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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)
Questions & Solutions

Date: 08 April, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | **FAX No. :** +91-022-39167222

To Know more : sms **RESO** at **56677** | **Website :** www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN :** U80302RJ2007PLC024029

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PART : MATHEMATICS

1. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6

(1) 782 (2) 792 (3) 752 (4) 772

NTA. **(2)**

RESO **(2)**

Sol. $n(A) = 5$, $n(B)=2$

$\therefore n(A\times B) = 5 \times 2 = 10$

Number of subsets $= {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$

$= 120+420 + 252 = 792$

2. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is

(1) 18000 (2) 33600 (3) 16800 (4) 14800

NTA. **(3)**

RESO **(3)**

Sol. Vol. I, , E, E, E, E

Constant N, D, P, N, D, N, C

No of ways $= \frac{8!}{3!2!}.\frac{5!}{4!} = 16800$

3. The number of ways, in which 5 girls and 7 boys can be seated at a round so that no two girls sit together, is

(1) $7(360)^2$ (2) $7(720)^2$ (3) $126(5!)^2$ (4) 720

NTA. **(3)**

RESO **(3)**

Sol. 7 boys can sit = 6 !

which create 7 gap between then in which 5 girls have to set

No of ways $= 6!. \times {}^7C_5 5! = 126(5!)^2$

4. If the equation of the plane containing the line x + 2y + 3z – 4 = 0 = 2x + y-z +5 and perpendicular to the plane $\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ is ax + by + cz = 4, than (a - b + c) is equal

(1) 20 (2) 22 (3) 21 (4) 18

NTA. **(2)**

RESO **(2)**

Sol. Equation of plane containing line of intersection of plane x + 2y + 3z – 4 = 0 and 2x + y – z+ 5 = 0 will be

$P_1 + \lambda P_2 = 0 \Rightarrow (1 + 2\lambda)x + (2 + \lambda)y + (3 - \lambda)z + (5\lambda - 4) = 0$ … (1)

This plane is ⊥ to plane

$\vec{r} = (\vec{i} - \vec{j}) + \lambda(\vec{i} + \vec{j} + \vec{k}) + \mu(\vec{i} - 2\vec{j} + 3\vec{k})$ …(2)

Normal of plane (2) = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = 5\hat{i} - 2\hat{j} - 3\hat{k}$

Now both plane (1) & (2) ⊥ Hence their normal also ⊥

$5(1 + 2\lambda) - 2(2 + \lambda) - 3(3 - \lambda) = 0$

$-8 + 11\lambda = 0 \Rightarrow \lambda = \frac{8}{11}$

Now equation of plane $\frac{27}{11}x + \frac{30}{11}y + \frac{25}{11}z - \frac{4}{11} = 0$

$27x + 30y + 25z = 4$

Hence a – b + c = 27 – 30 + 25 = 22

5. If the points with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to

(1) 25 (2) 16 (3) 49 (4) 36

NTA. **(4)**

RESO **(4)**

Sol. If $A(\vec{a})$, $B(\vec{b})$, $C(\vec{c})$ are collinear then

$\overrightarrow{AB} || \overrightarrow{BC} || \overrightarrow{AC}$

Now $\overrightarrow{AB} = (6-\alpha)\hat{i} + \hat{j} - 2\hat{k}$

$\overrightarrow{BC} = -\frac{3}{2}\hat{i} + (\beta - 11)\hat{j} - 19\hat{k}$

$\overrightarrow{AB} || \overrightarrow{BC} \Rightarrow \frac{6-\alpha}{-3/2} = \frac{1}{\beta - 11} = +\frac{2}{19}$

$\Rightarrow 6 - \alpha = \frac{-3}{19} \Rightarrow \alpha = 6 + \frac{3}{19} = \frac{117}{19} \Rightarrow 19\alpha = 117$

$\beta - 11 = +\frac{19}{2} \Rightarrow \beta = 11 + \frac{19}{2} = \frac{41}{2} \Rightarrow 6\beta = 123$

$(19\alpha - 6\beta)^2 = (-6)^2 = 36$

6. Let $S_K \frac{1+2+.....+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}(Bn^2 + Cn + D)$, where A,B,C,D $\in$N and A has least value. Than.

