Resonance®
Educating for better tomorrow

JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 08 April, 2023 (SHIFT-2) | TIME : (03.00 p.m. to 06.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | **FAX No. :** +91-022-39167222

To Know more : sms **RESO** at **56677** | **Website :** www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN :** U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | facebook.com/ResonanceEdu | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

PART : MATHEMATICS

1. The area of the quadrilateral ABCD with vertices
A (2, 1, 1), B(1, 2, 5), C (–2, –3, 5) and D(1, –6, –7) is equal toss

(1) $8\sqrt{38}$ (2) 48 (3) 54 (4) $9\sqrt{38}$

NTA Ans. **(1)**
Reso Ans. **(1)**

Sol. $\overrightarrow{AC} = -4\hat{i} - 4\hat{j} + 4\hat{k}$

$\overrightarrow{BD} = -0\hat{i} - 8\hat{j} + 12\hat{k}$

Area = $\frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}| = \frac{1}{2} \times 4 \times 4\sqrt{5^2 + 3^2 + 2^2}$

Area = $8\sqrt{38}$

2. The value of 36 (4 cos²9° – 1)(4 cos²27° – 1)(4 cos² 81° – 1) (4 cos² 243° –1) is

(1) 27 (2) 18 (3) 54 (4) 36

NTA Ans. **(4)**
Reso Ans. **(4)**

Sol. $36(4\cos^2 9°-1)(4\sin^2 9°-1)(4\cos^2 27°-1)(4\sin^2 27°-1)$

$36\left[(16\sin^2 9°\cos^2 9° - 4 + 1)(16\sin^2 27°\cos^2 27° - 4 + 1)\right]$

$36\left[(4\sin^2 18° - 3)(4\sin^2 54° - 3)\right]$

$36\left[4\left(\frac{\sqrt{5}-1}{4}\right)^2 - 3\right]\left[4\left(\frac{\sqrt{5}+1}{4}\right)^2 - 3\right]$

$36\left(\frac{6-2\sqrt{5}-12}{4}\right)\left(\frac{6+2\sqrt{5}-12}{4}\right)$

$9(-3-\sqrt{5})(-3+\sqrt{5})$

9(9–5) = 36

3. Let A = $\left\{\theta \in (0, 2\pi) : \frac{1+2i\sin\theta}{1-i\sin\theta} \text{ is purely imaginary}\right\}$ The sum of elements in A is

(1) π (2) 2π (3) 3π (4) 4π

NTA Ans. **(4)**
Reso Ans. **(4)**

Sol. $\text{Re}\left(\frac{1+2i\sin\theta}{1-i\sin\theta} \times \frac{1+i\sin\theta}{1+\sin\theta}\right) = 0$

$1-2\sin^2\theta = 0$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Sum is 4π

4. If the probability that the random variable X take values x is given by

$P(X = x) = k(x + 1)3^{-x}$, x = 0, 1, 2, 3, ……, where k is a constant, then $P(X \geq 2)$ is

(1) $\frac{20}{27}$ (2) $\frac{7}{18}$ (3) $\frac{7}{27}$ (4) $\frac{11}{18}$

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $P(x = 0) + P(x = 1) + p(x = 2) + p(x = 3) + ……. = 1$

$$\frac{K}{3^0} + \frac{2K}{3^1} + \frac{3K}{3^2} + \frac{4K}{3^3} + ...\infty = 1$$

$$K\left(1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + ...\infty\right) = 1$$

Now

$$\text{Let } S = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + ...\infty$$

$$\frac{S}{3} = 0 + \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + ...\infty$$

$$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ...\infty$$

$$= \frac{1}{1 - \frac{1}{3}}$$

$$S = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$$

$$\text{Now } K.\frac{9}{4} = 1$$

$$K = \frac{4}{9}$$

Now $P(x \geq 2) = P(2) + P(3) + ……. \infty$

$= 1 - P(0) - P(1)$

$$= 1 - \left(\frac{K}{1} + \frac{2K}{3}\right) = 1 - \frac{5K}{3}$$

$$1 - \frac{20}{27} = \frac{7}{27}$$

5. For a, b ∈ Z and a – b ≤ 10, let the angle between the plane P : ax + y – z = b and the line l : x – 1 = a – y = z + 1 be $\cos^{-1}\left(\frac{1}{3}\right)$ if the distance of the point (6, –6, 4) from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to

