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# JEE (Main)

PAPER-1 (B.E./B. TECH.)

# 2023

# COMPUTER BASED TEST (CBT)
## Questions & Solutions

**Date: 10 April, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)**

**Duration: 3 Hours | Max. Marks: 300**

## SUBJECT: MATHEMATICS


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# PART : MATHEMATICS

**1.** For the system of linear equation

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta,$

which of the following is NOT correct?

(1) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(2) The system is inconsistent for $\alpha = -5$ and $\beta = 8$

(3) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

(4) The system has a unique solution for $\alpha \neq 5$ and $\beta = 8$

**NTA Ans.** **(1)**

**Reso Ans.** **(1)**

**Sol.** For infinite solution

$\lambda_1 P_1 + \lambda_2 P_2 = P_3$

$\lambda_1(2x - y + 3z - 5) + (3x + 2y - z - 7)\lambda_2 = (4x + 5y + \alpha z - \beta)$

$$\left.\begin{aligned} 2\lambda_1 + 3\lambda_2 &= 4 \\ -\lambda_1 + 2\lambda_2 &= 5 \end{aligned}\right] \quad \begin{aligned} \lambda_1 &= -1 \\ \lambda_2 &= 2 \end{aligned}$$

$3\lambda_1 - \lambda_2 = \alpha \rightarrow \quad \alpha = -5$

$-5\lambda_1 - 7\lambda_2 = -\beta \rightarrow \beta = 9$

**2.** Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033, then the sum of these three terms is equal to

(1) 241 (2) 220 (3) 210 (4) 231

**NTA Ans.** **(4)**

**Reso Ans.** **(4)**

**Sol.** $a_1^2 + a_2^2 + a_3^2 = 33033$

$a^2(1+r^2+r^4) = 33033 = 11^2 \times 3 \times 7 \times 13$

$a^2(1 + r^2(1+r^2)) = 11^2.273$

$a^2(1+r^2+r^4) = 11^2.(16(16+1)+1)$

$\therefore$ a = 11 and r = 4

$a_1 + a_2 + a_3 = a + ar + ar^2 = 11(1 + 4 + 16) = 231$

**3.** A line segment AB of length $\lambda$ moves such that the points A and B remain on the periphery of a circle of radius $\lambda$. Then the locus of the point, that divides the line segment AB in the ratio 2:3, is a circle of radius

(1) $\frac{\sqrt{19}}{7}\lambda$ (2) $\frac{2}{3}\lambda$ (3) $\frac{\sqrt{19}}{5}\lambda$ (4) $\frac{3}{5}\lambda$

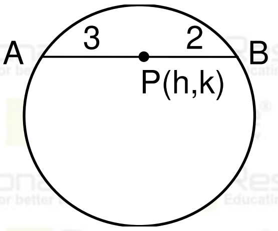
**NTA Ans.** **(3)**

**Reso Ans.** **(3)**

**Sol.**

$A\left(\frac{\lambda}{\sqrt{2}}\sin\theta, \frac{-\lambda}{\sqrt{2}}\cos\theta\right)$ ; $B\left(\frac{\lambda}{\sqrt{2}}\cos\theta, \frac{\lambda}{\sqrt{2}}\sin\theta\right)$

$$h = \frac{2.\frac{\lambda}{\sqrt{2}}\sin\theta + 3.\frac{\lambda}{\sqrt{2}}\cos\theta}{5}$$




$$K = \frac{-2.\frac{\lambda}{\sqrt{2}}\cos\theta + 3.\frac{\lambda}{\sqrt{2}}\sin\theta}{5}$$

$$h^2 + k^2 = \frac{19\lambda^2}{5}$$

$$\text{Radius} = \frac{\sqrt{19}\lambda}{5}$$

**4.** Let two vertices of a triangle ABC be (2, 4, 6) and (0, –2, –5), and its centroid be (2, 1, –1). If the image of the third vertex in the plane x + 2y + 4z = 11 is $(\alpha, \beta, \gamma)$, then $\alpha\beta + \beta\gamma + \gamma\alpha$ is equal to

