JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 11 April, 2023 (SHIFT-2) | TIME : (03.00 p.m. to 06.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x + 81)$, then λ, $\frac{\lambda}{3}$ are the roots of the equation

(1) $4x^2 - 24x + 27 = 0$ (2) $4x^2 - 24x - 27 = 0$ (3) $4x^2 + 24x + 27 = 0$ 4. $4x^2 + 24x - 27 = 0$

NTA. Ans. **(1)**

Reso Ans. **(1)**

Sol. Put x = 0 $\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{729}{8} \Rightarrow \lambda = \frac{9}{2}$

The equation whose roots are $\frac{9}{2}$ & $\frac{3}{2}$ is $x^2 - \frac{12x}{2} + \frac{27}{4} = 0 \Rightarrow 4x^2 - 24x + 27 = 0$

2. Let a, b, c and d be positive real numbers such that a + b + c + d = 11. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is

(1) 110 (2) 90 (3) 55 (4) 108

NTA. Ans. **(2)**

Reso Ans. **(2)**

Sol. By AM $\geq$ GM

$$\frac{5\left(\frac{a}{5}\right)+3\left(\frac{b}{3}\right)+2\left(\frac{c}{2}\right)+d}{11} \geq \left(\left(\frac{a}{5}\right)^5\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2 d\right)^{\frac{1}{11}}$$

$$1 \geq \frac{a^5b^3c^2d}{3125\times 27\times 4}$$

$3125 \times 108 \geq a^5b^3c^2d$

Now, $\beta(3750) = 3125 \times 108$

$\beta = 90$.

3. Let A= {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A × B such that $R = \{((a_1, b_1), (a_2, b_2)): a_1 \leq b_2 \text{ and } b_1 \leq a_2\}$. Then the number of elements in the set R is

(1) 180 (2) 26 (3) 160 (4) 52

NTA. Ans. **(3)**

Reso Ans. **(3)**

Sol. If $a_1 = 1$ then b_2 can be 2, 4, 5, 8, 10 = 5 cases

$a_1 = 3$ then b_2 = 4 cases

$a_1 = 4$ then $b_2 = 4$

$a_1 = 6$ then b_2 = 2 case

$a_1 = 9$ then b_2 = 1 case

total = 16 cases

also with

$a_2 = 3$ then b_1 = 1 case

$a_2 = 4$ then b_1 = 2 cases

$a_2 = 6$ then b_1 = 3cases

$a_2 = 9$ then b_1 = 4 cases

toal 10 cases

so total relations = 160

4. Let P be the plane passing through the points (5, 3, 0), (13, 3, –2) and (1, 6, 2). For $\alpha \in N$, if the distances of the points A (3, 4, α) and B (2, α, a) from the plane P are 2 and 3 respectively, then the positive value of a is

(1). 5 (2) 6 (3) 3 (4) 4

NTA. Ans. (4)

Reso Ans. (4)

Sol. Plane (Normal) = $\overrightarrow{AB} \times \overrightarrow{AC}$

$$= \begin{vmatrix} i & j & k \\ 8 & 0 & -2 \\ -4 & 3 & 2 \end{vmatrix} = i\,(6) - j(8) + k(24)$$

eqn. plane = 3(x – 5) – 4(y – 3) + 12 (z) = 0

3x – 4y + 12z – 3 = 0

Now $\pm 2 = \left|\dfrac{9-16+12\alpha-3}{13}\right|$

$\pm 26 = 12\alpha - 10$

$\alpha = 3, \dfrac{-4}{3}$

and $\pm 3 = \left|\dfrac{6-12+12a-3}{13}\right|$

$\pm 39 = 12a - 9$

a = 4 Ans.