(1) A + B =5(D-C)

(2) A + C+ D is not divisible by B

(3) A +B + C +D is divisible by 5

(4) A + B is divisible by D

NTA. (4)

RESO (4)

Sol. $S_k = \frac{\frac{k(k+1)}{2}}{k} = \frac{1}{2}(k+1)$

$\sum_{k=1}^{n} S_k^2 = \frac{1}{4}\sum_{k=1}^{n}(k+1)^2 = \frac{1}{4}\left(\left(2^2 + 3^3 + ...+(n+1)^2\right) + 1^2 - 1^2\right)$

$= \frac{1}{4}\left(\frac{(n+1)(n+2)(2(n+1)+1)}{6} - 1\right)$

$= \frac{1}{4}\left(\frac{(n+1)(n+2)(2n+3) - 6}{6}\right)$

$= \frac{(n+1)(n+2)(2n+3) - 6}{24} = \frac{n(2n^2 + 9n + 13)}{24}$

A = 24, B = 2, C= 9, D=13

7. Negation of $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$ is

(1) $(\sim p) \vee q$ (2) $q \wedge (\sim p)$

(3) $(\sim q) \wedge p$ (4) $p \vee (\sim q)$

NTA. **(2)**

RESO **(2)**

Sol.

P	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$	$\sim((p \to q) \to (q \to p))$
T	T	T	T	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	F

8. If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the

(1) $x^2 + 3x - 4 = 0$ (2) $x^2 + x - 12 = 0$

(3) $x^2 + 2x - 3 = 0$ (4) $x^2 + 7x + 12 = 0$

NTA. **(4)**

RESO **(4)**

Sol. $z = \alpha + i\beta$

$|z + 2| = z + 4(1 + i)$

$|(\alpha + 2) + i\beta| = \alpha + i\beta + 4 + 4i$

$\sqrt{(\alpha+2)^2 + \beta^2} = (\alpha + 4) + i(\beta + 4)$

compare real and imaginary part from both sides

$\beta + 4 = 0 \Rightarrow \beta = -4$

and $\sqrt{(\alpha+2)^2 + 16} = \alpha + 4$

$\Rightarrow \alpha^2 + 4 + 4\alpha + 16 = \alpha^2 + 16 + 8\alpha$

$\Rightarrow \alpha = 1, \beta = -4$

$\alpha + \beta = -3, \alpha\beta = -4$

Hence equation is $x^2 - (-3-4)x + (-3)(-4) = 0$

$x^2 + 7x + 12 = 0$

9. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1:5:20, then the coefficient of the fourth term is

(1) 3654 (2) 2436 (3) 1827 (4) 5481

NTA. **(1)**

RESO **(1)**

Sol. $a_r = {}^nC_r$

$$\frac{a_r+1}{a_r} = 5 \Rightarrow \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{(n+1)-(r+1)}{r+1} = 5 \Rightarrow n+1 = 6\,(r+1) \quad \_\_\_\_\_\_\_(1)$$

$$\frac{a_{r+2}}{a_{r+1}} = 4 \Rightarrow \frac{{}^nC_{r+2}}{{}^nC_{r+1}} = \frac{(n+1)-(r+2)}{r+2} = 4 \Rightarrow n+1 = 5(r+2) \quad \_\_\_\_\_\_\_(2)$$

(1) & (2) $6\,(r+1) = 5(r+2) \Rightarrow r = 4$

$$n+1 = 30 \Rightarrow n = 29$$

Hence consecutive terms are T5, T6, T7

Hence coefficient of T4 = ${}^{29}C_3 = 3654$

10 Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|adj(adj(adi2A)|\,| = (16)^n$, then n is equal to

(1) 9 (2) 12 (3) 8 (4) 10

NTA. **(4)**

RESO **(4)**

Sol. $|adj\ (adj(adj(2A)))| = |2A|\,|2A|^{(n-1)^3} = |\,2A\,|^8 = (2^3\,|\,A\,|^8) = (2^3.4)^8 = 16^{10}$

11. Let α,β,γ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

(1) 21 (2) 19 (3) $\frac{155}{2}$ (4) $\frac{169}{8}$

NTA. **(2)**

RESO **(2)**

Sol. $x^3 + bx + c = 0$ $\beta r = 1 = -\alpha$

$\alpha + \beta + \gamma = 0,\ \alpha\beta + \beta\gamma + \gamma\alpha = -b,\ \alpha\beta\gamma = -c$

