Resonance®
Educating for better tomorrow

JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 10 April, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 06.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | **FAX No. :** +91-022-39167222

To Know more : sms **RESO** at **56677** | **Website :** www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN :** U80302RJ2007PLC024029

Toll Free : 1800 258 5555 **7340010333** facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

PART : MATHEMATICS

1. If the points P and Q are respectively the circumcenter and the orthocentre of a ΔABC, then $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ is equal to

(1) $\overrightarrow{QP}$ (2) $2\overrightarrow{PQ}$ (3) $\overrightarrow{PQ}$ (4) $2\overrightarrow{QP}$

NTA Ans. (3)
Reso Ans. (3)

Sol. Let position vectors of vertices of Δ ABC be A ($\vec{a}$), B ($\vec{b}$) and C ($\vec{c}$)

Now

centroid G $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$

Now if circumcentre of & orthocentre of Δ ABC are $(\vec{p})$ and $Q(\vec{q})$ then

$$G\left(\frac{2\vec{p}+\vec{q}}{3}\right)$$

Now $\frac{2\vec{p}+\vec{q}}{3}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

$$2\vec{p}+\vec{q}=\vec{a}+\vec{b}+\vec{c} \quad \text{\_\_\_\_\_(1)}$$

Now $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{a}-\vec{p}+\vec{b}-\vec{p}+\vec{c}-\vec{p}$

$$=\left(2\vec{p}+\vec{q}\right)-3\vec{p}=\vec{q}-\vec{p}=\overrightarrow{PQ}$$

2. If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots\ldots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to

(A) 220 (2) 226 (3) 227 (4) 223

NTA Ans. (4)
Reso Ans. (4)

Sol. $S_n = 4 + 11 + 21 + 34 + 50 \ldots\ldots$

Difference $\Rightarrow$ 7 10 13 16

So, $T_n = 4+\frac{n-1}{2}(2\times 7+(n-1-1)3) = \frac{3n^2+5n}{2}$

So, $S_n = \sum T_n = \frac{1}{2}\left(3\Sigma n^2+5\Sigma n\right)$

$$=\frac{1}{2}\left(3\cdot\frac{n(n+1)(2n+1)}{6}+5\frac{n(n+1)}{2}\right)=\frac{n(n+1)(n+3)}{2}$$

So, $\frac{1}{60}(S_{29}-S_9)$

$$=\frac{1}{60}\left(\frac{29.30.32}{2}-\frac{9.10.12}{2}\right)=223$$

3. If $A = \frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then $|adj(adj)(2A))|$ is equal to

(1) 2^8 (2) 2^{16} (3) 2^{12} (4) 2^{20}

NTA Ans. (2)
Reso Ans. (2)

Sol. $|A| = \frac{1}{5!6!7!} \times 5!6!7!\begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_2$

$$|A| = \begin{vmatrix} 1 & 6 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} \Rightarrow |A| = 2$$

$|Adj\ (adj\ (2A)| = |2A|^{(n-1)^2} = |2A|^4 = 2^{12}|A|^4 = 2^{16}$

4. The statement $\sim [p \vee (\sim (p \wedge q))]$ is equivalent to

(1) $\sim (p \wedge q)$ (2) $\sim (p \vee q)$ (3) $(\sim (p \wedge q)) \wedge q$ (4) $(p \wedge q) \wedge (\sim p)$

NTA Ans. (4)
Reso Ans. (4)

Sol. $\sim [p \vee (\sim (p \wedge q))]$

$\sim p \wedge (p \wedge q)$

5. Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is

(1) 1120 (2) 1680 (3) 3360 (4) 560

NTA Ans. (3)
Reso Ans. (2)

Sol.