(1) 72 (2) 76 (3) 74 (4) 70

**NTA Ans.** **(3)**

**Reso Ans.** **(3)**

**Sol.** A(2, 4, 6) , B (0, –2, – 5), G (2, 1, –1)

Let C (x, y, z)

$$G \equiv (2,1,-1) \equiv \left(\frac{x+2+0}{3}, \frac{y+4-2}{3}, \frac{z+6-5}{3}\right) \Rightarrow x = 4,\ y = 1,\ z = -4$$

image of C in plane

$$\frac{\alpha-4}{1} = \frac{\beta-1}{2} = \frac{\lambda+4}{4} = \frac{-2(4+2-16-11)}{21}$$

$\alpha = 6,\ \beta = 5,\ \gamma = 4$

$\alpha\beta + \beta\gamma + \gamma\alpha = 74$

**5.** Let N denote the sum of the numbers obtained when two dice are rolled. If the m probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are co-prime , then 4m – 3n is equal to

(1) 8 (2) 6 (3) 12 (4) 10

**NTA Ans.** **(1)**

**Reso Ans.** **(1)**

**Sol.**

| **Sum** | $\rightarrow$ | **No. of Cases** |
|---|---|---|
| 2 | $\rightarrow$ | 1 |
| 3 | $\rightarrow$ | 2 |
| 4 | $\rightarrow$ | 3 |
| 5 | $\rightarrow$ | 4 |
| 6 | $\rightarrow$ | 5 |
| 7 | $\rightarrow$ | 6 |
| 8 | $\rightarrow$ | 5 |
| 9 | $\rightarrow$ | 4 |

favourable cases : sum 4 to 12

10 → 3

11 → 2

12 → 1

So probability = $\frac{33}{36} = \frac{11}{12} = \frac{m}{n}$

Now 4m – 3n = 44– 36 = 8

**6.** If $f(x)=\frac{(\tan 1^{\circ})x+\log_e(123)}{x\log_e(1234)-(\tan 1^{\circ})}$, x > 0 , then the least value of $f(f(x))+f\left(f\left(\frac{4}{x}\right)\right)$ is

(1) 8 (2) 2 (3) 4 (4) 0

**NTA Ans.** **(3)**

**Reso Ans.** **(3)**

**Sol.** $f(x) = \frac{ax+b}{cx+d}$ and d + a = 0 ∴ f(f(x)) = x

hence f(f(x)) + f(f(4/x)) = $x+\frac{4}{x} \geq 4$ (by use of AM ≥ GM)

**7.** Let f be a differentiable function such that $x^2f(x) - x = 4\int_0^x t\, f(t)\, dt$ , f(1) = $\frac{2}{3}$. Then 18 f(3) is equal to

(1) 210 (2) 160 (3) 180 (4) 150

**NTA Ans.** **(2)**

**Reso Ans.** **(2)**

**Sol.** Diff. W.r.t to x

$2xf(x) + x^2 f'(x)–1 = 4xf(x).\ 1 – 0$

$\frac{dy}{dx}-\frac{2y}{x}=\frac{1}{x^2}$

I.F = $e^{\int -\frac{2}{x}dx} = e^{-2\ell nx} = \frac{1}{x^2}$

So $y.\frac{1}{x^2} = \int \frac{1}{x^4}dx$

$\frac{y}{x^2} = -\frac{1}{3x^3}+c$

$y = -\frac{1}{3x}+cx^2$ ∴ $f(1) = \frac{2}{3} \Rightarrow \frac{2}{3} = -\frac{1}{3}+c.1 \Rightarrow c = 1$

$f(x) = x^2-\frac{1}{3x} \Rightarrow f(3) = 9-\frac{1}{9} = \frac{80}{9}$

18f(3) = 160

**8.** The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is

(1) 7 (2) 6 (3) 9 (4) 8

**NTA Ans.** **(3)**

**Reso Ans.** **(3)**

**Sol.** S.D. = $\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}\times\vec{d})}{|\vec{b}\times\vec{d}|}\right|$ where $\vec{b}\times\vec{d} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 1 & 2 & 0\end{vmatrix} = -4\hat{i}+2\hat{j}+4\hat{k}$