$\alpha = -c = -1$

$c = 1$

Now, $\alpha = -1 \Rightarrow \beta + \gamma = 1$

$-\beta - \gamma + \beta\gamma = -b$

$\beta\gamma = 1 - b = 1 \Rightarrow b = 0$

Now $x^3 + 0.x + 1 = 0 \Rightarrow x^3 + 1 = 0$

$(x + 1)(x^2 - x + 1) = 0$

$x = -1, -\omega, -\omega^2$

$\alpha = -1, \beta = -\omega, \gamma = -\omega^2, b = 0, c = 1$

$b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3 = 0 + 2 + 3 + 6 + 8 = 19$

12. In a bolt factory, machines A,B and C, manufacture respectively 20%, 30% and 50% of the total bolts, Of their output , 3, 4 and 2 percent are respectively defective bolts, A blot is drawn at random from the product. If the bolt drawn is found the defective, then the probability that is manufactured by the machine C is

(1) $\frac{9}{28}$ (2) $\frac{5}{14}$ (3) $\frac{2}{7}$ (4) $\frac{3}{7}$

NTA. **(2)**

RESO **(2)**

Sol. M : Bolt is defective

B_1 : Produced by A ; $P(B_1) = \frac{20}{100}$

B_2 : Produced by B ; $P(B_2) = \frac{30}{100}$

B_3 : Produced by C ; $P(B_3) = \frac{50}{100}$

$$P\left(\frac{B_3}{M}\right) = \frac{P(B_3)\ .\ P(M/B_3)}{\Sigma\ P(B_1)\ P(M/B_1)} = \frac{\frac{50}{100}\times\frac{2}{100}}{\frac{20}{100}\times\frac{3}{100}+\frac{30}{100}\times\frac{4}{100}+\frac{50}{100}\times\frac{2}{100}} = \frac{5}{14}$$

13. Let $f(x)=\dfrac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x\in[0,\pi]-\left\{\dfrac{\pi}{4}\right\}$, Then $f\left(\dfrac{7\pi}{12}\right)f''\left(\dfrac{7\pi}{12}\right)$ is equal to

(1) $\dfrac{-1}{3\sqrt{3}}$ (2) $\dfrac{2}{9}$ (3) $\dfrac{2}{3\sqrt{3}}$ (4) $\dfrac{-2}{3}$

NTA. **(2)**

RESO **(2)**

Sol. $f(x) = \sqrt{2}\,\dfrac{\left(\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)-1\right)}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)}=\dfrac{\sin(x+\pi/4)-1}{\sin(x-\pi/4)}$

$$f'(x) = \frac{\cos(x+\pi/4)\sin(x-\pi/4)-\cos(x-\pi/4)(\sin(x+\pi/4)-1)}{\sin^2(x-\pi/4)}$$

$$f'(x) = \frac{\cos(x-\pi/4)-1}{\sin^2(x-\pi/4)}=\frac{-(1-\cos(x-\pi/4))}{1-\cos^2(x-\pi/4)}$$

$$f'(x) = -\frac{1}{1+\cos(x-\pi/4)}$$

$$= f''(x)=-\frac{\sin(x-\pi/4)}{(1+\cos(x-\pi/4))^2}$$

$$f(7\pi/12) = -\frac{1}{\sqrt{3}}$$

$$f''(7\pi/12) = -\frac{2\sqrt{3}}{9}$$

$$f(7\pi/12)\, f''(7\pi/12) = \frac{2}{9}$$

14. The shortest distance between the lines $\dfrac{x-4}{4}=\dfrac{y+2}{5}=\dfrac{z+3}{3}$ and $\dfrac{x-1}{3}=\dfrac{y-3}{4}=\dfrac{z-4}{2}$ is

(1) $6\sqrt{3}$ (2) $3\sqrt{6}$ (3) $6\sqrt{2}$ (4) $2\sqrt{6}$

NTA. **(2)**

RESO (2)

Sol. $L_1 : \vec{r} = \left(4\hat{i} - 2\hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} + 5\hat{j} + 3\hat{k}\right)$

$L_2 : \vec{r} = \left(\hat{i} + 3\hat{j} + 4\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$

Shortest distance $\left|\dfrac{\left[\vec{a}_2 - \vec{a}_1\ \vec{b}_1\ \vec{b}_2\right]}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$\vec{a}_1 = 4\hat{i} - 2\hat{j} - 3\hat{k},\ \vec{b}_1 = 4\hat{i} + 5\hat{j} + 3\hat{k}$