5. The converse of $((\sim p) \wedge q) \Rightarrow r$ is

(1) $((\sim p) \vee q) \Rightarrow r$ (2) $(\sim r) \Rightarrow p \wedge q$ (3) $(\sim 1) \Rightarrow ((\sim p) \wedge q)$ (4) $(p \vee (\sim q)) \Rightarrow (\sim r)$

NTA. Ans. (4)

Reso Ans. (4)

Sol. Coverse of $((\sim p) \wedge q) \to r$

is $r \to ((\sim p) \wedge q)$

$(\sim r) \vee ((\sim p) \wedge q)$

or $((\sim p) \wedge q) \vee (\sim r)$

or $p \wedge (\sim q) \to (\sim r)$

6. The domain of the function $f(x) = \dfrac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is (where [x] denotes the greatest integer less than or equal to x)

(1) $(-\infty, -2) \cup [6, \infty)$ (2) $(-\infty, -2) \cup (5, \infty)$.(3) $(-\infty, -3] \cup [6, \infty)$ (4) $(-\infty, -3] \cup (5, \infty)$

NTA. Ans. (1)

Reso Ans. (1)

Sol. Let [x] = t

$t^2 - 3t - 10 > 0$

$(t-5)(t+2) > 0$

$t < -2$ or $t > 5$

$[x] < -2$ or $[x] > 5$

$x \in (-\infty, -2) \cup x \in [6, \infty)$

7. If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to

(1) 92 (2) 69 (3) 72 (4) 115

NTA. Ans. **(1)**

Reso Ans. **(1)**

Sol. Normal $\frac{36x}{6\cos\theta} - \frac{9y}{3\sin\theta} = 36 - 9$

it passes (2, 0)

$\frac{72}{6\cos\theta} = 27$

$\frac{4}{9} = \cos\theta$

Now $\sin\theta = \frac{\sqrt{65}}{9}$

Now rad. = $\sqrt{(6\cos\theta - 2)^2 (3\sin\theta)^2}$

$= \sqrt{\left(\frac{8}{3} - 2\right)^2 + 9 \times \frac{65}{81}}$

$= \sqrt{\frac{4+65}{9}} = \sqrt{\frac{69}{9}}$

Now $12r^2 = 12 \times \frac{69}{9} = 4 \times 23 = 92$

8. Let the function $f : [0, 2] \to R$ be defined as

$$f(x) = \begin{cases} e^{\min\{x^2,\, x - [x]\}}, & x \in [0, 1) \\ e^{[x - \log_e x]} & x \in [1, 2) \end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 xf(x)dx$ is

(1) $(e-1)\left(e^2 + \frac{1}{2}\right)$ (2) $2e - 1$ (3) $1 + \frac{3e}{2}$ (4) $2e - \frac{1}{2}$

NTA. Ans. **(4)**

Reso Ans. **(4)**

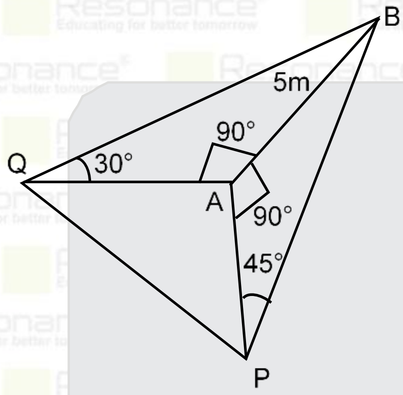
Sol. $f(x) = \begin{cases} e^{x^2} & x \in (0, 1) \\ e & x \in [1, 2) \end{cases}$

Now $\int_0^2 xf(x)dx = \int_0^1 xe^{x^2}dx + \int_1^2 x.\,e\,dx = 2e - \frac{1}{2}$

9. The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to

(1) $\frac{5}{2}\sqrt{5}$ (2) 10 (3) 5 (4) $5\sqrt{5}$

NTA. Ans. **(2)**
Reso Ans. **(2)**
Sol.



