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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)
Questions & Solutions

Date: 12 April, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. If $\frac{1}{n+1}{}^nC_n+\frac{1}{n}{}^nC_{n-1}+.....+\frac{1}{2}{}^nC_1+{}^nC_0=\frac{1023}{10}$ then n is equal to

(1) 8 (2) 7 (3) 9 (4) 6

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. since $\frac{1}{1}{}^nC_0+\frac{1}{2}{}^nC_1+\frac{1}{3}{}^nC_2+----+\frac{1}{n}{}^nC_{n-1}+\frac{1}{n+1}{}^nC_n=\frac{1023}{10}$

$\Rightarrow \sum_{r=0}^{n}\frac{1}{r+1}{}^nC_r=\frac{1023}{10}$ $\quad (\because {}^{n+1}C_{r+1}=\frac{n+1}{r+1}{}^nC_r)$

$\Rightarrow \sum_{r=0}^{n}\frac{1}{n+1}{}^{n+1}C_{r+1}=\frac{1023}{10}$

$\Rightarrow \frac{1}{n+1}\left[{}^{n+1}C_1+{}^{n+1}C_2+.........+{}^{n+1}C_{n+1}\right]=\frac{1023}{10}$

$\Rightarrow \frac{2^{n+1}-1}{n+1}=\frac{1023}{10}=\frac{2^{10}-1}{10}$

$\Rightarrow n + 1 = 10$

$\Rightarrow n = 9$

2. Let C be the circle in the complex plane with centre $z_0=\frac{1}{2}(1+3i)$ and radius r=1. Let z = 1 + i and the complex number z_2 be outside the circle C such that $|z_1-z_0|\,|z_2-z_0|=1$. If z_0, z_1 and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to

(1) $\frac{5}{2}$ (2) $\frac{7}{2}$ (3) $\frac{13}{2}$ (4) $\frac{3}{2}$

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. Eq^n of circle c is $|z - z_0| = 1$

$\Rightarrow \left|z-\frac{1}{2}(1+3i)\right|=1$ .... (1)

also $|z_1-z_0||z_2-z_0|=1$

$\Rightarrow |z_2-z_0|=\frac{1}{|z_1-z_0|}=\frac{1}{\left|(1+i)-\frac{1}{2}(1+3i)\right|}=\frac{1}{\left|\frac{1}{2}-\frac{1}{2}i\right|}=\sqrt{2}$ .... (2)

z_0, z_1, z_2 are collinear $\Rightarrow$ if $z_2 = x + iy$ then $y-1=\frac{\frac{3}{2}-1}{\frac{1}{2}-1}(x-1)$

$\Rightarrow y - 1 = -(x - 1)$

$\Rightarrow y = -x + 2$

by (2) $\left|x+iy-\frac{1}{2}-\frac{3}{2}i\right|=\sqrt{2}$

$$\Rightarrow \left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=2$$

$$\Rightarrow \left(x-\frac{1}{2}\right)^2+\left(-x+\frac{1}{2}\right)^2=2$$

$$\Rightarrow \left(x-\frac{1}{2}\right)^2=1 \Rightarrow x-\frac{1}{2}=\pm 1$$

$$\Rightarrow x=\frac{3}{2}=\text{or } x=-1$$

Points are $z_2 \left(\frac{3}{2},\frac{1}{2}\right) \text{or} \left(-\frac{1}{2},\frac{5}{2}\right)$

but z_2 be out side the circle C So by (1) $z_2 \equiv \frac{3}{2}+\frac{1}{2}i$

$$|z_2|^2=\frac{9}{4}+\frac{1}{4}=\frac{5}{2}$$

3. Among the two statements

(S1) : $(p \Rightarrow q) \wedge (p \wedge (\sim q))$ is a contradiction and

(S2) : $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$ is a tautology

(1) both are false (2) only (S2) is true (3) only (S1) is true (4) both are true

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. S1 : $(p \Rightarrow q) \wedge (p \wedge (\sim q))$

$\equiv (\sim p \vee q) \wedge (p \wedge \sim q)$

$\equiv \sim (p \wedge \sim q) \wedge (p \wedge \sim q)$

$\equiv$ c (contradiction)