(1) 32 (2) 25 (3) 48 (4) 85

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. ax+y–z=b

$$\frac{x-1}{1}=\frac{y-a}{-1}=\frac{z+1}{1}$$

$$\vec{n}=a\hat{i}+\hat{j}+\hat{k}$$

$$\vec{p}=\hat{i}-\hat{j}+\hat{k}$$

$$\sin\theta=\left|\frac{\vec{n}.\vec{p}}{|\vec{n}||\vec{p}|}\right|$$

$$\left|\frac{a-1-1}{\sqrt{a^2+2}\sqrt{3}}\right|=\frac{2\sqrt{2}}{3}$$

$3(a-2)^2 = 8 \times 3\,(a^2+2)$

$5a^2 + 12a + 4 = 0 \Rightarrow (5a+2)(a+2)=0$

a=–2 & a = –2/5 (reject)

a = –2, $a-b \le 10$

$b \ge -8$

Now distance of given point from the plane

–2x + y – z = b

2x – y + z + b = 0

$$\frac{12+6+4+6}{\sqrt{6}}=\pm 3\sqrt{6}$$

b+22= ±18

b = –4 and b = –40 (Reject)

$a^4 + b^2 = 16 + 16 = 32$

6. Let a_n be the n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 + ……. And $s_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to

(1) 11280 (2) 11290 (3) 11310 (4) 11260

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol; S_n = 5+8+14+23+35+………+a_n + a_n

S_n = 5+8 +14+23

a_n= 5+3+6+9+12+ ………

$$a_n = 5+\frac{n-1}{2}[6+(n-2)3]$$

$$a_n + 5 + a_n = 5+\left(\frac{n-1}{2}\right)(3x)=\frac{3}{2}x(x-1)+5$$

$$a_n=\frac{3x^2}{2}-\frac{3x}{2}+5$$

$$a_{40}=3\times\frac{40^2}{2}-3x\frac{40}{2}+5$$

= 2400 – 60 +5 = 2345

$a_{30} = \frac{3}{2}\times\frac{30\times31\times61}{2}6-\frac{3}{2}\frac{30\times31}{2}+5\times30$

$= \frac{30\times30\times31}{2}\frac{6}{6}-\frac{1}{2}+150$

= 13635

$S_{30} - q_{40}$ = 11290

$a_n = \frac{3}{2}n(n-1)+5$

$S_n = \frac{3}{2}\times\frac{1}{3}n(n-1)(n+1)+5n$

$S_{30} - a_{40} = \frac{1}{2}\times30\times29\times31\times150-\frac{3}{2}\times40\times39-5$

= 11290

7. Let A = {1, 2, 3, 4, 5, 6, 7}. Then the relation R = {(x, y) $\in$ A × A : x + y = 7} is

(1) reflexive but neither symmetric nor transitive

(2) transitive but neither symmetric nor reflexive

(3) symmetric but neither reflexive nor transitive

(4) an equivalence relation

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. Reflexive :- xRx $\Rightarrow$ x + x = 7. $\Rightarrow$ x $\notin$ A, R is not reflexive.

Symmetric :- xRy $\Rightarrow$ x + y = 7. $\Rightarrow$ y + x = 7 $\Rightarrow$ yRx $\Rightarrow$ R is symmetric

Transitive :- 1R6 & 6 R 1 but 1 is not related to 1 so not transitive

8. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2+\frac{1}{2x}\right)^{11}$ is equal to

(1) $10^3 - 10$ (2) $13^3 - 13$ (3) $11^3 - 11$ (4) $12^3 - 12$

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $T_{r+1} = {}^{11}C_r (2x^2)^{11-r}(2x)^{-r}$

for coeff. x^7 put 22 – 2r – r = 7

r = 5

and for coeff x^{10} put 22 – 3r = 10

r = 4

abs . diff. = $\left|{}^{11}C_5.\frac{2^6}{2^5}-{}^{11}C_4.\frac{2^7}{2^4}\right|$

|924 – 2640| = 1716 = $12^3 - 12$

9. The integral $\int\left(\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x\right)\log_2 x\, dx$ is equal to

(1) $\left(\frac{x}{2}\right)^x-\left(\frac{2}{x}\right)^x+C$ (2) $\left(\frac{x}{2}\right)^x\log_2\left(\frac{2}{x}\right)^x+C$ (3) $\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x$ (4) $\left(\frac{x}{2}\right)^x\log_2\left(\frac{x}{2}\right)^x$