Let tower be AB
A is foot of tower
Then AP = 5cot 45°
= 5
& AQ = 5cot 30°= $5\sqrt{3}$ m
Now PQ = $\sqrt{AP^2 + AQ^2}$ = $\sqrt{25+25\times 3}$ = 10

10. If f : R → R be a continuous function satisfying $\int_0^{\frac{\pi}{2}} f(\sin 2x)\sin x dx + \alpha\int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x dx = 0$, then the value of α is

(1) $-\sqrt{3}$ (2) $\sqrt{3}$ (3) $-\sqrt{2}$ (4) $\sqrt{2}$

NTA. Ans. **(3)**
Reso Ans. **(3)**

Sol. $I = \int_0^{\pi/2} f(\sin 2x)\sin x\, dx$

$\int_0^{\pi/4} f(\sin 2x)\sin x\, dx + \int_{\pi/4}^{\pi/2} f(\sin 2x)\sin x\, dx$

$\int_0^{\pi/4} f(\cos 2x)\sin\left(\frac{\pi}{4} - x\right)dx + \int_0^{\pi/4} f\left(\sin\left(2\left(\frac{\pi}{4}+x\right)\right)\right)\sin\left(\frac{\pi}{4}+x\right)dx$

$I = \int_0^{\pi/4} f(\cos 2x)\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)dx + \int_0^{\pi/4} f\left(\cos(2x)\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)\right)dx$

$I = \int_0^{\pi/4} f(\cos 2x)\left(\sqrt{2}\cos x\, dx\right)$

$\alpha = -\sqrt{2}$

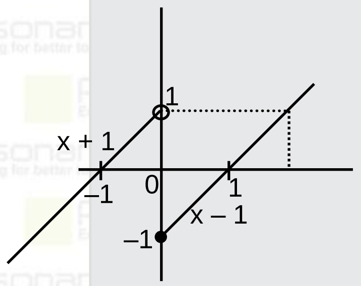
11. Let f and g be two function defined by

$$f(x) = \begin{cases} x+1, & x<0 \\ |x-1|, & x \ge 0 \end{cases} \text{ and } g(x) = \begin{cases} x+1, & x<0 \\ 1, & x \ge 0 \end{cases}$$

Then (gof) (x) is

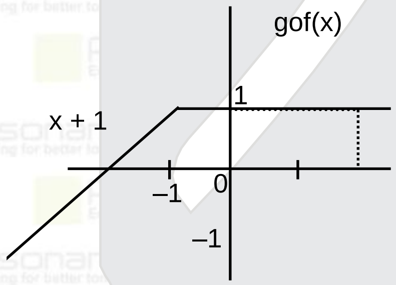
(1) Continuous everywhere but not differentiable at x = 1
(2) Continuous everywhere but not differentiable exactly at one point
(3) not continuous at x = – 1
(4) differentiable everywhere

NTA. Ans. **(2)**
Reso Ans. **(2)**
Sol.



$$g(f(x)) = \begin{cases} f(x)+1 & f(x)<0 \\ 1 & f(x) \ge 0 \end{cases}$$

$$g(f(x)) = \begin{cases} x+1+1 & ;x<-1 \\ 1 & ;-1 \le x < 0 \\ x-1+1 & ;0 \le x < 1 \\ 1 & ;x \ge 1 \end{cases} \Rightarrow h(x) = g(f(x)) = \begin{cases} x+2 & ;x<-1 \\ 1 & ;-1 \le x < 0 \\ 1 & ;0 \le x < 1 \\ 1 & ;x \ge 1 \end{cases}$$



12. If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial numbers, then the serial number of the word THAMS is

(1) 103 (2) 102 (3) 104 (4) 101

NTA. Ans. **(1)**
Reso Ans. **(1)**
Sol.

HAMS				

4 ↑ (first box), 4! (remaining boxes) = 4 × 4! = 96

T	A			

3! → 3! = 6

T	H	A	M	S

→ 1 = 103

13. Let y = y(x) be the solution of the differential equation

$\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^2+1)^2}{x^7}$, x > 0. If y(1) = 2, then y(2) is equal to

(1) $\frac{697}{128}$ (2) $\frac{693}{128}$ (3) $\frac{679}{128}$ (4) $\frac{637}{128}$