$\frac{8!}{3!\,3!\,2!2!}\ 3!$

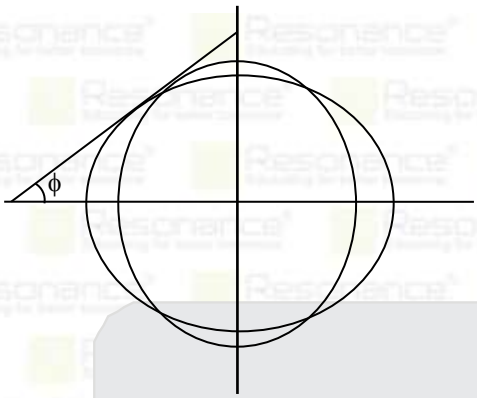
$= 1680$

6. Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{12}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

NTA Ans. (1)
Reso Ans. (1)

Sol.



equation of ellipse is $\frac{x^2}{19}+\frac{y^2}{15}=1$ ..... (1)

and equation of circle is $x^2 + y^2 = 16$ ..... (2)

let common tangent be $y = mx + \sqrt{19m^2+15}$

It's also tangent of circle (2) So

$\frac{\sqrt{19m^2+15}}{\sqrt{1+m^2}} = 4 \Rightarrow m = \frac{1}{\sqrt{3}}$

$\Rightarrow \phi = \frac{\pi}{6}$

$\Rightarrow$ required angle = $\frac{\pi}{3}$

7. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7 . Then $(\alpha^2 + \beta^2)$ is equal to :

(1) 5 (2) 20 (3) 10 (4) 13

NTA Ans. (1)

Reso Ans. (1)

Sol. $(22)^{2022} + (2022)^{22}$

$(21 + 1)^{2022} + (2022)^{22}$

$= 21\lambda + 1 + 3\mu \Rightarrow \alpha = 1$

Now $(21 + 1)^{2022} + (2023 - 1)^{22}$

$= 21\lambda + 2023\mu + 2 \Rightarrow \beta = 2$

Now, $\alpha^2 + \beta^2 = 1 + 4 = 5$

8. Let $S=\left\{z=x+iy:\dfrac{2z-3i}{4z+2i}\text{is real number}\right\}$. Then which of the following is NOT correct ?

(1) $(x,y)=\left(0,-\dfrac{1}{2}\right)$ (2) $y+x^2+y^2\neq-\dfrac{1}{4}$ (3) $x=0$ (4) $y\in\left(-\infty,-\dfrac{1}{2}\right)\cup\left(-\dfrac{1}{2},\infty\right)$

NTA Ans. (1)
Reso Ans. (1)

Sol. $\dfrac{2z-3i}{4z+2i}=\dfrac{2x+i(2y-3)}{4x+i(4y+2)}$ ..... (1)

$=\dfrac{\{2x+i(2y-3)\}\{4x-i(4y+2)\}}{16x^2+(4y+2)^2}$ is a real number

So $-2x(4y+2)+4x(2y-3)=0\Rightarrow x=0$

and $4y+2\neq0$

$\Rightarrow y\in\left(-\infty-\dfrac{1}{2}\right)\cup\left(-\dfrac{1}{2},\infty\right)$

option (1) is not correct

9. For $\alpha,\beta,\gamma,\delta\in\mathbb{N}$, if $\int\left(\left(\dfrac{x}{e}\right)^{2x}+\left(\dfrac{e}{x}\right)^{2x}\right)\log_e x\,dx=\dfrac{1}{\alpha}\left(\dfrac{x}{e}\right)^{\beta x}-\dfrac{1}{\gamma}\left(\dfrac{e}{x}\right)^{\delta x}+C$, where $e=\sum_{n=0}^{\infty}\dfrac{1}{n!}$ and C is constant of integration, then $\alpha+2\beta+3\gamma-4\delta$ is equal to

(1) 4 (2) –4 (3) –8 (4) 1

NTA Ans. (1)
Reso Ans. (1)

Sol. Put $\left(\dfrac{x}{e}\right)^x=t$

So, $\ell nt=x\cdot\ell n\left(\dfrac{x}{e}\right)$

$=x(\ell nx-1)$

$\Rightarrow\quad\dfrac{dt}{t}=\ell nx\,dx$

So, $\int\left(t^2+\dfrac{1}{t^2}\right)\dfrac{dt}{t}$

$=\int\left(t+\dfrac{1}{t^3}\right)dt$

$\dfrac{t^2}{2}+\dfrac{t^{-2}}{-2}+C$

$=\dfrac{1}{2}\left(t^2-t^{-2}\right)+C$

$=\dfrac{1}{2}\left(\left(\dfrac{x}{e}\right)^{2x}-\left(\dfrac{e}{x}\right)^{2x}\right)+C=\dfrac{1}{\alpha}\left(\dfrac{x}{e}\right)^{\beta x}-\dfrac{1}{\gamma}\left(\dfrac{e}{x}\right)^{\delta x}+C\Rightarrow\alpha=2,\ \beta=2,\ \gamma=2,\delta=2$