NTA Ans. **(1)**
Reso Ans. **(Bonus)**

10. If $A=\begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}$, $A^{-1}=\alpha A+\beta I$ and $\alpha+\beta=-2$, then $4\alpha^2+\beta^2+\lambda^2$ is equal to :

(1) 10 (2) 19 (3) 14 (4) 12

NTA Ans. **(3)**
Reso Ans. **(3)**

Sol. $A=\begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}$

Characteristic Equation: $A^2-11x+(10-5\lambda)I=0$

given $A^{-1}=\alpha A+\beta I$

$\alpha A^2+\beta A-I=0$

$A^2-11A+(10-5\lambda)I=0$

$\frac{\alpha}{1}=\frac{\beta}{-11}=\frac{+1}{5\lambda-10}=K(\text{let})$

$\alpha+\beta=-2$

$K-11K=-2$

$K=\frac{1}{5}$

$\alpha=\frac{1}{5}$

$\beta=\frac{-11}{5}$

$5\lambda-10=5$

$\lambda=3$

$4\alpha^2+\beta^2+\lambda^2$

$=4\times\frac{1}{25}+\frac{121}{25}+9=14$

11. The negation of $(p\wedge(\sim q))\vee(\sim p)$ is equivalent to

(1) $p\vee(q\vee(\sim p))$ (2) $p\wedge q$ (3) $p\wedge(q\wedge(\sim p))$ (4) $p\wedge(\sim p)$

NTA Ans. **(2)**
Reso Ans. **(2)**

Sol. $\sim[(p\wedge\sim q)\vee(\sim p)]$

$\sim[(p\wedge\sim q)\wedge(p)]$

$(\sim p\vee q)\wedge(p)=(\sim p\wedge p)\vee(q\wedge p)$

$C\vee(p\wedge q)=(p\wedge q)$

12. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and

$$\lim_{x \to \frac{1}{\alpha}} \left(\frac{1 - \cos(x^2 + bx + a)}{2(1 - \alpha x)^2} \right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha} \right)$$, then k is equal to

(1) 2α (2) 2β (3) β (4) α

NTA Ans. (1)
Reso Ans. (1)

Sol. Equations whose roots and $\frac{1}{\alpha}, \frac{1}{\beta}$ is $a + bx + x^2 = \left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)$

$$\ell t_{x \to \frac{1}{\alpha}} \left(\frac{1 - \cos\left(\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)\right)}{2\alpha^2 \left(x - \frac{1}{\alpha}\right)^2 \left(x - \frac{1}{\beta}\right)^2} \right)^{\frac{1}{2}} \times \left| x - \frac{1}{\beta} \right| = \left(\frac{1}{2.2\alpha^2} \right)^{\frac{1}{2}} \left| \left(\frac{1}{\alpha} - \frac{1}{\beta} \right) \right| = \frac{1}{2\alpha}\left(\frac{1}{\beta} - \frac{1}{\alpha} \right)$$

13. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is (6!)k, then k is equal to

(1) 1890 (2) 2835 (3) 5670 (4) 945

NTA Ans. (3)
Reso Ans. (3)

Sol. Required no. of ways $= \frac{11!}{2!\,2!\,2!} - \frac{10!}{2!\,2!\,2!} \times 2!$

$= \frac{9 \times 10!}{8}$

now $\frac{9 \times 10!}{8} = k \times 6!$

$k = 9 \times 9 \times 10 \times 7$

$= 5670$

14. Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$x + y + \sqrt{3}\, z = 0$

$-x + (\tan\theta)\, y + \sqrt{7}\, z = 0$

$x + y + (\tan\theta)\, z = 0$

has non-trivial solution. Then $\frac{120}{\pi} \sum_{\theta \in S} \theta$ is equal to

(1) 20 (2) 30 (3) 10 (4) 10

NTA Ans. (1)
Reso Ans. (1)

Sol. $\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix}$

$\tan^2\theta - \sqrt{3} + \sqrt{7} - \sqrt{3}\tan\theta - \sqrt{7} + \tan\theta = 0$