S2 : $(p \wedge q) \vee ((\sim p) \wedge q)) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$

$\equiv ((p \vee \sim p) \wedge q) \vee (p \vee (\sim p)) \wedge (\sim q))$

$\equiv (t \vee q) \vee (t \wedge (\sim q))$

$\equiv q \vee \sim q$

$\equiv$ t (tautology)

4. The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1-x)^{100}$, is equal to

(1) ${}^{101}C_{50}$ (2) $-{}^{101}C_{50}$ (3) ${}^{99}C_{49}$ (4) $-{}^{99}C_{49}$

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $(1\text{-}x)^{100} = \sum_{r=0}^{100} {}^{100}c_r (-1)^r x^r$

sum of 1st fifty coefficients

$= {}^{100}c_0 - {}^{100}c_1 + {}^{100}c_2 - {}^{100}c_3 + \ldots\ldots - {}^{100}c_{49}$

$= \frac{1}{2}\left[\left({}^{100}c_0 - {}^{100}c_1 + {}^{100}c_2 - {}^{100}c_3 + \ldots\ldots + {}^{100}c_{100}\right) - {}^{100}C_{50}\right]$

$= \frac{1}{2}\left[0 - {}^{100}c_{50}\right] = -\frac{1}{2}{}^{100}c_{50} = -{}^{99}C_{49}$

5. Let D be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$.

If the range of the function $g: D \to R$ defined by $g(x) = x - [x]$, ([x] is the greatest integer function), is (α, β) then $\alpha^2 + \frac{5}{\beta}$ is equal to

(1) 136 (2) 45 (3) 46 (4) 135

NTA Ans **(4)**
Reso. Ans. **(BONUS)**

6. In a triangle ABC, if cos A+2 cos B + cos C=2 and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then cos A –cos C is equal to

(1) $\frac{10}{7}$ (2) $\frac{3}{7}$ (3) $\frac{5}{7}$ (4) $\frac{9}{7}$

NTA Ans. **(1)**
Reso Ans. **(1)**

Sol. cosA + 2cosB + cosC = 2, c = 3, a = 7

$$\frac{b^2+c^2-a^2}{2bc} + \frac{2(c^2+a^2-b^2)}{2ac} + \frac{a^2+b^2-c^2}{2ab} = 2$$

$$\frac{49+b^2-9}{14b} + \frac{2(49+9-b^2)}{2.3.7} + \frac{b^2+9-49}{6b} = 2$$

$$\frac{40+b^2}{14b} + \frac{58-b^2}{21} + \frac{b^2-40}{6b} = 2$$

$\Rightarrow$ b = 5

$$\cos A - \cos C = \frac{b^2+c^2-a^2}{2bc} - \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{ab^3+ac^2-a^3-ca^2-cb^2+c^3}{2abc} = \frac{10}{7}$$

7. Let a, b, c be three distinct real numbers , none equal to one. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is equal to

(1) –1 (2) 1 (3) –2 (4) 2

NTA Ans. **(2)**
Reso Ans. **(2)**

Sol. $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$$\begin{vmatrix} \frac{a}{1-a} & \frac{1}{1-a} & \frac{1}{1-a} \\ \frac{1}{1-b} & \frac{b}{1-b} & \frac{1}{1-b} \\ \frac{1}{1-c} & \frac{1}{1-c} & \frac{c}{1-c} \end{vmatrix} = 0$$

$$\begin{vmatrix} \frac{1}{1-a}-1 & \frac{1}{1-a} & \frac{1}{1-a} \\ \frac{1}{1-b} & \frac{b}{1-b}-1 & \frac{1}{1-b} \\ \frac{1}{1-c} & \frac{1}{1-c} & \frac{1}{1-c}-1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} \left(\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}-1\right) & \left(\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}-1\right) & \left(\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}-1\right) \\ \frac{1}{1-b} & \frac{b}{1-b}-1 & \frac{1}{1-b} \\ \frac{1}{1-c} & \frac{1}{1-c} & \frac{1}{1-c}-1 \end{vmatrix} = 0$$

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

8. Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$ If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the element of the matrix $\sum_{n=1}^{50} B^n$ is equal to