$\Rightarrow\alpha+2\beta+3\gamma-4\delta=4$

10. If the coefficients of x and x^2 in $(1 + x)^p (1-x)^q$ are 4 and –5 respectively, then 2p + 3q is equal to

(1) 69 (2) 60 (3) 66 (4) 63

NTA Ans. (4)

Reso Ans. (4)

Sol. $(1 + x)^p (1 - x)^q = (1 + {}^pC_1x + ...) (1 - {}^qC_1 x .....)$

coefficient of $x^1 = {}^pC_1 - {}^qC_1 = p - q = 4$ .....(1)

coefficient of $x^2 = - {}^pC_1 {}^qC_1 + {}^pC_2 + {}^qC_2$

$= - p.q + \frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -5 \Rightarrow (p - q)^2 - p - q = -10 \Rightarrow p + q = 26$ .....(2)

by (1) and (2) $\Rightarrow$ p = 15 and q = 11 $\Rightarrow$ 2p + 3q = 63

11. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1$ divides b_2 and a_2 divides $b_1\}$ is :

(1) 18 (2) 36 (3) 12 (4) 24

NTA Ans. (2)

Reso Ans. (2)

Case I

$a_1 = a_2 = 2 \Rightarrow b_1, b_2 \in \{8, 12\} \Rightarrow$ 4 elements

Case II

$a_1 = a_2 = 3 \Rightarrow b_1, b_2 \in \{9, 12\} \Rightarrow$ 4 elements

Case III

$a_1 = a_2 = 4 \Rightarrow b_1, b_2 \in \{8, 12\} \Rightarrow$ 4 elements

Case IV

$a_1 = 2, a_2 = 3 \Rightarrow b_1 \in \{9, 12\}, b_2 \in \{8, 12\} \Rightarrow$ 4 elements

Case V

$a_1 = 3, a_2 = 2 \Rightarrow b_1 \in \{8, 12\}, b_2 \in \{9, 12\} \Rightarrow$ 4 elements

Case VI

$a_1 = 2, a_2 = 4 \Rightarrow b_1 \in \{8, 12\}, b_2 \in \{8, 12\} \Rightarrow$ 4 elements

Case VII

$a_1 = 4, a_2 = 2 \Rightarrow b_1 \in \{8, 12\}, b_2 \in \{8, 12\} \Rightarrow$ 4 elements

Case VIII

$a_1 = 3, a_2 = 4 \Rightarrow b_1 \in \{8, 12\}, b_2 \in \{9, 12\} \Rightarrow$ 4 elements

Case IX

$a_1 = 4, a_2 = 3 \Rightarrow b_1 \in \{9, 12\}, b_2 \in \{8, 12\} \Rightarrow$ 4 elements

Total elements = 4 × 9 = 36

12. Let μ be the mean and σ be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	k+2	2k	k^2-1	k^2-1	k^2+1	k–3

where $\sum f_i = 62$. If [x] denotes the greatest integer $\le x$, then $[\mu^2 + \sigma^2]$ is equal to

(1) 8 (2) 7 (3) 6 (4) 9

NTA Ans. (1)
Reso Ans. (1)

Sol. $\Sigma F_i = 3k^2 + 4k - 2 = 62$

$\Rightarrow 3k^2 + 4k - 64 = 0 \Rightarrow k = (3k + 16)(k - 4) = 0 \Rightarrow k = 4$

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \mu^2$$

$$\sigma^2 + \mu^2 = \frac{\sum f_i x_i^2}{\sum f_i}$$

$$= \frac{29k^2 + 27k - 72}{62} = \frac{500}{62} = 8.06$$

$[\sigma^2 + \mu^2] = 8$

13. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{C} = \hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d} = 12$. Then $(-\hat{i} + \hat{j} - \hat{k}).(\vec{c} \times \vec{d})$ is equal to

(1) 48 (2) 42 (3) 44 (4) 24

NTA Ans. (3)
Reso Ans. (3)