$\vec{a}_2 = \hat{i} + 3\hat{j} + 4\hat{k},\ \vec{b}_2 = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 5\hat{j} + 7\hat{k}$

$$\left[\vec{a}_2 - \vec{a}_1\ \vec{b}_1\ \vec{b}_2\right] = \begin{vmatrix} -3 & 5 & 7 \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = 18$$

$$\vec{b}_1 \times \vec{b}_2 == \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = -2\hat{i} + \hat{j} + \hat{k}$$

$-3\hat{i} + 5\hat{j} + 7\hat{k},\ \vec{b}_2 = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{6}$

Shortest distance = $\dfrac{18}{\sqrt{6}} = 3\sqrt{6}$

15. Let C (α, β) be the circumcenter of the triangle formed by the lines

$4x+3y = 69$,

$4y-3x=17$, and

$x+7y=61$.

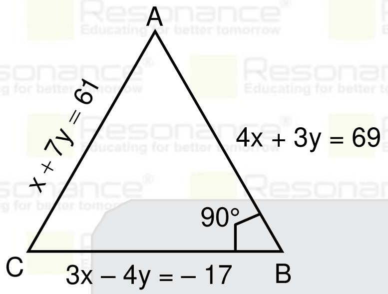
Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

(1) 18 (2) 15 (3) 16 (4) 17

NTA. (4)

RESO (4)

Sol.



Triangle is right angle at vertex B. Hence circum centre is mid point of AC

For A solve $x + 7y = 61$ and

$$\begin{aligned} 4x + 3y &= 69 \\ 4x + 28y &= 244 \\ - \quad - \quad & \quad - \\ \hline -25y &= -175 \end{aligned}$$

$y=7$

$x=12$

A(12, 7)

For C solve $x + 7y = 61$ and

$$\begin{aligned} 3x - 4y &= -17 \\ 3x + 21y &= 183 \\ - \quad - \quad & \quad - \\ \hline -25y &= -200 \end{aligned}$$

$y=8$

$x=5$

C(5, 8)

Curcumcenter $\equiv \left(\frac{5+12}{2}, \frac{7+8}{2}\right) \equiv \left(\frac{17}{2}, \frac{15}{2}\right)$

$\alpha = \frac{17}{2}, \beta = \frac{15}{2}$ Hence $(\alpha - \beta)^2 + \alpha + \beta = 1 + 16 = 17$

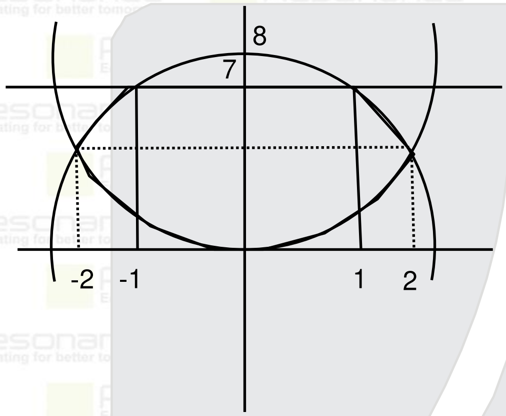
16. The area of the region $\{(x, y) : x^2 \le y \le 8 - x^2, y \le 7\}$ is

(1) 18 (2) 20 (3) 21 (4) 24

NTA. **(2)**

RESO **(2)**

Sol.



$y \le 7$, $x^2 \le y \le 8 - x^2$

$x^2 = y$ & $y = 8 - x^2$

Point of intersection

$x^2 = 8 - x^2$

$2x^2 = 8$

$x = \pm 2$

$$A = 2\int_0^1 (7 - x^2)dx + 2\int_1^2 (8 - x^2 - x^2)dx$$

$$= 2\left[7x - \frac{x^2}{3}\right]_0^1 + 2\left[8x - \frac{2x^3}{3}\right]_1^2$$

$$= 2\left(7 - \frac{1}{3} + 8(2-1) - \frac{2}{3}(8-1)\right) = 2\left(7 - \frac{1}{3} + 8 - \frac{14}{3}\right) = 2\ (15 - 5) = 20$$

17. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$. If $P^TQ^{2007}P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equal to

(1) 2005 (2) 2004 (3) 2007 (4) 2006

NTA. **(1)**

RESO **(1)**

Sol. $PQ^{2007}P^T = P(P^T AP)(P^T AP)(P^T AP) \text{------} (P^T AP)P^T$

$= IAIA \text{-----} IAI$

$= A^{2007}$

$= \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$PP^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

....