NTA. Ans. **(2)**

Reso Ans. **(2)**

Sol. I.F = $e^{\int \frac{5}{x(1+x^5)}dx}$

$e^{\int \frac{5}{x^6(\frac{1}{x^5}+1)}dx}$

Put $\frac{1}{x^5}+1=t$

$\frac{-5}{x^6}dx = dt$

$e^{\int \frac{-dt}{t}dx} = e^{-\ln t}\ \frac{1}{t}$

$= \frac{x^5}{1+x^5}$

Solution is $y\left(\frac{x^5}{1+x^5}\right) = \int \frac{x^5}{1+x^5}\cdot\frac{(1+x^5)^2}{x^7}dx + c$

$y\left(\frac{x^5}{1+x^5}\right) = \frac{-1}{x} + \frac{x^4}{4} + c$

Now y(1) = 2 then $\frac{2}{2} = \frac{-1}{1} + \frac{1}{4} + c$

$C = 2 - \frac{1}{4} = \frac{7}{4}$

Now $y(2) = \left(\frac{\frac{-1}{2} + 4 + \frac{7}{4}}{1}\right) \frac{33}{32} = \frac{693}{128}$

14. The sum of the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to

(1) 25 (2) 63 (3) 92 (4) 41

NTA. Ans. **(2)**

Reso Ans. **(2)**

Sol. Coefficients are ${}^{n+2}C_{r-1}$, ${}^{n+2}C_r$, ${}^{n+2}C_{r+1}$

Now $\frac{{}^{n+2}C_{r-1}}{{}^{n+2}C_r} = \frac{1}{3} \Rightarrow n + 3 = 4r$ … (1)

$\frac{{}^{n+2}C_r}{{}^{n+2}C_{r+1}} = \frac{3}{5} \Rightarrow 3n + 1 = 8r$ … (2)

By (1) & (2) n = 5, r = 2

Now coefficients are ${}^7C_1$, ${}^7C_2$, ${}^7C_3$

sum = 63 Ans.

15. For a∈C, let A = {z∈C : Re $(a + \bar{z}) > \text{Im}(\bar{a} + z)$} and B = {z∈C : Re $(a + \bar{z}) < \text{Im}(\bar{a} + z)$}. Then among the two statements:

(S1) : If Re (a), Im (a) > 0, then the set A contains all the real numbers

(S2): If Re (a), Im (a) < 0, then the set B contains all the real numbers,

(1) only (S2) is true (2) only (S1) is true

(3) both are true (4) both are false

NTA. Ans. **(4)**

Reso Ans. **(4)**

Sol **Let a=α + iβ**

& z=x +iy

From S1 : $A \rightarrow \alpha + x > -\beta + y$

From S2 : $B \rightarrow \alpha + x < -\beta + y$

By observing both are false.

16. Let the line passing through the points P (2, –1, 2) and Q (5, 3, 4) meet the plane x – y + z = 4 at the point R. Then the distance of the point R from the plane x + 2y + 3z + 2 = 0 measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is equal to

(1) $\sqrt{189}$ (2) $\sqrt{31}$ (3) $\sqrt{61}$ (4) 3

NTA. Ans. **(4)**

Reso Ans. **(4)**

Sol. equ. of line from p,q

$\frac{x-2}{+3} = \frac{y+1}{+4} = \frac{z-2}{+2} = \lambda$

Let R (3λ + 2, 4λ –1, 2λ + 2)

Now It lies on plane x – y + z = 4

$3\lambda + 2 - 4\lambda + 1 + 2\lambda + 2 = 4$

$\lambda = -1$

R (–1, – 5, 0)

also

Line parallel to $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$

& passes through (-1, -5, 0) 4

$\frac{x+1}{2} = \frac{y+5}{2} = \frac{z}{1} = t(\text{let})$

Let pt(s) on plane x + 2y + 3z + 2 = 0

is S(2t –1, 2t – 5, t)

2t – 1 + 4t – 10 + 3t + 2 = 0

9t = 9

t = 1

So S(1, – 3, 1) Now distance

$= \sqrt{4+4+1} = 3$

17. Let the mean of 6 observations 1, 2, 4, 5, x and y be 5 and their variance be 10. Then their mean deviation about the mean is equal to