$= \left|\frac{(6\hat{i}+\hat{j}-\hat{k}).(-4\hat{i}+2\hat{j}+4\hat{k})}{\sqrt{16+4+16}}\right|$

= 9

**9.** Let the ellipse E: $x^2 + 9y^2 = 9$ intersect the positive x- and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are co-prime, then m – n is equal to

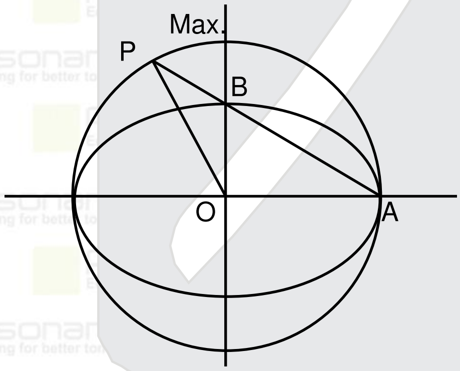
(1) 15 (2) 18 (3) 16 (4) 17

**NTA Ans.** **(4)**

**Reso Ans.** **(4)**

**Sol.** A (-3, 0) & B (3, 0)

For maximum area P must lie on y– axis so Area of DPAB = $\frac{1}{2}\times 6\times 3 = 9$ sq units




line AB : $x + 3y = 3$ ; circle $x^2 + y^2 = 9$

$(3-3y)^2 + y^2 = 9 \Rightarrow y = 0, \frac{9}{5}$

Area = $\frac{1}{2}\times 3\times\frac{9}{5} = \frac{27}{10}$

m – n = 27 – 10 = 17

**10.** If A is a 3 × 3 matrix and | A | = 2, then 3 adj (|3A|$A^2$) is equal to

(1) $3^{11}.6^{10}$ (2) $3^{12}.6^{11}$ (3) $3^{12}.6^{10}$ (4) $3^{10}.6^{11}$

**NTA Ans.** **(1)**

**Reso Ans.** **(1)**

**Sol.** Given $|A_{3\times3}| = 2$

Now $|3\text{adj}(|3A|A^2)|$

$= 3^3\left|\text{adj}\left(|3A|A^2\right)\right| = 3^3\left||3A|A^2\right|^2$

$= 3^3\left(\left(|3A|\right)^3|A^2|\right)^2 = 3^3.\left(3^9|A|^3.|A|^2\right)^2$

$= 3^3.3^{18}|A|^{10} = 3^{21}2^{10} = 3^{11} \times 6^{10}$

**11.** Let the complex number z = x + iy be such that $\dfrac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to :

(1) $\dfrac{2}{3}$ (2) $\dfrac{3}{2}$ (3) $\dfrac{4}{3}$ (4) $\dfrac{3}{4}$

**NTA Ans.** **(4)**

**Reso Ans.** **(4)**

**Sol.** $\dfrac{2z-3i}{2z+i} + \dfrac{2\bar{z}+3i}{2\bar{z}-i} = 0$

put z = x + iy equation becomes

$4x^2 + 4y^2 - 4y - 3 = 0$

$4(-y^2)^2 + 4y^2 - 4y - 3 = 0$

$y^4 + y^2 - y = \dfrac{3}{4}$

**12.** $96\cos\dfrac{\pi}{33}\cos\dfrac{2\pi}{33}\cos\dfrac{4\pi}{33}\cos\dfrac{8\pi}{33}\cos\dfrac{16\pi}{33}$ is equal to

(1) 3 (2) 1 (3) 2 (4) 4

**NTA Ans.** **(1)**

**Reso Ans.** **(1)**

**Sol.** Let $\dfrac{\pi}{33} = A$

$96\cos A\cos 2A\cos 2^2A\cos 2^3A\cos 2^4A$

$= 96\dfrac{\sin 2^5 A}{2^5 \sin A} = \dfrac{96.\sin\dfrac{32\pi}{33}}{32\sin\dfrac{\pi}{33}} = \dfrac{96.\sin\left(\pi - \dfrac{\pi}{33}\right)}{32\sin\dfrac{\pi}{33}} = \dfrac{96.\sin\dfrac{\pi}{33}}{32\sin\dfrac{\pi}{33}} = 3$ Ans.