$\tan\theta\,[\tan\theta + 1] - \sqrt{3}\,[\tan\theta + 1] = 0$

$\tan\theta = \begin{matrix} \to \sqrt{3} \\ \to -1 \end{matrix}$

$$\sum\theta = \frac{\pi}{3} + (-\pi) + \frac{\pi}{3} + \left(\frac{\pi}{4}\right) + (+\pi) - \frac{\pi}{4}$$

$$\sum\frac{2\pi}{3} + \frac{\pi}{2} = \frac{\pi}{6}$$

$$\frac{120}{\pi}\sum\theta = \frac{120}{\pi} \times \frac{\pi}{6} = 20$$

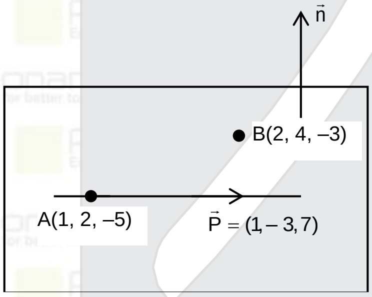
15. Let P be the plane passing through the line $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ and the point (2, 4, –3). If the image of the point (–1, 3, 4) in the plane P is (α, β, λ) then $\alpha + \beta + \lambda$ is equal to

(1) 12 (2) 9 (3) 10 (4) 11

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol.



$\vec{P} = \hat{i} - 3\hat{j} + 7\hat{k}$

$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{P} \times \overrightarrow{AB} = -20\hat{i} + 5\hat{j} + 5\hat{k}$

$\vec{n} = 4\hat{i} - \hat{j} - \hat{k}$

eq^n of plane $\Rightarrow$ 4(x – 1) + (–1)(y – 2) + (–1)(z + 5) = 0

4x – y – z – 7 = 0

Image of (–1, 3, 4) is (α, β, γ)

$$\frac{\alpha+1}{4} = \frac{\beta-13}{-1} = \frac{\gamma-4}{-1} = \frac{-2(-4-3-4-7)}{16+1+1} = 2$$

$\alpha = 7,\quad \beta = 1,\quad \gamma = 2$

$\alpha + \beta + \gamma = 10$

16. Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha, \frac{1}{2}\right)$ then a value of α is

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$ (3) 1 (4) $\frac{5}{2}$

NTA Ans. **(4)**
Reso Ans. **(4)**

Sol. Equation of circumcircle whose diametric end point is (3,– 2) & (0,0)

$x(x - 3) + y(y + 2) = 0$

$x^2 + y^2 - 3x + 2y = 0$

pt $(\alpha, \frac{1}{2})$ is on circle

$\alpha^2 + \frac{1}{4} - 3\alpha + 1 = 0$

$\alpha^2 - 3\alpha + \frac{5}{4} = 0$

$4\alpha^2 - 12\alpha + 5 = 0$

$\alpha = \frac{10}{4}, \alpha = \frac{2}{4}$

$\alpha = \frac{5}{2}, \frac{1}{2}$

17. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$,where m and n are coprime, then m + n is equal to

(1) 316 (2) 317 (3) 314 (4) 315

NTA Ans. **(2)**
Reso Ans. **(2)**

Sol. $\frac{\sum x_i}{12} = \frac{9}{2} \Rightarrow \sum x_i = 54.$

correct $\sum x_i = 54 + 7 + 14 - 9 - 10 = 56.$

correct $\bar{x}_i = \frac{56}{12} = \frac{14}{3}$

$\sigma^2 = \frac{\sum x_i^2}{12} - \left(\frac{9}{2}\right)^2 = 4$

$\Rightarrow \frac{\sum x^2}{12} = \frac{97}{4} \Rightarrow \sum x_i^2 = 291$

corr. $\sum x_i^2 = 291 + 64 = 355$

corr $\sigma^2 = \frac{\sum x^2}{12} - (x)^2$

$= \frac{355}{12} - \left(\frac{14}{3}\right)^2$

$= \frac{281}{36} = \frac{m}{n}$

m + n = 281 + 36 = 317

18. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}$, $\vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a + b)\,\hat{i} + c\,\hat{j} + c\,\hat{k}$, $\vec{v}_2 = a\,\hat{i} + (b + c)\,\hat{j} + a\,\hat{k}$ and $\vec{v}_3 = b\,\hat{i} + b\,\hat{j} + (c + a)\,\hat{k}$ are also coplanar, then 6 (a + b + c) is equal to

(1) 4 (2) 0 (3) 12 (4) 6

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $\begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0$

a + b + c – 1 –1– abc =0

& $\begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & a+c \end{vmatrix} = 0$