Sol. $\vec{d} = \lambda(\vec{a} \times \vec{b})$

$\Rightarrow \vec{c}.\vec{d} = \lambda[\vec{a}\,\vec{b}\,\vec{c}] = 12$

$$\lambda \begin{vmatrix} 2 & 7 & -1 \\ 3 & 0 & 5 \\ 1 & -1 & 2 \end{vmatrix} = 12 \Rightarrow \lambda = 2 \Rightarrow \vec{d} = 2(\vec{a} \times \vec{b})$$

Now $\vec{c} \times \vec{d} = 2[\vec{c} \times (\vec{a} \times \vec{b})] = 2[\vec{a}(\vec{c}.\vec{b}) - \vec{b}(\vec{c}.\vec{a})]$

$\Rightarrow \vec{c} \times \vec{d} = 2(47\hat{i} + 91\hat{j} + 22\hat{k})$

$\Rightarrow (-\hat{i} + \hat{j} - \hat{k})(\vec{c} \times \vec{d}) = 2[-47 + 91 - 22] = 44$

14. Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to :

(1) 90 (2) B (3) 60 (4) 15

NTA Ans. (3)
Reso Ans. (3)

Sol. $^{n}C_7\left(\frac{1}{2}\right)^7.\left(\frac{1}{2}\right)^{n-7} = {}^{n}C_9\left(\frac{1}{2}\right)^9.\left(\frac{1}{2}\right)^{n-9}$

So $^{n}C_7 = {}^{n}C_9$

So $n = 16$

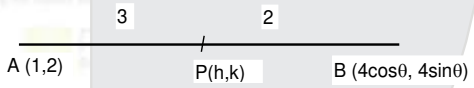
So, Probability of getting even numbers twice = $^{16}C_2\left(\frac{1}{2}\right)^{16} = \frac{K}{2^{15}}$

So $K = 16$.

15. Let A be the point (1, 2) and B be any point on the curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point C (α, β), then the length of the line segment AC is

(1) $\frac{4\sqrt{5}}{5}$ (2) $\frac{2\sqrt{5}}{5}$ (3) $\frac{3\sqrt{5}}{5}$ (4) $\frac{6\sqrt{5}}{5}$

NTA Ans. (3)
Reso Ans. (3)
Sol.



$h = \frac{12\cos\theta + 2}{5}$, $k = \frac{12\sin\theta + 4}{5}$

$5h - 2 = 12\cos\theta$, $5k - 4 = 12\sin\theta$

locus of point P is

$(5x - 2)^2 + (5y - 4)^2 = 144$.

its centre : C (2/5, 4/5)

AC = $\sqrt{(1-2/5)^2 + (2-4/5)^2} = \sqrt{\frac{9}{25} + \frac{36}{25}} = \sqrt{\frac{45}{25}} = \frac{3}{\sqrt{5}}$

16. Let $S = \left\{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\right\}$ and $\beta = \sum_{x \in S} \tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta - 14)^2$ is equal to

(1) 8 (2) 16 (3) 32 (4) 64

NTA Ans. (3)
Reso Ans. (3)

Sol. $9^{1-\tan^2 x} + 9^{\tan^2 x} = 10$

$\Rightarrow \frac{9}{y} + y = 10$ (where $9^{\tan^2 x} = y$)

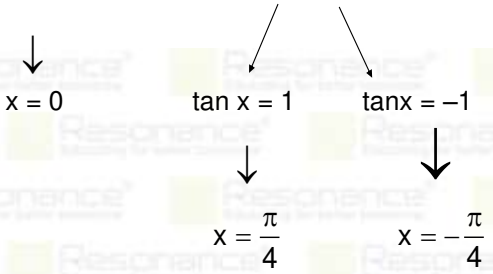
So, $9 + y^2 = 10y$

$y^2 - 10y + 9 = 0$

$(y - 1)(y - 9) = 0$

So, $9^{\tan^2 x} = 1$ & $9^{\tan^2 x} = 9$

$\tan^2 x = 0$ $\tan^2 x = 1$



Now $\beta = \sum \tan^2 \frac{x}{3} = \tan^2 \frac{0}{3} + \tan^2 \frac{\pi}{12} + \tan^2 \frac{\pi}{12}$