$A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$a = 1, b = 2007, c = 0, d = 1$

$2a + b - 3c - 4d = 2005$

18. Let $I(x) = \int \frac{(x+1)}{x(1+xe^x)^2} dx$, x>0. If $\lim_{x\to\infty} I(x) = 0$, then I (1) is equal to

(1) $\frac{e+1}{e+2} - \log_e(e+1)$ (2) $\frac{e+1}{e+2} + \log_e(e+1)$ (3) $\frac{e+2}{e+1} - \log_e(e+1)$ (4) $\frac{e+2}{e+1} + \log_e(e+1)$

NTA. **(3)**

RESO **(3)**

Sol. $I(x) = \int \frac{(x+1)}{x(1+xe^x)^2} dx$

$= \int \frac{(x+1)e^x dx}{xe^x(1+xe^x)^2}$

Let $1 + xe^x = t$

$e^x(x + 1)\, dx = dt$

$I(x) = \int \frac{dt}{(t-1)t^2}$

$I(x) = \int \left(\frac{A}{t} + \frac{B}{t^2} + \frac{C}{t-1}\right) dt$

$A\, t(t-1) + B(t-1) + Ct^2 = 1$

Put $t = 0 \Rightarrow B = -1$

$t = 1 \Rightarrow C = 1$

Compare coff of $t^2 \Rightarrow A + C = 0 \Rightarrow A = -1$

$I(x) = \int \left(-\frac{1}{t} - \frac{1}{t^2} + \frac{1}{t-1}\right) dt$

$I(x) = -\ln(t) + \frac{1}{t} + \ln(t-1) + k$

$I(x) = \frac{1}{(1+xe^x)} + \ln\left(\frac{xe^x}{1+xe^x}\right) + k$

Now $\lim_{x\to\infty} I(x) = \lim_{x\to\infty} \frac{1}{(1+xe^x)} + \ln\left(\frac{1}{\frac{1}{xe^x}+1}\right) + k = 0$

$= 0 + 0 + k = 0$

Now $I(1) = \frac{1}{1+e} + \ell n\left(\frac{e}{1+e}\right) = \frac{1}{1+e} + 1 - \ell n(1+e)$

$= \frac{2+e}{1+e} = -\ell n(1+e)$

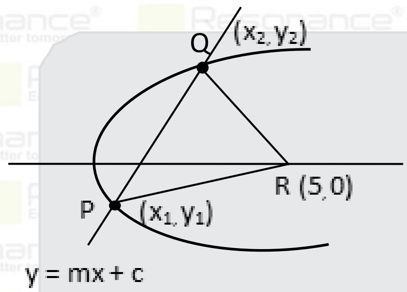
19. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the point G (10,10) be the centroid of the triangle PQR. If c-m=6, then $(PQ)^2$ is

(1) 325 (2) 296 (3) 317 (4) 316

NTA. **(1)**

RESO **(1)**

Sol. $y^2 = 20x$ focus is R(a, 0) = R(5, 0)



Centroid of $\Delta PQR \equiv G\left(\frac{x_1+x_2+5}{3}, \frac{y_1+y_2+0}{3}\right) \equiv (10, 10)$

$x_1 + x_2 = 25, \quad y_1 + y_2 = 30$

Now, $y^2 = 20x, \quad y = mx + C \quad \because C = m + 6$

$y = mx + (m + 6)$

$y^2 = 20\left(y - \frac{(m+6)}{m}\right)$

$my^2 - 20y + 20(m + 6) = 0$

$y_1 + y_2 = \frac{20}{m} = 30 \Rightarrow m = 2/3, C = \frac{2}{3} + 6 = \frac{20}{3}$

$PQ^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

Now for $m = 2/3 \Rightarrow \frac{2}{3}y^2 - 20y + 20\left(\frac{2}{3}+6\right) = 0 \Rightarrow y^2 - 30y + 200 = 0$