$R_3 \rightarrow R_3 - (R_1 + R_2)$

$\begin{vmatrix} a+b & c & c \\ a & b+c & a \\ -2a & -2c & 0 \end{vmatrix} = 0$

$-2ac^2 - 2a^2c - 2ac(b + c) - 2ac(a + b) = 0$

$ac^2 + a^2c - abc - ac^2 - a^2c - abc = 0$

$-2abc = 0$

$abc = 0$

$2 - a - b - c = 0$

$a + b + c = 12$ $\quad$ $6(a + b + c) = 12$

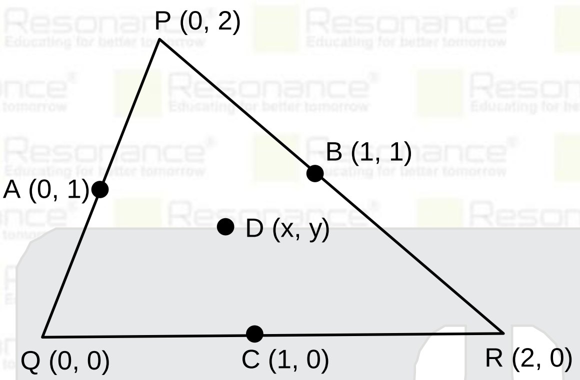
19. Let A (0, 1), B(1, 1) and C(1, 0) be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{2}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

(1) 8 (2) 6 (3) $\frac{9}{2}$ (4) 12

NTA Ans. (1)

Reso Ans. (1)

Sol.



$$D \equiv \left(\frac{0+4+0}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$$

$$D \equiv \left(\frac{2}{2+\sqrt{2}}, \frac{2}{2+\sqrt{2}}\right)$$

$$D \equiv \left(2-\sqrt{2}, 2-\sqrt{2},\right)$$

$y^2 = 4ax$

$$\left(2-\sqrt{2}\right)^2 = 4a\left(2-\sqrt{2}\right)$$

$$a = \frac{2-\sqrt{2}}{4}$$

focus $(a, 0)$

$$\frac{1}{2} - \frac{1}{4}\sqrt{2} = \alpha + \beta\sqrt{2}$$

$$\alpha = \frac{1}{2}, b = -\frac{1}{4}$$

$$\frac{\alpha}{\beta^2} = \frac{1/2}{1/16} = 8$$

20. $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by

(1) 34 but not by 14 (2) neither 14 nor 34 (3) 14 but not by 34 (4) both 14 and 34

NTA Ans. (1)

Reso Ans. (1)

Sol. $(25^{190} - 19^{190}) - (8^{190} - 2^{190})$ is divisible by 6.

$(25^{190} - 8^{190}) - (19^{190} - 2^{190})$ is divisible by 17.

$25^{190} - 8^{190}$ is not divisible by 7

but $19^{190} - 2^{190}$ is divisible by 7

So, $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by 34 but not 14

21. Let the solution curve x = x(y), $0 < y < \frac{\pi}{2}$, of the differential equation

$$(\log_e \cos y)^2 \cos y\, dx - (1 + 3x \log_e(\cos y)) \sin y\, dy = 0$$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$,

where m and n are coprime, then mn is equal to

NTA Ans. **(12)**

Reso Ans. **(12)**

Sol. $\frac{dy}{dx} - \frac{3\sin y}{(\ln(\cos y))\cos y} x = \frac{\sin y}{(\ln\cos y)^2 \cos y}$

I. f. = $e^{-\int 3 \frac{\sin y}{\cos y . \ln \cos y} dy}$

ln cosy = t

$-\frac{1}{\cos y}.\sin y\, dy = dt$

$= e^{3\int \frac{1}{t} dt} = e^{3\ln t} = t^3 = (\ln \cos y)^3$

Sol $x.(\ln\cos y)^3 = \int \ln\cos y \cdot \frac{\sin y}{\cos y} dy + C$

$x(\ln\cos y)^3 = -\frac{-(\ln\cos y)^2}{2} + C$

$y = \frac{\pi}{3},\ x = \frac{1}{2\ln 2}$

$\frac{1}{2\ln 2}\left(\ln\frac{1}{2}\right)^3 = -\frac{\left(\ln\frac{1}{2}\right)^3}{2} + C$