$= 2\tan^2 \frac{\pi}{12}$

$= 2\left(2-\sqrt{3}\right)^2$

$= 2\left(7-4\sqrt{3}\right)$

$\beta = 14 - 8\sqrt{3} \Rightarrow \frac{1}{6}(\beta-14)^2 = 32$

17. Let g(x) = f(x) + f(1–x) and f''(x) > 0, x ∈ (0, 1), If g is decreasing in the interval (0, α) and increasing in the interval (α, 1) then $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{1}{\alpha}\right) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to

(1) $\frac{3\pi}{4}$ (2) $\frac{3\pi}{2}$ (3) π (4) $\frac{5\pi}{4}$

NTA Ans. (3)
Reso Ans. (3)

Sol. g'(x) = f'(x) – f'(1–x)

If x > 1 – x

$\Rightarrow$ f' (x) > f' (1 – x) $\because$ f'(x) is strictly increasing

$\Rightarrow$ g'(x) > 0 $\Rightarrow$ so g(x) is increasing when $x > 1 - x \Rightarrow x > \frac{1}{2}$

$\Rightarrow \alpha = \frac{1}{2}$

similarly when x < 1 – x $\Rightarrow$ g(x) is decreeing

Now

$\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{1}{\alpha}\right) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$

$= \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$ ($\because$ 1 + 2 + 3 = 1.2.3)

18. Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is :

(1) 3 (2) $\frac{10}{3}$ (3) 4 (4) $\frac{11}{3}$

NTA Ans. (3)
Reso Ans.(3)

Sol. Let A $(r, 6 - 2r, 5r - 8)$

So $\frac{r-5}{4}=\frac{6-2r-7}{3}=\frac{5r-8+2}{1} \Rightarrow r = 1$

$\Rightarrow$ A(1, 4 – 3)

Similarly B (r, 6– 2r, 5r – 8)

$\Rightarrow \frac{r+3}{6}=\frac{2r-3}{3}=\frac{5r-14}{1}$

$\Rightarrow r = 3$

$\Rightarrow$ B(3, 0, 7)

mid-point of AB is M (2,2,2)

$\therefore$ Distance of M(2,2,2) from the plane $2x – 2y + z = 14$ is equal to $=\left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right| = 4$

19. Let f be continuous function satisfying $\int_0^{t^2}(f(x)+x^2)dx=\frac{4}{3}t^3, \forall t>0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to

(1) $\pi\left(1-\frac{\pi^3}{16}\right)$ (2) $\pi^2\left(1-\frac{\pi^2}{16}\right)$ (3) $-\pi^2\left(1+\frac{\pi^2}{16}\right)$ (4) $-\pi\left(1+\frac{\pi^3}{16}\right)$

NTA Ans. (1)
Reso Ans. (1)

Sol. $\int_0^{t^2}(f(x)+x^2)dx=\frac{4}{3}t^3$

Diff. on both sides

$(f(t^2) + t^4)\, 2t = 4t^2$

$f(t^2) = 2t – t^4$

Put $t = \pi/2$

$f(\pi^2/4) = 2 \times \frac{\pi}{2}-\frac{\pi^4}{16} = \pi - \frac{\pi^4}{16}$

20. Let the image of the point P(1, 2, 6) in the plane passing through the points A(1, 2, 0), B(1, 4, 1) and C (0, 5,1) be Q (α, β, γ). Then ($\alpha^2 + \beta^2 + \gamma^2$) is equal to

(1) 65 (2) 70 (3) 62 (4) 76

NTA Ans. (1)
Reso Ans. (1)

Sol. equation of plane

$$\begin{vmatrix} x-1 & y-2 & z-0 \\ 1-1 & 4-2 & 1-0 \\ 0-1 & 5-2 & 1-0 \end{vmatrix} = 0$$

$\Rightarrow x + y - 2z - 3 = 0$ ..... (1)

so image of P(1,2,6) about the plane (1)

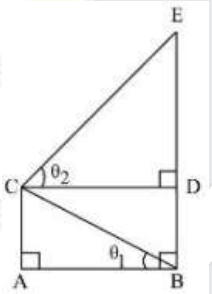
$$\frac{\alpha-1}{1} = \frac{\beta-2}{1} = \frac{\gamma-6}{-2}$$

$$= -2\left(\frac{-12}{6}\right)$$

$\Rightarrow \alpha = 5, \beta = 6, \gamma = -2$

$\Rightarrow \alpha^2 + \beta^2 \gamma^2 = 65$

21. In the figure, $\theta_1 + \theta_2 = \frac{\pi}{2}$ and $\sqrt{3}$ (BE) = 4(AB). If the area of ΔCAB is $2\sqrt{3} - 3$ unit^2, when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in unit) of ΔCED is equal to _____.