**13.** If $I(x) = \int e^{\sin^2 x}(\cos x\sin 2x - \sin x)dx$ and $I(0) = 1$, then $I\left(\dfrac{\pi}{3}\right)$ is equal to

(1) $e^{\frac{3}{4}}$ (2) $-e^{\frac{3}{4}}$ (3) $-\dfrac{1}{2}e^{\frac{3}{4}}$ (4) $\dfrac{1}{2}e^{\frac{3}{4}}$

**NTA Ans.** **(4)**

**Reso Ans.** **(4)**

**Sol.** $\because I = \int e^{f(x)}(g'(x)+f'(x).g(x))dx = e^{f(x)}.g(x)$

$\therefore\ I = e^{\sin^2 x}.\cos x + c$

but I(0) = 1 $\Rightarrow$ c = 0

$I = e^{\sin^2 x}\cos x$

$I\left(\frac{\pi}{3}\right) = \frac{1}{2}e^{\frac{3}{4}}$

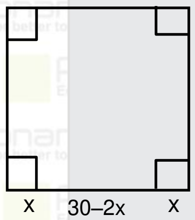
**14.** A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm²) is equal to

(1) 800 (2) 1025 (3) 675 (4) 900

**NTA Ans.** **(1)**

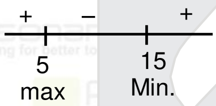
**Reso Ans.** **(1)**

**Sol.**




$V = (30-2x)^2 x$

$\frac{dv}{dx} = (2x-30)(6x-30)$




Now T.S.A = $(30-2x)^2 + 4x(30-2x) = 400 + 400 = 800cm^2$

**15.** The negation of the statement $(p \vee q) \wedge (q \vee (\sim r))$ is

(1) $((\sim p) \vee r)) \wedge (\sim q)$ (2) $((\sim p) \vee (\sim q)) \vee (\sim r)$ (3) $(p \vee r) \wedge (\sim q)$ (4) $((\sim p) \vee (\sim q)) \wedge (\sim r)$

**NTA Ans.** **(1)**

**Reso Ans.** **(1)**

**Sol.** $\sim((p \vee q) \wedge (q \vee (\sim r)))$

$= \sim(p \vee q) \vee \sim(q \vee (\sim r))$

$= (\sim p \wedge \sim q) \vee (\sim q \wedge r)$

$= (\sim p \vee (\sim q \wedge r)) \wedge (\sim q \vee (\sim q \wedge r)$

$= (\sim p \vee (\sim q \wedge r)) \wedge \sim q$

$= (\sim p \vee \sim q) \wedge (\sim p \vee r) \wedge (\sim q)$

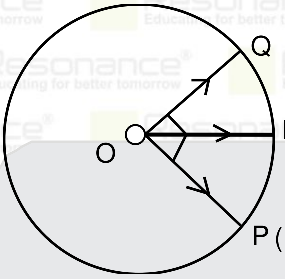
$= (\sim p \vee r) \wedge (\sim q)$

**16.** An arc PQ of a circle subtends a right angle at its centre O. The mid-point of the arc PQ is R. If $\overrightarrow{OP} = \vec{u}, \overrightarrow{OR} = \vec{v}$ and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then $\alpha$, $\beta^2$ are the roots of the equation

(1) $3x^2 + 2x - 1 = 0$ (2) $x^2 + x - 2 = 0$ (3) $3x^2 - 2x - 1 = 0$ (4) $x^2 - x - 2 = 0$