$y = 20, 10$

For $y = 10, x = 5$

$y = 20, x = 20$

$PQ^2 = 15^2 + 10^2 = 225 + 100 = 325$

20. $\lim\limits_{x \to 0}\left(\left(\dfrac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\dfrac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ is equal to ________

(1) 15 (2) 9 (3) 24 (4) 18

NTA. **(4)**

RESO **(4)**

Sol. $\lim\limits_{x\to0}\dfrac{1-\cos^2 3x}{\cos^3(4x)}\dfrac{\sin^3(4x)}{(\ell n(1+2x))^5}$

$\lim\limits_{x\to0}\dfrac{\sin^2(3x)}{(3x)^2}\dfrac{\sin^3(4x)}{(4x)^3}\dfrac{1}{\cos^3(4x)}\dfrac{1}{\left(\dfrac{\ell n(1+2x)}{(2x)}\right)^5}\dfrac{3^2.4^3}{2^5}$

$\Rightarrow \dfrac{3^2.4^3}{2^5} = 9\times2=18$

21. If a_n is the greatest term in the sequence $a_n = \dfrac{n^3}{n^4+147}$, n = 1.2,3…, then a is equal to _____

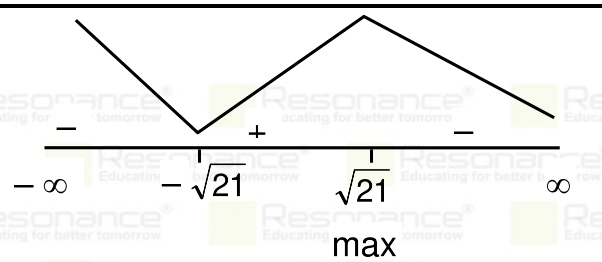
NTA. **(5)**

RESO **(5) (In question a should be n)**

Sol. $f'(n) = \dfrac{3n^2(n^4+147)-4n^3(n^3)}{(n^4+147)^2}$

$f'(n) = \dfrac{n^2[3n^4+441-4n^4]}{(n^4+147)^2}$

$f'(n) = \dfrac{-n^2(n^4-441)}{(n^4+147)^2} = \dfrac{-n^2(n^2+21)(n^2-21)}{(n^4+147)^2} = \dfrac{-n^2(n^2+21)}{(n^4+147)^2}\left(n+\sqrt{21}\right)\left(n-\sqrt{21}\right)$



When n = 4 $\quad a_4 = \frac{4^3}{4^4+147} = \frac{64}{256+147} = \frac{64}{403} =$

When n = 5 $\quad a_5 = \frac{125}{625+147} = \frac{125}{772} = \frac{125}{772}$

a_5 is max = $\frac{125}{772}$ hence n = 5

22. Let the mean and variance of 8 number x,y,10,12,6,12,4,8 be 9 and 9.25 respectively, If x>y, then 3x – 2y is equal to_____.

NTA. (25)

RESO (25)

Sol. $\frac{x+y+12+6+4+8+12+10}{8} = 9 \Rightarrow x + y = 20$ _____ (1)

$\sigma^2 + \bar{x}^2 = \frac{1}{n}\sum x_i^2$

$\Rightarrow \quad 9.25 + 9^2 = \frac{1}{8}(x^2 + y^2 + 12^2 + 6^2 + 4^2 + 8^2 + 12^2 + 10^2)$

$74 + 8 \times 81 = x^2 + y^2 + 504$

$218 = x^2 + y^2$ __________(2)

From (1) & (2) $\Rightarrow x^2 + (20 - x)^2 = 218$

$\Rightarrow x^2 - 20x + 91 = 0$

$x = 7, 13$

$\therefore \quad x = 13 , y = 7$

$3x - 2y = 39 - 14 = 25$

23. Let [t] denote the greatest integer $\leq t$. If the constant term in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ is a , then [a] is equal to _____.

NTA. **(1275)**

RESO **(1275)**

Sol. $T_{r+1} = {}^7C_r\,(3x^2)^{7-r}\left(-\frac{1}{2x^5}\right)^r$

$T_{r+1} = {}^7C_r\,3^{7-r}\left(-\frac{1}{2}\right)^r x^{14-7r}.$

For term independent of x : $14 - 7r = 0 \Rightarrow r = 2$

Required coefficient $= {}^7C_2\,3^{7-2}\left(-\frac{1}{2}\right)^2 = \frac{7.6}{2}.3^5.\frac{1}{2^2}$

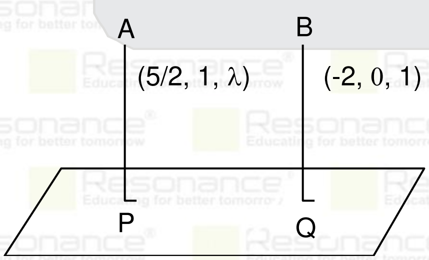
$= \frac{7.3^6}{4} = 1275.85$

$[1275.85] = 1275$

24. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and (-2,0,1) are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is _______.