$-\frac{(\ln 2)^2}{2} = -\frac{(\ln 2)^2}{2} + C$

C = 0

$x = \frac{1}{2\ln\cos y}$ $\quad y = \frac{\pi}{6}$

$x = -\frac{1}{2\ln\frac{\sqrt{3}}{2}}$

$x = \frac{1}{\ln 4 - \ln 3}$, m = 4, n = 3, m·n = 12

22. Let 0 < z < y < x be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and x, $\sqrt{2}$ y, z are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}} xyz$, then $3(x + y + z)^2$ is equal to ________

NTA Ans. **(150)**

Reso Ans. **(150)**

Sol. $\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$ ......(1)

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{\sqrt{2}} \Rightarrow \frac{3}{y} = \frac{3}{\sqrt{2}}$

$y = \sqrt{2}$ ......(2)

x, $\sqrt{2}y$, z are in G. P.

$2y^2 = xz$

xz = 4........(3)

from (1) $x + z = \sqrt{2}\, xz$

$x + z = 4\sqrt{2}$

now $3(x + y + z)^2 = 3(4\sqrt{2} + \sqrt{2})^2 = 150$

23. Let k and m be positive real numbers such that the function $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$ is differentiable for all x > 0. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to ________.

NTA Ans. **(309)**

Reso Ans. **(309)**

Sol. $f(1^-) = f(1) = f(1^+)$

$3 + k\sqrt{2} = m + k^2$ --------(1)

$f'(1^-) = f^1(1^+)$

$6 + \frac{k}{2\sqrt{2}} = 2m$ ---------(2)

from (1) & (2)

$3 + k\sqrt{2} = 3 + \frac{k}{4\sqrt{2}} + k^2 \quad \Rightarrow \quad k^2 + k\left(\frac{1}{4\sqrt{2}} - \sqrt{2}\right) = 0$

$k = 0, k = \frac{7}{4\sqrt{2}} \Rightarrow \quad$ If $k = \frac{7}{4\sqrt{2}}$, $m = 3 + \frac{7}{32} = \frac{103}{32}$

$f'(x) = \begin{cases} 6x + \frac{k}{2\sqrt{x+1}} \\ 2mx \end{cases}$

$f'(8) = \frac{103}{2}$

$f'\left(\frac{1}{8}\right) = \frac{6}{8} + \frac{k.2\sqrt{2}}{2.3} = \frac{3}{4} + \frac{\sqrt{2}.k}{3} = \frac{4}{3}$ now $\frac{8.f'(8)}{f'\left(\frac{1}{8}\right)} = 309$

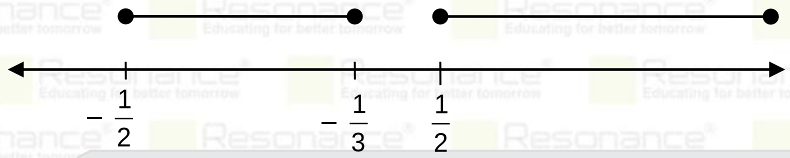
24. If domain of the function $\log_e\left(\frac{6x^2+5x+1}{2x-1}\right) + \cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then $18(\alpha^2 + \beta^2 + \lambda^2 + \delta^2)$, is equal to ________.

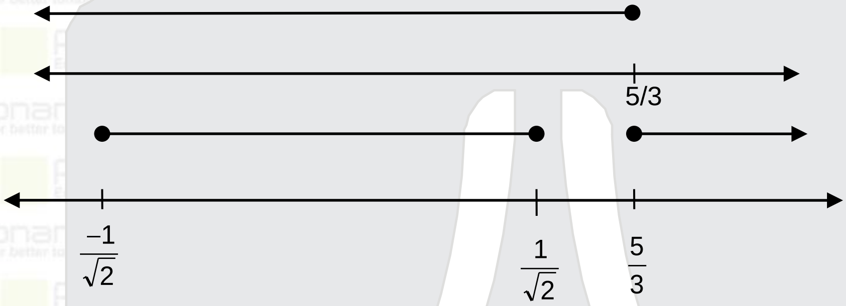
NTA Ans. **(20)**

Reso Ans. **(20)**

Sol. $\dfrac{6x^2+5x+1}{2x-1} > 0$ & $-1 \le \dfrac{2x^2-3x+4}{3x-5} \le 1$

$\dfrac{(3x+1)(2x+1)}{2x-1} > 0$ & $\dfrac{2x^2-6x+9}{3x-5} \le 0$ & $\dfrac{2x^2-1}{3x-5} \ge 0$