**NTA Ans.** **(4)**

**Reso Ans.** **(4)**




$|\vec{u}| = |\vec{v}| = |\alpha\vec{u} + \beta\vec{v}| = r$

**Sol.**

Given $\overrightarrow{OP}.\overrightarrow{OQ} = 0$

$\alpha\ |u|^2 + \beta(\vec{u}.\vec{v}) = 0$

$\alpha r^2 + \beta r.r \cos 45^\circ = 0 \Rightarrow \alpha + \dfrac{\beta}{\sqrt{2}} = 0$ ….. (1)

Again $|\vec{u}| = |\alpha\vec{u} + \beta\vec{u}|$ $\Rightarrow r^2 = \alpha^2 r^2 + \beta^2 r^2 + 2\alpha\beta \dfrac{r^2}{\sqrt{2}}$

$1 = \alpha^2 + \beta^2 + \sqrt{2}\alpha\beta$

$\Rightarrow 1 = \dfrac{\beta^2}{2} + \beta^2 - \beta^2 \Rightarrow \beta^2 = 2$

equations (1) $\alpha = 1, -1$

equations whose roots are 1, 2 $\rightarrow x^2 - 3x + 2 = 0$

equations whose roots are –1, 2 $\rightarrow x^2 - x - 2 = 0$

**17.** The slope of tangent at any point (x, y) on a curve y = y(x) is $\dfrac{x^2+y^2}{2xy}, x > 0$. If y(2)=0, then a value of y(8) is

(1) $4\sqrt{3}$ (2) $2\sqrt{3}$ (3) $-4\sqrt{2}$ (4) $-2\sqrt{3}$

**NTA Ans.** **(1)**

**Reso Ans.** **(1)**

**Sol.** $\dfrac{dy}{dx} = \dfrac{x^2+y^2}{2xy}$

Let $y = \upsilon x \therefore \dfrac{dy}{dx} = \upsilon + x\dfrac{d\upsilon}{dx}$ $\therefore \upsilon + x\dfrac{d\upsilon}{dx} = \dfrac{x^2 + \upsilon^2 x^2}{2x.\upsilon x}$

$x\dfrac{d\upsilon}{dx} = \dfrac{1+\upsilon^2}{2\upsilon} - \upsilon = \dfrac{1-\upsilon^2}{2\upsilon} \Rightarrow \dfrac{2\upsilon}{1-\upsilon^2}d\upsilon = \dfrac{dx}{x} \Rightarrow \int \dfrac{2\upsilon\ell\upsilon}{\upsilon^2 - 1} + \int \dfrac{\ell x}{x} = 0$

$\Rightarrow \ell n(\upsilon^2 - 1) + \ell nx = \ell nc \Rightarrow x(\upsilon^2 - 1) = c \Rightarrow x\dfrac{y^2 - x^2}{x^2} = c \Rightarrow y^2 - x^2 = cx$

Now f(2) = 0 $\therefore$ c = –2

$\therefore y^2 - x^2 = -2x$ $\therefore$ x = 8 then $y^2 - 64 = -16$

$y^2 = 48$

$y = f(8) = 4\sqrt{3}$

**18.** Let O be the origin and the position vector of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the points A, B and C are $-2\hat{i}+\hat{j}-3\hat{k}$, $2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

(1) $\frac{8}{3}$ (2) $\frac{10}{3}$ (3) 3 (4) $\frac{7}{3}$

**NTA Ans.** **(3)**
**Reso Ans.** **(3)**

**Sol.** $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 3 & 1\\ -2 & 1 & -3\end{vmatrix}=5\left(\hat{i}-2\hat{j}+2\hat{k}\right)$

Projection = $\frac{\overrightarrow{OP}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)}{\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|}$ = $\frac{5(-1+4+6)}{5\sqrt{1+4+4}}=\frac{9}{3}=3$

**19.** If the coefficient of $x^7$ in $\left(ax-\frac{1}{bx^2}\right)^{13}$ and the coefficient of $x^{-5}$ in $\left(ax+\frac{1}{bx^2}\right)^{13}$ are equal, then $a^4b^4$ is equal to :