(1) $\frac{10}{3}$ 2. 3 3. $\frac{7}{3}$ 4. $\frac{8}{3}$

NTA. Ans. **(4)**

Reso Ans. **(4)**

Sol. $\frac{1+2+4+5+x+y}{6} = 5 \Rightarrow x + y = 18$ … (1)

$\frac{1^2+2^2+4^2+5^2+x^2+y^2}{6} - (5)^2 = 10$

$\Rightarrow x^2 + y^2 = 164$ … (2)

By (1) & (2) x = 8 y = 10

Now mean deviation about mean = $\frac{|x-\bar{x}|}{6} = \frac{4+3+1+0+3+5}{6} = \frac{8}{3}$

18. If four distinct points with position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar, then $[\vec{a}\,\vec{b}\,\vec{c}]$ is equal to

(1) $[\vec{a}\,\vec{d}\,\vec{b}] + [\vec{d}\,\vec{c}\,\vec{a}] + [\vec{d}\,\vec{b}\,\vec{c}]$ (2) $[\vec{b}\,\vec{c}\,\vec{d}] + [\vec{d}\,\vec{a}\,\vec{c}] + [\vec{d}\,\vec{b}\,\vec{a}]$

(3) $[\vec{d}\,\vec{b}\,\vec{a}] + [\vec{a}\,\vec{c}\,\vec{d}] + [\vec{d}\,\vec{b}\,\vec{c}]$ (4) $[\vec{d}\,\vec{c}\,\vec{a}] + [\vec{b}\,\vec{d}\,\vec{a}] + [\vec{c}\,\vec{d}\,\vec{b}]$

NTA. Ans. **(4)**

Reso Ans. **(4)**

Sol. $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar point then

vectors $\vec{b} - \vec{a}$, $\vec{c} - \vec{a}$, $\vec{d} - \vec{a}$ are also coplanar

$[\vec{b}-\vec{a} \quad \vec{c}-\vec{a} \quad \vec{d}-\vec{a}] = 0$

$[\vec{b} \quad \vec{c} \quad \vec{d}] - [\vec{b} \quad \vec{c} \quad \vec{a}] - [\vec{b} \quad \vec{a} \quad \vec{d}] - [\vec{a} \quad \vec{c} \quad \vec{d}] = 0$

$[\vec{a} \quad \vec{b} \quad \vec{c}] = [\vec{b} \quad \vec{c} \quad \vec{d}] + [\vec{a} \quad \vec{b} \quad \vec{d}] + [\vec{a} \quad \vec{d} \quad \vec{c}]$

19. If the system of linear equations
$7x + 11y + \alpha z = 13$
$5x + 4y + 7z = \beta$
$175x + 194y + 57z = 361$
has infinitely many solutions, then $\alpha + \beta + 2$ is equal to:
(1) 3 (2) 4 (3) 6 (4) 5

NTA. Ans. **(2)**
Reso Ans. **(2)**

Sol. For infinitely many solution.

$$D = \begin{vmatrix} 7 & 11 & \alpha \\ 5 & 4 & 7 \\ 175 & 194 & 57 \end{vmatrix} = 0$$

$7(228 - 1358) - 11(285 - 1225) + \alpha(970 - 700) = 0$
$-7 \times 1130 + 11 \times 940 + \alpha \times 270 = 0$
$1034 - 7910 = 27\alpha$
$\frac{-243}{27} = \alpha$
$\alpha = -9$

Also $D_x = \begin{vmatrix} 13 & 11 & -9 \\ \beta & 4 & 7 \\ 361 & 194 & 57 \end{vmatrix} = 0$

$\beta = 11$

20. If the 1011th term from the end in the binominal expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011th term from the beginning, then $|x|$ is equal to
(1) 10 (2) 8 (3) 15 (4) 12

NTA. Ans. **(1)**
Reso Ans. **(Bonus)**

Sol. $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$

T_{1011} from end $= 1024\ T_{1011}^{th}$ term from beginning

$${}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010} = 1024 \times {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$$

$$\left(\frac{5}{2x}\right)^2 = \left(\frac{4x}{5}\right)^2 \times 1024 \Rightarrow x^4 = \frac{25 \times 25}{4 \times 16 \times 1024} \Rightarrow |x| = \frac{5}{2^4} = \frac{5}{16}$$

21. Let the tangent to the parabola $y^2 = 12x$ at the point $(3, a)$ be perpendicular to the line $2x + 2y = 3$. Then the square of distance of the point $(6, -4)$ from the normal to the hyperbola $\alpha^2x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to ________.