Common $\left(-\dfrac{1}{2},-\dfrac{1}{3}\right) \cup \left(\dfrac{1}{2},\dfrac{1}{\sqrt{2}}\right]$

$(\alpha, \beta) \cup (\gamma, \delta]$

$18(\alpha^2+\beta^2+\gamma^2+\delta^2) = \left(\dfrac{1}{4}+\dfrac{1}{9}+\dfrac{1}{4}+\dfrac{1}{2}\right)18 = \dfrac{10}{9} \times 18 = 20$

25. Let [t] denote the greatest integer function. If $\int_0^{2.4}\left[x^2\right]dx = \alpha+\beta\sqrt{2}+\gamma\sqrt{3}+\delta\sqrt{5}$, then $\alpha+\beta+\gamma+\delta$ is equal to ________.

NTA Ans. **(6)**

Reso Ans. **(6)**

Sol. $= \int_0^1 0.dx + \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2.dx + \int_{\sqrt{3}}^{2} 3.dx + \int_2^{\sqrt{5}} 4.dx + x\int_{\sqrt{5}}^{2.4} 5.dx = (x)_1^{\sqrt{2}} + 2(x)_{\sqrt{2}}^{\sqrt{3}} + 3(x)_{\sqrt{3}}^{2} + 4(x)_2^{\sqrt{5}} + 5(x)_{\sqrt{5}}^{2.4}$

$= \left(\sqrt{2}-1\right) + 2\left(\sqrt{3}-\sqrt{2}\right) + 3\left(2-\sqrt{3}\right) + 4\left(\sqrt{5}-2\right) + 5\left(2.4-\sqrt{5}\right)$

$= -1-\sqrt{2}-\sqrt{3}-2+12-\sqrt{5}$

$= 9-\sqrt{2}-\sqrt{3}-\sqrt{5}$

$= \alpha = 9, \beta = -1, \gamma = -1, \delta = -1$

$= \alpha+\beta+\gamma+\delta = 6$

26. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points (2, –1, 0), (2, 0, –1), and (5, 1, 1). If the foot of the perpendicular drawn from the point (7, 4, –1) on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to__________.

NTA Ans. **(11)**

Reso Ans. **(11)**

Sol. $P_1 : 3x - y - 7z = 11$

$$P_2 : \begin{vmatrix} x-2 & y+1 & z \\ 0 & 1 & -1 \\ 3 & +2 & 1 \end{vmatrix} = 0$$

$(x - 2)\ -(y + 1)\ (+3)\ +z\ (+3) = 0$

$P_2 : x - y - z - 3$

$\vec{P} = \vec{n_1} \times \vec{n_2}$

$\vec{n_1} = 3\hat{i} - \hat{j} - 7\hat{k}$

$\vec{n_2} = \hat{i} - \hat{j} - \hat{k}$

$\vec{n_1} \times \vec{n_2} = -6\hat{i} - 4\hat{j} - 2\hat{k}$

$\vec{P} = 3\hat{i} + 2\hat{j} + \hat{k}$ common point on the planes

Put $z = 0$

$3x - y = 11$

$x - y = 3$

$x = 4,\ y = 1, z = 0$

Line is $\dfrac{x-4}{3} = \dfrac{y-1}{2} = \dfrac{z-0}{1}$

Let point on this line $\equiv (3\lambda + y, 2\lambda + 1, \lambda) \equiv N$

given point is $A \equiv (7,4,-1)$

$\overrightarrow{AN} = (3\lambda - 3)\hat{i} + (2\lambda - 3)\hat{j} + (\lambda + 1) = 0$

$\overrightarrow{AN}.\vec{P} = 0$

$3(3\lambda - 3) + 2(2\lambda - 3) + 1\ (\lambda+1) = 0$

$14\lambda - 14 = 0$ $\qquad$ $N \equiv (7, 3, 1)$

$\lambda = 1$ $\qquad$ (α, β, γ)

$\alpha, \beta, \gamma = 11$

27. The ordinates of the points P and Q on the parabola with focus (3, 0) and directrix $x = -3$ are in the ratio 3 : 1. If R (α, β) is the point of intersection of the tangents to the parabola at P and Q, then $\dfrac{\beta^2}{\alpha}$ is equal to ____________.