(1) 44 (2) 11 (3) 33 (4) 22

**NTA Ans.** **(4)**
**Reso Ans.** **(4)**

**Sol.** $\left(ax-\frac{1}{bx^3}\right)^{13}$ $T_{r+1}={}^{13}C_r(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$

$T_{r+1}={}^{13}C_r.\,a^{13-r}\left(-\frac{1}{b}\right)^r x^{13-3r}$

coeff. of $x^7$ = ${}^{13}C_2.a^{11}\left(-\frac{1}{b}\right)^2$ $\quad\left|\begin{array}{l}13-3r=7\\ 3r=6\\ r=2\end{array}\right.$

$\left(ax+\frac{1}{bx^2}\right)^{13}$ $T_{r+1}={}^{13}C_r(ax)^{13-r}\left(\frac{1}{bx^2}\right)^r$

$T_{r+1}={}^{13}C_r.a^{13-r}\left(\frac{1}{b}\right)^r x^{13-3r}$

coeff. of $x^{-5}$ = ${}^{13}C_6.a^7\left(\frac{1}{b}\right)^6$ $\quad\left|\begin{array}{l}13-3r=-5\\ 3r=18\\ r=6\end{array}\right.$

Now ${}^{13}C_6a^7\left(\frac{1}{b}\right)^6={}^{13}C_2a^{11}\left(-\frac{1}{b}\right)^2$

$a^4b^4=\frac{{}^{13}C_6}{{}^{13}C_2}=\frac{\lfloor 13}{\lfloor 6\lfloor 7}\times\frac{\lfloor 2\lfloor 11}{\lfloor 13}$

$a^4\,b^4 = 22$

**20.** Let P be the point of intersection of the line $\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$ and the plane x + y + z = 2. If the distance of the point P from the plane 3x – 4y + 12z = 32 is q, then q and 2q are the roots of the equation

(1) $x^2 - 18x - 72 = 0$ (2) $x^2 + 18x + 72 = 0$ (3) $x^2 + 18x - 72 = 0$ (4) $x^2 - 18x + 72 = 0$

**NTA Ans.** **(4)**

**Reso Ans.** **(4)**

**Sol.** Let point on line is P(3λ – 3, λ – 2 , –2λ + 1)

Now it also lies on plane

x + y + z = 2

3λ – 3 + λ – 2 – 2λ + 1 = 2

λ = 3

P(6,1, –5)

so distance from plane

3x – 4y + 12z = 32 is

$$\Rightarrow\left|\frac{18-4-60-32}{\sqrt{9+16+144}}\right|=6$$

∴ q = 6, 2q = 12

and equation $x^2 - 18x + 72 = 0$

**21.** If the mean of the frequency distribution

| Class : | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---|---|---|---|---|---|
| Frequency : | 2 | 3 | $x$ | 5 | 4 |

is 28, then its variance is –

**NTA Ans.** **(151)**

**Reso Ans.** **(151)**

**Sol.** Mean= 28 = $\frac{5\times2+15\times3+25\times x+35\times5+45\times4}{14+x}$

392 + 28x = 410 + 25x

3x = 18

x = 6

Now variance

$$=\frac{\sum f_i(x_i)^2}{\sum f_i}-(\bar{x})^2$$

$$=\frac{2(25)+3(15)^2+6(25)^2+5(35)^2+4(45)^5}{20}-28^2$$

$$=\frac{50+675+3750+6125+8100}{20}-784$$

$$=\frac{18700}{20}-784=935-784$$

= 151

**22.** The number of permutations, of the digits 1, 2, 3, ..., 7 without repetition, which neither contain the string 153 nor the string 2467, is__________