NTA Ans. (6)
Reso Ans. (6)

Sol. $\tan\theta_2 = \frac{DE}{CD} = \frac{BE - AC}{AB}$

$\tan\theta_2 = \frac{4}{\sqrt{3}} - \tan\theta_1 \Rightarrow \tan\theta_1 + \tan\theta_2 = \frac{4}{\sqrt{3}}$ ..... (1)

Now $\theta_1 + \theta_2 = \frac{\pi}{2} \Rightarrow \tan\theta_1 \tan\theta_2 = 1$ ..... (2)

Let $\tan\theta_1 = m_1$ and $\tan\theta_2 = m_2$ and $\frac{\theta_2}{\theta_1}$ is the largest

So $m_1 + m_2 = \frac{4}{\sqrt{3}}$, $m_1 m_2 = 1$

$\Rightarrow m_2 = \sqrt{3}$, $m_1 \frac{1}{\sqrt{3}}$

Now area of $\Delta CAB = \frac{1}{2} AB.AB \tan 30^\circ = \sqrt{3}\left(2-\sqrt{3}\right) \Rightarrow AB = 3-\sqrt{3}$

Now perimeter of ΔCED = CD + DE + CE AB + AB tan 60+ AB Sec60º

= AB $\left(1+\sqrt{3}+2\right)$

$\left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right) = 6$

22. Let the foot of perpendicular from the point A (4,3,1) on the plane P : x – y + 2z + 3 = 0 be N. If B(5, α, β), $\alpha,\beta \in \mathbb{Z}$ is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to _____.

NTA Ans. (7)
Reso Ans. (7)

Sol. Distance of A(4,3,1) from the plane x – y +2z +3 = 0

is AN = $\left|\frac{4-3+2+3}{\sqrt{1+1+4}}\right| = \sqrt{6}$

Area of $\Delta ABN = \frac{1}{2}$ AN. BN = $3\sqrt{2}$

$\Rightarrow$ BN = $2\sqrt{3}$

So AB = $\sqrt{6+12} = 3\sqrt{2}$

$\because$ B(5, α, β) lies on x – y + 2z +3 = 0

$\Rightarrow \alpha = 8+2\beta$ …… (1)

and AB = $3\sqrt{2} \Rightarrow$

$1 + (3 - \alpha)^2 + (1-\beta)^2 = 18$ …… (2)

by (1) and (2)

$\alpha = 2$ $\beta = -3$ ($\because \alpha,\beta \in \mathbb{Z}$)

$\because \alpha^2 + \beta^2 + \alpha\beta = 7$

23. Le S be the set of values of λ, for which the system of equations $6\lambda x - 3y + 3z = 4\lambda^2$, $2x + 6\lambda y + 4z = 1$, $3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12\sum_{\lambda \in S} |\lambda|$ is equal to _____.

NTA Ans. (24)
Reso Ans. (24)

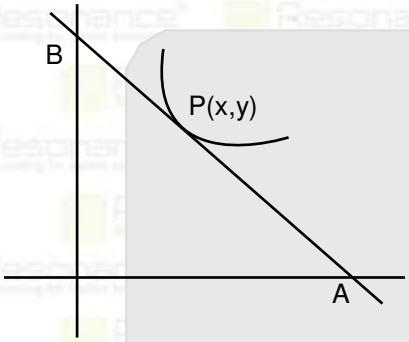
Sol. $D = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0 \Rightarrow (\lambda - 1)(3\lambda + 2)(3\lambda+1) = 0$

$\Rightarrow \lambda = \frac{-2}{3}, \frac{-1}{3}, 1$

for all value of λ given system of equations has no solution so $12\sum_{\lambda \in S} |\lambda| = 12\left(\frac{2}{3}+\frac{1}{3}+1\right) = 24$

24. Let the tangent at any point P on a curve passing through the points (1,1) and $\left(\frac{1}{10},100\right)$, intersect positive x-axis and y-axis at the point A and B respectively. If PA : PB = 1 : k and y = y(x) is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}$, y(0) = k, then $4y(1) - 5\log_e 3$ is equal to _____.