NTA. Ans. **(116)**
Reso Ans. **(116)**

Sol. $(3, \alpha)$ lies on $y^2 = 12x$
$a = \pm 6$
Now Tang $y.\alpha = 6(x+3)$
Slope $= \frac{6}{\alpha} = 1$

$\alpha = 6$

Hyperbola $36x^2 - 9y^2 = 324$

$$\frac{x^2}{9} - \frac{y^2}{36} = 1$$

Normal [5,8) is $\frac{9x}{5} + \frac{36y}{8} = 9 + 36$

$$\frac{x}{5} + \frac{4y}{8} = 5$$

$2x + 5y = 50$

dis = $\left|\frac{12 - 20 - 50}{\sqrt{29}}\right| = 2\sqrt{29}$

22. The number of points, where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1$, $x \in R$ cuts x – axis, is equal to __________.

NTA. Ans. **(2)**

Reso Ans. **(2)**

Sol. $e^{4x} - e^{2x} - 3 - \frac{1}{e^{2x}} + \frac{1}{e^{4x}} = 0$

Let $e^{2x} + \frac{1}{e^{2x}} = t$ then

$t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) = 3$

$\left(t + \frac{1}{t}\right)^2 - 2 - \left(t + \frac{1}{t}\right) = 3$

Let $u = t + \frac{1}{t}$

Then $u^2 - u - 5 = 0$

Now, $u = \frac{1 + \sqrt{21}}{2}$

$t + \frac{1}{t} = \frac{1 + \sqrt{21}}{2}$

$2t^2 - \left(1 + \sqrt{21}\right)t + 2 = 0$

Here t = 2 values

So, number of solutions = 2

23. For $k \in N$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \ldots$ is 10, then the value of k is __________.

NTA. Ans. **(2)**

Reso Ans. **(2)**

Sol. $10 = 1 + \frac{4}{K} + \frac{8}{K^2} + \frac{13}{K^3} + \ldots\ldots\ldots\ \ldots\ldots$

$9 = \frac{4}{K} + \frac{8}{K^2} + \frac{13}{K^3} + \ldots\ldots\ldots\ \ldots\ldots$

$$\frac{9}{K} = 0 + \frac{4}{K^2} + \frac{8}{K^3} + \text{.......... ....}$$

$$9\left(\frac{K-1}{K}\right) = \frac{4}{K} + \frac{4}{K^2} + \frac{5}{K^3} + \text{.......... .....}$$

$$9 - \frac{10}{K} = \frac{3}{K} + \frac{4}{K^2} + .\frac{5}{K^3} + \text{.......... ....}$$

$$\frac{9}{K} - \frac{10}{K^2} = 0 + \frac{3}{K^2} - \frac{4}{K^3} + \text{.......... ....}$$

$$9 - \frac{19}{K} + \frac{10}{K^2} = \frac{3}{K} + \left(\frac{1}{K^2} + \frac{1}{K^3} + \text{..........} \infty\right)$$

$$9 - \frac{22}{K} + \frac{10}{K^2} = \frac{1}{K(K-1)}$$

$(K – 1) (9K^2 – 22k + 10) = k$

$9K^3 – 31K^2 + 31K – 10 = 0$

$(K – 2) (9K^2 – 13k + 5) = 0$

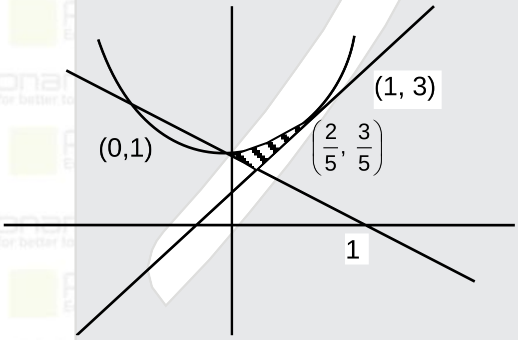
$K = 2$

24. If A is the area in the first quadrant enclosed by the curve C: $2x^2 – y + 1 = 0$, the tangent to C at the point (1, 3) and the line x + y = 1, then the value of 60A is ________ .