(1) 125 (2) 50 (3) 100 (4) 75

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. Let $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$DC = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1+2 & -2+2 \\ 1-1 & 2-1 \end{bmatrix}$

$DC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\because B = (CAD)$

$B^n = (CAD)^n = CA\,(DC) \cdot A(DC) \cdot A(DC) \cdot \ldots\ldots\ldots\ldots A \cdot D$

$B^n = C \cdot A^n \cdot D$

$\because A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix} = I + E$ $\quad \left[E = \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix}\right]$

$E^2 = \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$I \cdot E = E$

$A^n = (I + E)^n = I^n + {}^nC_1\, E \cdot I + \ldots\ldots\ldots\ldots\ldots$

$A^n = (I + E)^n = I + nE = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$$B^n = C\cdot A^n\cdot D = \begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & \frac{n}{51}\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$$

$$B^n = C\cdot A^n\cdot D = \begin{bmatrix}1+\frac{n}{51} & \frac{n}{51}\\ \frac{-n}{51} & 1-\frac{n}{51}\end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix}50+\frac{50\cdot 51}{2\cdot 51} & \frac{50\cdot 51}{2\cdot 51}\\ \frac{-50\cdot 51}{2\cdot 51} & 50-\frac{50\cdot 51}{2\cdot 51}\end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix}50+25 & 25\\ -25 & 50-25\end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix}75 & 25\\ -25 & 25\end{bmatrix}$$

sum of all the element of the matrix $\sum_{n=1}^{50} B^n$ is 100

9. Let <a_n> be a sequence such that $a_1 + a_2 + ... + a_n = \frac{n^2+3n}{(n+1)(n+2)}$ If $28\sum_{k-1}^{10}\frac{1}{a_k} = p_1 p_2 p_3 ... p_m$, where $p_1, p_2, ...p_m$ are the first m prime numbers, then m is equal to

(1) 6 (2) 7 (3) 8 (4) 5

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. $a_n = \frac{n^2+3n}{(n+1)(n+2)} - \frac{(n-1)^2+3(n-1)}{n(n+1)}$

$$a_n = \frac{n^3+3n^2-(n+2)(n^2+n-2)}{n(n+1)(n+2)}$$

$$a_n = \frac{4}{n(n+1)(n+2)}$$

$$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{4}\sum_{k=1}^{10}(k(k+1)(k+2))$$

$$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{16}\sum_{k=1}^{10}(k(k+1)(k+2)(k+3)-(k-1)k(k+1)(k+2))$$

$$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{16}(10\cdot 11\cdot 12\cdot 13)$$

$$28\sum_{k=1}^{10}\frac{1}{a_k} = \frac{28}{16}\cdot 2\cdot 5\cdot 11\cdot 3\cdot 4\cdot 13$$

$$28\sum_{k=1}^{10}\frac{1}{a_k} = 2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13 \qquad \Rightarrow m = 6$$

10. Let y=y(x), y>0, be a solution curve of the differential equation $(1+x^2)dy=y(x-y)\,dx$, If y(0)=1 and $y\left(2\sqrt{2}\right)=\beta$, then

(1) $e^{\beta^{-1}}=e^{-2}\left(3+2\sqrt{2}\right)$ (2) $e^{\beta^{-1}}=e^{-2}\left(5+\sqrt{2}\right)$

(3) $e^{3\beta^{-1}}=e\left(5+\sqrt{2}\right)$ (4) $e^{3\beta^{-1}}=e\left(3+2\sqrt{2}\right)$

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $y(x-y)\,dx = (1+x^2)\,dy$

$$\frac{y(x-y)}{1+x^2}=\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{x}{1+x^2}y-\frac{y^2}{1+x^2}$$

$$\frac{dy}{dx}-\frac{x}{1+x^2}y=\frac{-y^2}{1+x^2}$$

$$\frac{-1}{y^2}\frac{dy}{dx}+\frac{x}{1+x^2}\frac{1}{y}=\frac{1}{1+x^2}$$

Let $\frac{1}{y}=t$

$$\therefore \frac{-1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$$

$$\therefore \frac{dt}{dx}+\frac{x}{1+x^2}t=\frac{1}{1+x^2} \text{ linear in t}$$

$$\text{I. F.} = e^{\int\frac{x}{1+x^2}dx} = e^{\frac{1}{2}\ell n\left(1+x^2\right)} = \sqrt{1+x^2}$$