NTA Ans. (6)
Reso Ans. (5)
Sol.



tangent at P(x,y)

$Y - y = m(X - x)$

point at x axis $A\left(x - \frac{y}{m}, 0\right)$

point at y axis B(0, y – mx)

$\because$ P(x,y) divides AB in the ratio 1 : K

So, $\frac{K\left(x - \frac{y}{m}\right)}{k+1} = x \Rightarrow m + \frac{ky}{x} = 0$

$\Rightarrow \frac{dy}{dx} + \frac{ky}{x} = 0$

$\Rightarrow$ Solution is $x^k y = C$ .....(1)

$\because$ curve (1) is passing through (1, 1) and $\left(\frac{1}{10},100\right)$

So, C = 1, K = 2

Now differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2} \Rightarrow \frac{dy}{dx} = \ln(2x + 1)$

$\Rightarrow$ its solution is of the form

$y(x) = \left(x + \frac{1}{2}\right)\ln(2x + 1) - x + C$

Now y (0) = C = K = 2

$\therefore y(x) = \left(x + \frac{1}{2}\right)\ln(2x + 1) - x + 2 \Rightarrow 4y(1) = 4 + 6\ln 3$

Now $4y(1) - 5\ln 3 = 4 + \ln 3$

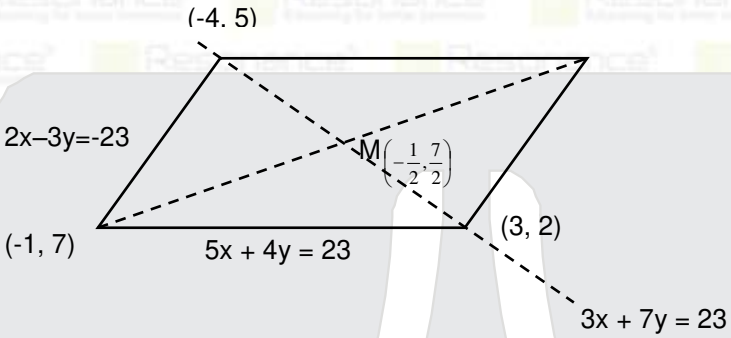
Approximate value = 5

25. Let the equation of two adjacent sides of a parallelogram ABCD be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of its one diagonal AC is $3x + 7y = 23$ and the distance of A from the other diagonal is d, then $50\,d^2$ is equal to _____.

NTA Ans. (529)

Reso Ans. (529)

Sol.



equation of other diagonal joining points (–1, 7) and $\left(-\frac{1}{2},\frac{7}{2}\right)$ is $7x + y = 0$

So distance of A from the other diagonal = $d = \left|\frac{7\times -4+5}{\sqrt{50}}\right|$

$\Rightarrow 50d^2 = (23)^2 = 529$

26. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _____.

NTA Ans. (26664)

Reso Ans. (26664)

Sol. Sum of all 4 digit numbers

= sum of digit at unit place × 10^0

+ sum of digit at term place × 10^1

+ sum of digit at hundred place × 10^2

+ sum of digit at thousand place × 10^3

$= \left(1\times\frac{3!}{2!}+2\times 3!+3\times\frac{3!}{2!}\right)(10^0+10^1+10^2+10^3)$

= 24 × (1111)

= 26664

27. If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta) \cup (\gamma, \delta]$, then $|3\alpha + 10(\beta + \gamma) + 21\delta|$ is equal to _____.