**NTA Ans.** **(4898)**

**Reso Ans.** **(4898)**

**Sol.** $n(\bar{A} \cap \bar{B}) = n(\cup) - n(A \cup B)$    n(A) = 5! ; $\boxed{153}$ , 2, 4, 6 , 7

$7! - (5! + 4! - 2!) = 4898$    n (B) = 4! ; $\boxed{2467}$ , 1, 3, 5

**23.** The coefficient of $x^7$ in $(1-x+2x^3)^{10}$ is _______

**NTA Ans.** **(960)**

**Reso Ans.** **(960)**

**Sol.** General term = $\frac{10!}{r_1!r_2!r_3!}(-1)^{r_2}2^{r_3}.x^{r_2+3r_3}$ : $r_1 + r_2 + r_3 = 10$

for coefficient of $x^7$ : $r_2 + 3r_3 = 7$ ; $r_1 + r_2 + r_3 = 10$

| $r_3$ | $r_2$ | $r_1$ |
|---|---|---|
| 0 | 7 | 3 |
| 1 | 4 | 5 |
| 2 | 1 | 7 |

Required coefficient = $\frac{10!}{0!7!3!}(-1)^7(2)^0 + \frac{10!}{1!4!5!}(-1)^4(2)^1 + \frac{10!(-1)^1(2)^2}{2!1!71}$

$= -120 + 2520 - 1440 = 960$

**24.** The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is ________

**NTA Ans.** **(6)**

**Reso Ans.** **(6)**

**Sol.** $|n^2 - 10n + 19| < 6$

$-6 < n^2 - 10n + 19 < 6$

$+6 \quad +6 \quad +6$

$0 < (n-5)^2 < 12$

$0 < n - 5 < \sqrt{12}$    $-\sqrt{12} < n - 5 < 0$

$5 < n < 8.5$    $1.5 < n < 5$

{6,7,8}    $n \in \{2, 3, 4\}$

number of values of n = 6

**25.** Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple played in a match, is 840, then the total numbers of persons, who participated in the tournament, is___________

**NTA Ans.** **(16)**

**Reso Ans.** **(16)**

**Sol** Let number of couples is n

$^nC_2\ ^{n-2}C_2 . 2 = 840$

$^nC_2\ ^{n-2}C_2 . = 420$

$\frac{n(n-1)}{2}.\frac{(n-2)(n-3)}{2} = 420.$

n(n–1) (n–2) (n–3) = 1680 = 8.7.6.5 = n = 8

number of person = 2 × 8 = 16

**26.** Let y = p(x) be the parabola passing through the points (–1, 0), (0, 1) and (1, 0). If the area of the region $\{(x, y): (x + 1)^2 + (y – 1)^2 \le 1, y \le p(x)\}$ is A, then 12(π – 4A) is equal to ____________.

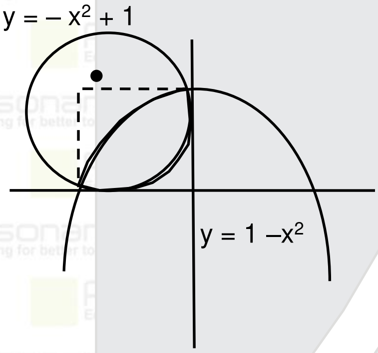
**NTA Ans.** **(16)**

**Reso Ans.** **(16)**

**Sol.** Let $y = ax^2 + bx + c$

$$\left.\begin{aligned}\text{Pass}(-1,0):&\quad a-b+c=0\\ \text{Pass}(0,1):&\quad c=1\\ \text{Pass}(1,0):&\quad a+b+c=0\end{aligned}\right\} 2(a + c) = 0,\ a = -1,\ b = 0$$

$y = -x^2 + 1$




$A = \left(\frac{1}{2}.1^2.\frac{\pi}{2}-\left(1-\frac{2}{3}\right)\right)=\frac{\pi}{4}-\frac{1}{3}$ Now $12(\pi - 4A) = 12(\pi - \pi + \frac{4}{3}) = 16$

**27.** Let a common tangent to the curves $y^2 = 4x$ and $(x – 4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to__________