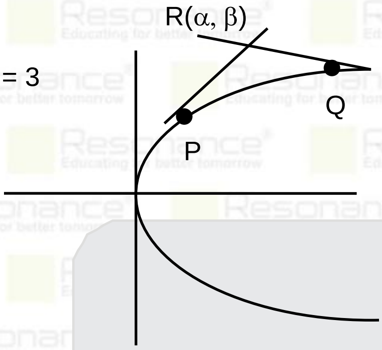
NTA Ans. (16)

Reso Ans. (16)

Sol. Parabola is

$y^2 = 12x$

$a = 3$



$P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ $\alpha = at_1t_2$ and $\beta = a(t_1+t_2)$

$2at_2 = 3.2\ at_1$

$t_2 = 3t_1$

$$\frac{\beta^2}{\alpha} = \frac{a^2(t_1+t_2)}{at_1t_2} = \frac{a(16t_1^2)}{3t_1^2} = 16$$

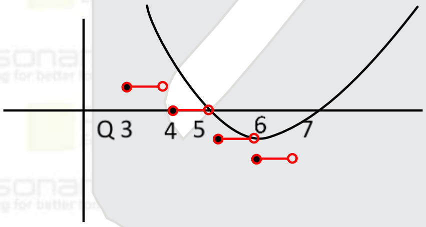
28. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x + 2| - 4 = 0$ respectively, where [x] denotes the greatest integer ≤ x. Then $m^2 + mn + n^2$ is equal to __________.

NTA Ans. **(9)**

Reso Ans. **(9)**

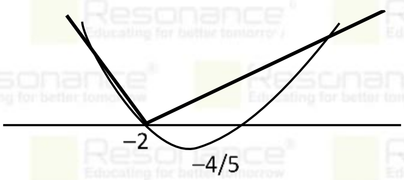
Sol. $x^2 - 12x + 35 = 4-[x]$

$(x-5)(x-7) = 4-[x]$



No solution so m = 0

$$\frac{x^2-4}{5} = |x+2|$$



Number of solution are 3 so n = 3

$m^2 + mn + n^2 = 0 + 0 + 9 = 9$

29. Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto functions f : R → S such that f(a) ≠ 1, is equal to ________.

NTA Ans. **(384)**

Reso Ans. **(180)**

Sol. If f(a) ≠ 1

For a [remaining 4 to all 4 + remaining 4 to remaining 3

$$^3C_1 \left[4! + \frac{4!}{2!\ 2!} \times 3! \right]$$

3 × (24 + 36) = 180

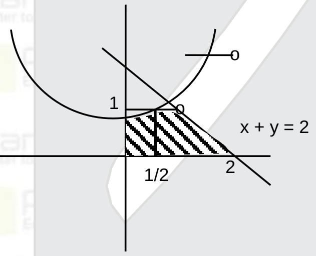
30. Let the area enclosed by the lines x + y = 2, y = 0, x = 0 and the curve f(x) = min $\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$ where] denotes the greatest integer ≤ x, be A. Then the value of 12 A is __________.

NTA Ans. **(17)**

Reso Ans. **(17)**

Sol. $A = \int_0^{\frac{1}{2}} \left(x^2 + \frac{3}{4}\right) dx + \frac{1}{2}\left(\frac{3}{2} + \frac{1}{2}\right).^2$

$= \frac{1}{3}.\frac{1}{8} + \frac{3}{4}.\frac{1}{2} + 1$



$12\text{ A} = \frac{1}{2} + \frac{9}{2} + 12 = 17$

Resonance®
Educating for better tomorrow
JEE (ADVANCED) 2022 RESULT
RESONites ने फिर लहराया सफलता का परचम
STUDENTS FROM CLASSROOM PROGRAM (OFFLINE/ ONLINE)
AIR 6
KARTHIKEYA POLISETTY
AIR-1 GEN-EWS
AIR 8
DHEERAJ KURUKUNDA
Students in TOP-100 All India Ranks (AIRs)
AIR-11
AIR-15
AIR-35
AIR-50
AIR-54
AIR-58
ADMISSIONS OPEN
Academic Session 2023-24
Class: V to XII & XII+
JEE (Advanced)
JEE (Main)
NEET (UG)
SCHOLARSHIP UPTO
100%
Based on ResoNET (Scholarship Test)
REGISTERED & CORPORATE OFFICE (CIN: U80302RJ2007PLC024029):
CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Rajasthan) - 324005
0744-2777777 | 73400 10345 | contact@resonance.ac.in | www.resonance.ac.in
Follow Us: @ResonanceEdu | @Resonance_Edu