$\therefore$ solution is

$$t\sqrt{1+x^2}=\int\frac{\sqrt{1+x^2}}{1+x^2}dx+c$$

$$\frac{1}{y}\sqrt{1+x^2}=\int\frac{1}{\sqrt{1+x^2}}dx+c$$

$$\frac{1}{y}\sqrt{1+x^2}=\ell n\left(x+\sqrt{x^2+1}\right)+c$$

y (0) = 1 $\Rightarrow$ 1 = 0 + c $\Rightarrow$ c = 1

So at $x=2\sqrt{2}$ we have $\frac{3}{\beta}=\ln\left(2\sqrt{2}+3\right)+1 \Rightarrow 3\beta^{-1}=\ln\ e\left(2\sqrt{2}+3\right)$

$$\Rightarrow e^{3\beta^{-1}}=e\left(3+2\sqrt{2}\right)$$

11. Two dice A and B are rolled. Let the numbers obtained on A and B be α and β respectively. If the variance of $\alpha-\beta$ is $\frac{p}{q}$, where p and q are co-prime , then the sum of the positive divisors of p is equal to

(1) 48 (2) 36 (3) 72 (4) 31

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. Values of $(\alpha-\beta)$ are

0, – 1, – 2, – 3, – 4, – 5, 1, 0, – 1, – 2, – 3, – 4, 2, 1, 0, – 1, – 2, – 3, 3, 2, 1, 0, – 1, – 2, 4, 3, 2,1, 0,– 1, 5, 4, 3, 2, 1, 0,

$\Rightarrow$ mean = 0

and variance = $\frac{\Sigma x_i^2}{36} - (\bar{x})^2 = \frac{10.(1)^2 + 8(2)^2 + 6(3)^2 + 4(4)^2 + 2(5)^2}{36}$

$= \frac{10+32+54+64+50}{36}$

$= \frac{210}{36} = \frac{35}{6} = \frac{p}{q}$

$\Rightarrow$ p = 35

hence positive divisors of p are = 1, 5, 7, 35

Sum = 1 + 5 + 7 + 35 = 48

12. If the point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on the curve traced by the mid –points of the line segments of the lines x cos θ + ysin θ=7, $\theta \in \left(0, \frac{\pi}{2}\right)$ between the co-ordinates axes, then α is equal to

(1) $7\sqrt{3}$ (2) –7 (3) 7 (4) $-7\sqrt{3}$

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $x\cos\theta + y\sin\theta = 7$; $\theta \in \left(0, \frac{\pi}{2}\right)$

Mid point of line segment between coordinate axes

$(h, k) \equiv \left(\frac{7}{2\cos\theta}, \frac{7}{2\sin\theta}\right)$

$\Rightarrow$ Locus of (h, k) is $\frac{49}{y^2} + \frac{49}{x^2} = 4 \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{49}$

Point $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$ lies on it

$\Rightarrow \frac{1}{\alpha^2} + \frac{3}{49} = \frac{4}{49}$

$\Rightarrow \frac{1}{\alpha^2} = \frac{1}{49}$

$\Rightarrow \alpha = 7$

13. let α, β be the roots of the quadratic equation $x^2+\sqrt{6}x+3=0$ then $\dfrac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}$ is equal to

(1) 729 (2) 9 (3) 81 (4) 72

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $\alpha+\beta=-\sqrt{6}$, $\alpha\beta=3$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=0$

$\alpha^{14}+\beta^{14}=(\alpha^2)^7+(\beta^2)^7=(\alpha^2+\beta^2)\,(\ldots\ldots\ldots\ldots)=0$

$\alpha^{10}+\beta^{10}=(\alpha^2)^5+(\beta^2)^5=(\alpha^2+\beta^2)\,(\ldots\ldots\ldots\ldots)=0$