NTA. Ans. **(16)**

Reso Ans. **(16)**

Sol.



Tangent at (1,3) = $2x.1 - \frac{y+3}{2} + 1 = 0$

$4x – y – 3 + 2 = 0$

$4x – y = 1$

Line x + y = 1

Solving $= x = \frac{2}{5}$

Area. $\frac{2}{5}$, $y = \frac{3}{5}$

$$= \int_0^{2/5} (2x^2 + 1 - (1 - x))dx + \int_{2/5}^{1} (2x^2 + 1 - (4x - 1))dx$$

$= \left(\frac{2x^3}{3}+\frac{x^2}{2}\right)_0^{2/5} + \left(\frac{2x^3}{3}-2x^2+2x\right)_{2/5}^{1}$

$= \frac{16}{125\times 3}+\frac{4}{50}+\frac{2}{3}-\frac{16}{125\times 3}+\frac{2\times 4}{125}-\frac{4}{5}$

$= \frac{24+200+96-240}{300} = \frac{80}{300}$

60A = 16

25. Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6). Then the number of functions f: A→B satisfying f(1) + f(2) = f(4)-1 is equal to _________ .

NTA. Ans. **(360)**

Reso Ans. **(360)**

Sol. f(1) + f(2) + 1 = f(4) ≤ 6

f(1) + f(2) ≤ 5

Case I: f(1) = 1 ⇒ f(2) = 1, 2, 3, 4 (4 ways), f(3) = (6 ways), f(5) = (6 ways)

Case II: f(1) = 2 ⇒ f(2) = 1, 2, 3 (3 ways), f(3) = (6 ways), f(5) = (6 ways)

Case III: f(1) = 3 ⇒ f(2) = 1, 2 (2 ways), f(3) = (6 ways), f(5) = (6 ways)

Case IV: f(1) = 4 ⇒ f(2) = 1 (1 ways), f(3) = (6 ways), f(5) = (6 ways)

Total functions = 10 × 6 × 6 = 360

26. Let S = $\left\{z \in C - \{i, 2i\} : \frac{z^2+8iz-15}{z^2-3iz-2} \in R\right\}$. If $\alpha - \frac{13}{11}i \in S$, $a \in R - \{0\}$, then $242\alpha^2$ is equal to _________ .

NTA. Ans. **(1680)**

Reso Ans. **(1680)**

Sol. $\frac{z^2+8iz-15}{z^2-3iz-2} = \frac{\bar{z}^2-8i\bar{z}-15}{\bar{z}^2+3i\bar{z}-2}$

$z^2\bar{z}^2+3\bar{z}z^2i-2z^2+8iz\bar{z}^2-24z\bar{z}-16iz$

$-15\bar{z}^2-45i\bar{z}+30$

$= z^2\bar{z}^2-8izz^2-15z^2$

$-3iz\bar{z}^2 \;\; 24z\bar{z}+45iz$

$-2\bar{z}^2+16i\bar{z}+30$

$11i\bar{z}z^2+13z^2+11iz\bar{z}^2-61iz-13\bar{z}^2-61i\bar{z}=0$

$11i(z^2\bar{z}+\bar{z}^2z)+13(z-\bar{z})-61i=0$

$z + \bar{z} \pm 0$ So $11i\,(z\,\bar{z}) + 13\,(z-\bar{z}) - 61i = 0$

$11i\,(x^2 + y^2) + 13(2iy) - 61i = 0$

$11x^2 + 11y^2 + 26y - 61 = 0$

$y = \frac{-13}{11}$ then $11x^2 + \frac{169}{11} - \frac{338}{11} - 61 = 0$

$\Rightarrow \alpha^2 = \frac{840}{121}$

27. Let the line $l : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}$, $\lambda \in R$ meet the plane P : x + 2y + 3z = 4 at the point (α, β, γ). If the angle between the line l and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to ______.