**NTA Ans.** **(32)**

**Reso Ans.** **(32)**

**Sol.** Tangent of parable $y = mx + \frac{1}{m}$ ......... (1)

touches circle : $\left|\frac{4m+\frac{1}{m}-0}{\sqrt{1+m^2}}\right| = 4$

$16m^2 + 8 + \frac{1}{m^2} = 16m^2 + 16$

$m^2 = \frac{1}{8} \Rightarrow m = \pm\frac{1}{\sqrt{32}}$

Point of contact on parable P(8, $4\sqrt{2}$ )

Length of tangent PQ = $\sqrt{(8-4)^2+(4\sqrt{2})^2-16}=\sqrt{32}\Rightarrow$ $(PQ)^2 = 32$

**28.** Let f : (–2,2) → IR be defined by

$$f(x)=\begin{cases} x[x] & , \; -2<x<0 \\ (x-1)[x], & 0\le x<2 \end{cases}$$

where [x] denotes the greatest integer function. If m and n respectively are the number of points in (–2, 2) at which y = |f(x)| is not continuous and not differentiable, then m + n is equal to________.

**NTA Ans.** **(4)**

**Reso Ans.** **(4)**

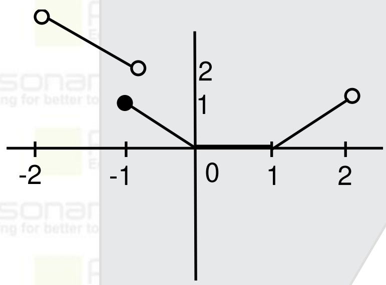
.

**Sol.**

$$\begin{cases} -2x & :-2<x<-1 \\ -x & :-1\le x<0 \\ 0 & :0\le x\le 1 \\ x-1 & :1\le x<2 \end{cases}$$

point of discontinuity → -1

point of non-differentiability → -1, 0, 1




**29.** Let a, b, c be three distinct positive real number such that $(2a)^{\log_e a}=(bc)^{\log_e b}$ and $b^{\log_e 2}=a^{\log_e c}$.

Then 6a + 5bc is equal to ____________

**NTA Ans.** **(8)**

**Reso Ans.** **(8)**

**Sol.** $(2a)^{\ell na} = (b,c)^{\ell nb}$ $\qquad$ $b^{\ell n2} = a^{\ell nc}$

$\ell na(\ell n2 + \ell na) = \ell nb\,(\ell nb + \ell nc)$ …… (1) $\qquad$ $\ell n2.\,\ell nb = \ell nc.\,\ell na$ ……..(2)

$$(\ell na)^2-(\ell nb)^2=\frac{\ell nb.\ell nb.\ell n2}{\ell na}-\ell n2\ell na$$

$$(\ell na)^2-(\ell nb)^2\left(1+\frac{\ell n2}{\ell na}\right)=0$$

$$1+\frac{\ell n2}{\ell na}=0$$

$$a=\frac{1}{2}$$

equation (2) $\quad \ell n2\,.\,\ell nb = \ell nc.(-\ell n2)$

$\ell nb = -\ell nc$

$bc = 1$

$\therefore$ 6a + 5bc = 3 + 5 = 8

**30.** The sum of all those terms, of the arithmetic progression 3, 8, 13, ........ 373, which are not divisible by 3, is equal to ________.

**NTA Ans.** **(9525)**

**Reso Ans.** **(9525)**

**Sol.** 3, 8, 13, ….. 373

$t_n = 3 + (n - 1)5 = 373$

$(n - 1) = \frac{370}{5} = 74$

$n = 75$

$\text{sum} = \frac{75}{2}(3+373) = 14100$

Terms divisible by 3 : 3, 18,33 …….

$t_n = 3 + (n - 1)\,15 \le 373$

$\Rightarrow n - 1 \le \frac{370}{15}$

$\Rightarrow n \le 25.66$

$n = 25$

$\text{Sum} = \frac{25}{2}[2.3 + (25-1)15]$

$= \frac{25}{2}(6 + 24 \times 15)$

$= 4575$

$\therefore$ Request sum = 14100 – 4575

= 9525 Ans.