Now

$$\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\alpha^{15}+\beta^{15}+\alpha^{10}+\beta^{10}}=\frac{\alpha^{23}+\beta(-\alpha^2)^{11}+0}{\alpha^{15}+\beta(-\alpha^2)^7+0}$$

$$=\frac{\alpha^{22}(\alpha-\beta)}{\alpha^{14}(\alpha-\beta)}=\alpha^8=(-9)^2=81$$

(since $\alpha^2+3=-\sqrt{6}\alpha \Rightarrow \alpha^4=-9$)

14. The area of the region enclosed by the curve $y=x^3$ and its tangent at the point (–1, –1) is

(1) $\dfrac{31}{4}$ (2) $\dfrac{27}{4}$ (3) $\dfrac{23}{4}$ (4) $\dfrac{19}{4}$

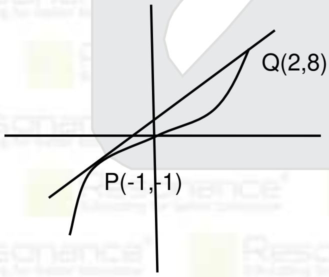
NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. Given $y=x^3$ ………………… (1)

$\Rightarrow \dfrac{dy}{dx}=3x^2$

$\left(\dfrac{dy}{dx}\right)_{(-1,-1)}=3$



Equation of tangent at (–1, –1)

$(y+1)=3(x+1)$

$y=3x+2$ ……….. (2)

Solving (1) & (2)

$x^3=3x+2$

$x^3 - 3x - 2 = 0$ with roots -1, -1, α

$-2 + \alpha = 0$

$\alpha = 2$

$Q \equiv (2, 8)$

Required area = $\int_{-1}^{2}\left(3x + 2 - x^3\right)dx$

$= \frac{3}{2}(4 - 1) + 2\,(2 + 1) - \frac{1}{4}(16 - 1)$

$= \frac{9}{2} + 6 - \frac{15}{4} = \frac{18 + 24 - 15}{4} = \frac{27}{4}$

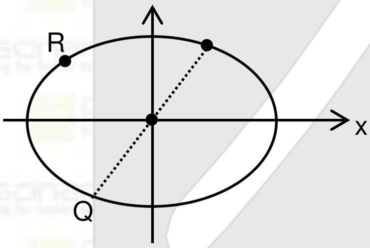
15. Let $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R, and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually Perpendicular and pass through the origin .If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, where p and q are coprime, then p + q is equal to

(1) 157 (2) 143 (3) 137 (4) 147

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. Ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$



PQ passes through origin and ellipse is symmetrical in both axes $\Rightarrow Q\left(-\frac{2\sqrt{e}}{\sqrt{7}} - \frac{6}{\sqrt{7}}\right)$

$\Rightarrow (PQ)^2 = \left(\frac{4\sqrt{3}}{\sqrt{7}}\right)^2 + \left(\frac{2 \times 6}{\sqrt{7}}\right)^2 = \frac{48 + 144}{7} = \frac{192}{7}$

Slope of PQ is $m_1 = \frac{6}{2\sqrt{3}} = \sqrt{3}$

Equation RS is $y = -\frac{1}{\sqrt{3}}x$

By solving with $9x^2 + 4y^2 = 36$ we have $x = \pm\frac{6\sqrt{3}}{\sqrt{31}}$ & $y = \mp\frac{6}{\sqrt{31}}$

$\Rightarrow (RS)^2 = \left(\frac{12\sqrt{3}}{\sqrt{31}}\right)^2 + \left(\frac{12}{\sqrt{31}}\right)^2 = \frac{576}{31}$

Hence $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{7}{192} + \frac{31}{576} = \frac{21+31}{576} = \frac{52}{576} = \frac{13}{144} = \frac{p}{q}$

$\Rightarrow$ p + q = 157 Ans.