NTA. Ans. **(11)**
Reso Ans. **(11)**

Sol. Angle = $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right) = \sin^{-1}\frac{3}{\sqrt{14}}$

Now, $\frac{3}{\sqrt{14}} = \pm\left(\frac{1+(2)(2)+3(\lambda)}{\sqrt{14}\sqrt{\lambda^2+4+1}}\right)$

$9(\lambda^2 + 5) = (3\lambda + 5)^2$

$\lambda = \frac{2}{3}$

Now let point of intersection be $(k, 2k + 1, \frac{2}{3}k + 3)$

$\Rightarrow k + 4k + 2 + 2k + 9 = 4 \Rightarrow k = -1$

Point of intersection (α, β, γ) is $\left(-1, -1, \frac{7}{3}\right)$

$\Rightarrow \alpha + 2\beta + 6\gamma = 11$

28. Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to ______.

NTA. Ans. **(27)**
Reso Ans. **(27)**

Sol. $64x^2 + 5nx + 1 = 0$

$D < 0$

$25N^2 - 256 < 0$

$-\frac{16}{5} < N < \frac{16}{5}$

$N = 1, 2, 3$

$P = \frac{1}{4} + \frac{3}{4}\cdot\frac{1}{4} + \left(\frac{3}{4}\right)^2\frac{1}{4}$

$= \frac{16+12+9}{64} = \frac{37}{64}$

$q - p = 27$

29. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = 11$, $\vec{b}\cdot(\vec{a}\times\vec{c}) = 27$ and $\vec{b}\cdot\vec{c} = -\sqrt{3}\,|\vec{b}|$, then $|\vec{a}\times\vec{c}|^2$ is equal to ________.

NTA. Ans. **(285)**
Reso Ans. **(285)**

Sol. $$\left(\vec{b}.\left(\vec{a}\times\vec{c}\right)\right)^2 = \left(\vec{a}.\left(\vec{b}\times\vec{c}\right)\right)^2 = \begin{vmatrix} a^2 & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & b^2 & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & c^2 \end{vmatrix}$$

$$3^6 = \begin{vmatrix} 14 & 0 & \vec{a}.\vec{c} \\ 0 & 3 & \vec{b}.\vec{c} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & c^2 \end{vmatrix} = \begin{vmatrix} 14 & 0 & 11 \\ 0 & 3 & -3 \\ 11 & -3 & c^2 \end{vmatrix} = 3^6$$

$14(3c^2 - 9) + 11(-33) = 3^6$

$42c^2 - 126 - 363 = 729$

$c^2 = 29$

$\left(\vec{a}\times\vec{c}\right)^2 = a^2c^2 - \left(\vec{a}.\vec{c}\right)^2$

$= 14 \times 29 - 11^2$

$= 285$

30. If the line $l_1 : 3y - 2x = 3$ is the angular bisector of the lines $l_2 : x - y + 1 = 0$ and $l_3 : \alpha x + \beta y + 17 = 0$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to ________.

NTA. Ans. **(348)**
Reso Ans. **(348)**

Sol. Point of intersection of l_1 & l_2

$= 3(x+1) - 2x = 3$

$x = 0$

and $y = 1$

Now let slope of $l_3 = m$

Then $$\frac{m - \frac{2}{3}}{1 + \frac{2m}{3}} = \pm\left(\frac{\frac{2}{3} - 1}{1 + \frac{2}{3}\times 1}\right)$$

$$\frac{3m - 2}{3 + 2m} = \pm\frac{1}{5}$$

$15m - 10 = \pm(3 + 2m)$

$13m = 13$ or $17m = 7$

$m = 1$, $m = 7/17$

Now equ. $y - 1 = \frac{7}{17}(x - 0)$

$7x - 17y + 17 = 0$

So $\alpha^2 + \beta^2 - \alpha - \beta = 49 + 289 - 7 + 17$

$= 348$

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