NTA. **(9)**

RESO **(9)**



Sol.

$2x + 3y - 6z + 7 = 0$

Given AP = BQ $\Rightarrow \left|\frac{5+3-6\lambda+7}{7}\right| = \left|\frac{-4-6+7}{7}\right|$

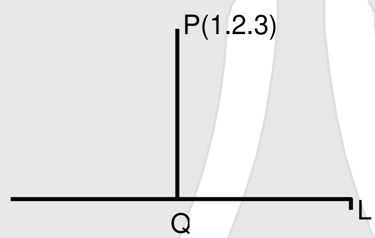
$\Rightarrow 15 - 6\lambda = \pm 3$

$6\lambda = 12, 18$

$\lambda = 2, 3$

$\lambda_1 = 3$, $\lambda_2 = 2$

hence point is P(1,2,3)



Q (t + 5, 2t + 1, 2t – 7) $\qquad \frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2} = t$

D'r (PQ) : t + 4 , 2t – 1 , 2t – 10

D'r (L) : 1 , 2, 2

PQ$\perp$L $\Rightarrow$ (t +4) + 2(2t –1) +2(2t –10) = 0

9t – 18 = 0 $\Rightarrow$ t = 2 $\Rightarrow$ Q (7, 5, -3)

PQ = $\sqrt{36+9+36} = 9$

25. The largest natural number n such that 3^n divides 66! is ________.

NTA. **(31)**

RESO **(31)**

Sol. Exponent of 3 in 66! is $\left[\frac{66}{3}\right] + \left[\frac{66}{9}\right] + \left[\frac{66}{27}\right] + \left[\frac{66}{81}\right]$

= 22 +7 + 2 + 0 = 31

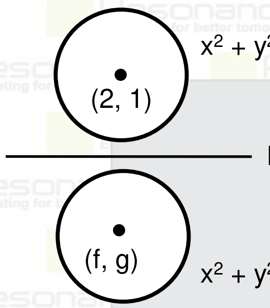
$66! = 3^{31}$

n = 31

26. Consider a circle C_1: $x^2 + y^2 - 4x - 2y = \alpha - 5$, Let its mirror image in the line $y=2x+1$ be another circle C_2: $5x^2 + 5y^2 - 10f\,x - 10gy + 36 = 0$. Let r be the radius of C_2, then $\alpha + r$ is equal to ___________

NTA. **(2)**

RESO **(2)**



$x^2 + y^{2-} - 4x - 2y + (5 - \alpha) = 0$

L $2x - y + 1 = 0$

$x^2 + y^{2-} - 2fx - 2gy + \dfrac{36}{5} = 0$

image of (2, 1) in line $2x - y + 1 = 0$ will be (f, g)

$$\frac{x-2}{2} = \frac{y-1}{-1} = -2\left(\frac{4-1+1}{5}\right) = -\frac{8}{5}$$

$$x = 2 - \frac{16}{5} = -\frac{6}{5} = g$$

$$y = 1 + \frac{8}{5} = \frac{13}{5} = f$$

radius of both circle will be same

$$4 + 1 - (5-\alpha) = f^2 + g^2 - \frac{36}{5}$$

$\alpha = 1$

Now radius of c_2 is $r = \sqrt{\dfrac{169}{25} + \dfrac{36}{25} - \dfrac{36}{5}} = 1$

$\alpha + r = 2$

27. If the solution curve of the differential equation $(y - 2\log_e x)dx + (x\log_e x^2)dy = 0, x>1$ passes through the points $\left(e, \frac{4}{3}\right)$ and (e^4, a) then a is equal to ________.

