ∞) pass through the points $P\left(1, \frac{3}{2}\right)$ and $Q\left(\alpha, \frac{1}{2}\right)$. If the tangent at any point R(b, f(b)) to the given curve cuts the y-axis at the point S(0, c) such that bc = 3, then $(PQ)^2$ is equal to ________ .

NTA Ans. (5)

Reso Ans.(5)

Sol. Equation of tangent at R (b, f(b))

$y - f(b) = f'(b)(x - b)$

passes through (0, c) so

$c - f(b) = f'(b)(-b)$

$bf'(b) - f(b) = -c$ s

$bf'(b) - f(b) = -\frac{3}{b}$

$$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = -\frac{3}{b^3}$$

$$\Rightarrow \frac{f(b)}{b} = +\frac{3}{2b^2} + \lambda$$

it passes through $P\left(1, \frac{3}{2}\right)$

$\Rightarrow \frac{3}{2} = +\frac{3}{2} + \lambda \Rightarrow \lambda = 0$

So, f(b) = $\frac{3}{2b}$

it passes through $\left(a, \frac{1}{2}\right)$

So, $\frac{1}{2} = \frac{3}{2a} \Rightarrow a = 3$

So, $P\left(1, \frac{3}{2}\right)$ and $Q\left(3, \frac{1}{2}\right)$

So, $PQ^2 = 4 + 1 = 5$

30. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is ______.

NTA Ans. (432)

Reso Ans.(432)

sol. Vowels — Consonants

UIEE — NVRS

All different = ${}^3C_2 . {}^4C_2 \times 4! = 432$