NTA Ans. (24)

Reso Ans. (24)

Sol. $\frac{2x}{5x+3} \le -1$ or $\frac{2x}{5x+3} \ge 1$

$\frac{2x}{5x+3} + 1 \le 0$ or $\frac{2x}{5x+3} - 1 \ge 0$

$\frac{7x+3}{5x+3} \le 0$ or $\frac{3(x+1)}{5x+3} \le 0 \Rightarrow x \in [-1, -3/5) \cup (-3/5, -3/7]$

$\alpha = -1, \beta = -3/5, \quad \gamma = -3/5, \ \delta = -3/7$

$$\left|3(-1) + 10\left(-\frac{6}{5}\right) + 21\left(-\frac{3}{7}\right)\right| = |-3 - 12 - 9| = 24$$

28. Let the quadratic curve passing through the point (–1, 0) and touching the line y = x at (1,1) be y = f(x). Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is _____.

NTA Ans. (11)

Reso Ans. (11)

Sol. Let quadratic curve passing through (–1, 0) is $y - (x + 1)(ax - b)$

$\therefore y = ax^2 + (a - b)x - b$

so $\left(\frac{dy}{dx}\right)_{(1,1)} = \{2ax + (a + b)\}_{(1,1)}$

$= 3a - b = 1$ …… (2)

and curve (1) is passing through (1,1) So $2(a - b) = 1$ …… (3)

by (2) and (3) $a = \frac{1}{4}$, $b = -\frac{1}{4}$

So curve $y = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$ ….. (4)

now point $(\alpha, \alpha + 1)$ lies on curve (4) so $\alpha = 3$ (point $(\alpha, \alpha + 1)$ in the first quadrant)

normal to the curve (4) at the point $(3, 4) \Rightarrow x + 2y = 11$

$\Rightarrow$ its x intercept = 11

29. Suppose $a_1, a_2, 2, a_3, a_4$, be in an arithmetic co-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetic co-geometric progression is $\frac{49}{2}$, then a_4 is equal to _____.

NTA Ans. (16)

Reso Ans. (16)

Sol.

a_1	a_2	2	a_3	a_4
↓	↓	↓	↓	↓
a	2(a +d)	4(a + 2d)	8(a + 3d)	16(a + 4d)

So $4(a + 2d) = 2$

$2a + 4d = 1$ ……(1)

Sum = $a + 2a + 2d + 4a + 8d + 8a + 24d + 16a + 64d$

$= 31\,a + 98d = \frac{49}{2}$ ……..(2)

by equation (1) and (2) $\Rightarrow a = 0$ and $d = \frac{1}{4}$

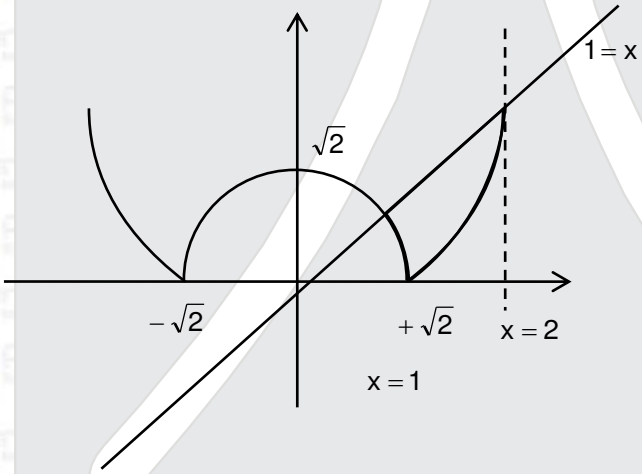
So, $a_4 = 16(a + 4d) = 16$

30. If the area of the region $\{x, y : |x^2 - 2| \le y \le x\}$ is A, then $6A + 16\sqrt{2}$ is equal to _____.

NTA Ans. (27)

Reso Ans. (27)

Sol. $= |x^2 - 2| \le y \le x$



So Area $\int_1^{\sqrt{2}} \left(x - (2 - x^2)\right)dx + \int_{\sqrt{2}}^{2} \left(x - (x^2 - 2)\right)dx$

$= \left[\frac{x^2}{2} - 2x + \frac{x^3}{3}\right]_1^{\sqrt{2}} + \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{\sqrt{2}}^{2}$

$= \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(2 - \frac{8}{3} + 4\right) - \left(1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$

$= 1 - \frac{4\sqrt{2}}{3} + \frac{7}{6} + \frac{10}{3} - 1 - \frac{4\sqrt{2}}{3}$

$A = \frac{9}{2} - \frac{8\sqrt{2}}{3}$ So $6A + 16\sqrt{2} = 27$