NTA. **(3)**

RESO **(3)**

Sol. $2x \ln x \frac{dy}{dx} + y - 2\ln x = 0$

$$\frac{dy}{dx} + \frac{y}{2x\ln x} = \frac{1}{x}$$

$$IF = e^{\int \frac{1}{2x\ln x}dx} = e^{\frac{1}{2}\ln(\ln x)} = \sqrt{\ln x}$$

$$y\left(\sqrt{\ln x}\right) = \int \frac{1}{x}\sqrt{\ln x}\,dx$$

$$y\left(\sqrt{\ln x}\right) = \frac{(\ln x)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

It passes through $\left(e, \frac{4}{3}\right)$

$$\frac{4}{3} = \frac{2}{3} + C \Rightarrow C = \frac{2}{3}$$

It also passes through (e^4, α)

$$\alpha(2) = \frac{16}{3} + \frac{2}{3} = 6$$

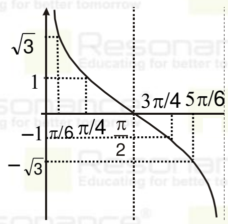
$\alpha = 3$

28. Let [t] denote the greatest integer $\leq t$, Then $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6}(8[\operatorname{cosec} x] - 5[\cot x])dx$ is equal to ________,

NTA. **(14)**

RESO **(14)**

Sol.



$$\mathbf{I} = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6}(8[\operatorname{cosec} x]-5[\cot x])dx$$

For $x \in (\pi/6, 5\pi/6)$

$\operatorname{cosec}x \in [1,2)$,

$[\operatorname{cosec}x] = 1$

$$I = \frac{2}{\pi}\left[\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 8(1)dx - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 5(1)dx - \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} 5(0)dx - \int_{\frac{\pi}{6}}^{\frac{3\pi}{4}} 5(-1)dx - \int_{\frac{3\pi}{4}}^{\frac{5\pi}{6}} 5(-2)dx\right] = 14$$

29. Let A = $\{0.3,4,6,7,8,9,10\}$ and R be the relation defined on A such that R = $\{(x,y) \in A \times A : x-y \text{ is odd positive integer or } x-y=2\}$. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation , is equal to ______.

NTA. **(19)**

RESO **(19)**

Sol. A = $\{0.3,4,6,7,8,9,10\}$

xRy ⇒ x – y = odd positive integer or x – y = 2

R = {(6, 4), (8, 6), (9, 7), (10, 8), (3, 0), (7, 0), (9, 0), (4, 3), (6, 3), (8, 3), (10, 3), (7, 4), (9, 4), (7, 6), (9, 6), (8, 7), (10, 7), (9, 8), (10, 9)}

Hence 19 elements should be add in R for making it symmetric

30. Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}, \vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a}\times\vec{c} = \vec{a}\times\vec{b}$. If $\vec{a}.\vec{c} = -12$, $\vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c}.(\hat{i} + \hat{j} + \hat{k})$ is equal to _______.

NTA. (11)

RESO (11)

Sol. $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}$ $\qquad \vec{a}.\vec{b} = 6\alpha + 99 - 24 = 6\alpha + 75$

$\vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ $\qquad |\vec{a}| = \sqrt{36+81+144} = \sqrt{261}$

$\vec{a}\times\vec{c} = \vec{a}\times\vec{b}.$, $\qquad \vec{a}.\vec{c} = -12, \quad \vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5,$

$\vec{a}\times\vec{c} = \vec{a}\times\vec{b}. \Rightarrow \vec{a}\times\vec{c} - \vec{a}\times\vec{b}. = 0$

$\Rightarrow \vec{a}\times(\vec{c} - \vec{b}) = 0$

$\Rightarrow \vec{c} - \vec{b} \,||\, \vec{a}$

$\Rightarrow \vec{c} = \vec{b} + \lambda\vec{a}$

Now $\Rightarrow \vec{c}.\vec{a} = \vec{b}.\vec{a} + \lambda(\vec{a}.\vec{a}) = -12$

$6\alpha + 75 + \lambda(261) = -12$

$6\alpha + 261\lambda = -87$ ....(i)

$\vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5, \Rightarrow (\alpha - 22 - 2) + \lambda(6 - 18 + 12) = 5$

$\Rightarrow \alpha = 29$

From equation (i) $\lambda = -1$

Hence $= \vec{c}.(\hat{i} + \hat{j} + \hat{k}) = (\vec{b} - \vec{a}).\ (\hat{i} + \hat{j} + \hat{k}) = 11$