16. Let $\lambda \in Z, \vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \vec{0}, \vec{a}\cdot\vec{c} = -17$ and $\vec{b}\cdot\vec{c} = -20$. Then $|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})|^2$ is equal to

(1) 46 (2) 49 (3) 53 (4) 62

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = 0$

$\Rightarrow (\vec{a} + \vec{b}) \times \vec{c} = \vec{0}$

$\vec{c} = \alpha(\vec{a} + \vec{b})$, α is scalar

Now $\vec{a}\cdot\vec{c} = -17$

$\therefore$ $(\lambda i + j - k) . \alpha((\lambda + 3) i + k) = -17$

$(\lambda(\lambda + 3) - 1) \alpha = -17$-----------(1)

also $\vec{b}\cdot\vec{c} = -20 \Rightarrow \alpha((\lambda + 3) i + k).(3i - j + 2k) = -20$

$\Rightarrow$ $\alpha(3(\lambda + 3) + 2) = -20$ ___________(2)

by (1) and (2) $\frac{\lambda^2 + 3\lambda - 1}{3\lambda + 11} = \frac{17}{20}$

$\Rightarrow$ $20\lambda^2 + 60\lambda - 20 = 51\lambda + 187$

$\Rightarrow$ $20\lambda^2 + 9\lambda - 207 = 0$

$\Rightarrow$ $\lambda = 3$

$\therefore$ by (2) $\alpha(20) = -20 \Rightarrow \alpha = -1$

$\vec{c} = -(6i + k)$

Now $\vec{c} \times (\lambda i + j + k) = \begin{vmatrix} i & j & k \\ -6 & 0 & -1 \\ 3 & 1 & 1 \end{vmatrix}$

$= i(0+1) - j(-6 + 3) + k(-6 - 0) = i + 3j - 6k$

$\therefore$ $|\vec{c} \times (\lambda i + j + k)|^2 = 1 + 9 + 36 = 46$

17. If the local maximum values of the function $f(x) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}, x \in \left(0, \frac{\pi}{2}\right)$, is $\frac{k}{e}$, then $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$ is equal to

(1) $e^3 + e^5 + e^{11}$ (2) $e^3 + e^6 + e^{11}$

(3) $e^5 + e^6 + e^{11}$ (4) $e^3 + e^6 + e^{10}$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $f(x) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x} : x \in \left(0, \frac{\pi}{2}\right) \Rightarrow \ell nf(x) = \sin^2 x \ell n\left(\frac{\sqrt{3e}}{2\sin x}\right)$

$= \sin^2 x\, [\, [\ell n\sqrt{3e} - \ell n(2\sin x)]$

$\Rightarrow \frac{f'(x)}{f(x)} = \sin 2x[\ell n\sqrt{3e} - \ell n(2\sin x))] + \sin^2 x(0 - \cot x)$

$\Rightarrow f'(x) = f(x)[\sin 2x[\ell n\sqrt{3e} - \ell n(2\sin x)) - \sin x\cos x]$

$= \sin 2x.f(x)\left(\ell n\sqrt{3e} - \ell n(2\sin x) - \frac{1}{2}\right)$

$f'(x) = 0 \Rightarrow \ell n\left(\frac{\sqrt{3e}}{2\sin x}\right) = \frac{1}{2}$

$\Rightarrow \frac{\sqrt{3e}}{2\sin x} = \sqrt{e} \Rightarrow \sin x = \frac{\sqrt{3}}{2} \Rightarrow x = 60°$

Local maximum at $x = \frac{\pi}{3}$

$f\left(\frac{\pi}{3}\right) = \left(\frac{\sqrt{3e}}{\sqrt{3}}\right)^{\frac{3}{4}} = (e)^{3/8} = \frac{k}{e} \Rightarrow k = e^{11/8}$

Hence $\left(\frac{k}{e}\right)^8 + \frac{k}{e^5}^8 + k^8 = k^8\left(\frac{1+e^3+e^8}{e^8}\right) = e^{11}\left(\frac{1+e^3+e^8}{e^8}\right)$

$= e^3 + e^6 + e^{11}$

18. Let the plane P: $4x - y + z = 10$ be rotated by an angle $\frac{\pi}{2}$ about its line of intersection with the plane $x+y-z=4$. If α is the distance of the point (2, 3,-4) from the new position of the plane P, then 35α is equal to

(1) 105 (2) 90 (3) 126 (4) 85

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. By family of plane P_1 : $(4x - y + z - 10) + \lambda (x + y - z - 4) = 0$

$(4 + \lambda) x + (-1 + \lambda) \gamma + (1 - \lambda)z - (10 + 4\lambda) = 0$

angle between given plane P and P_1 is $\frac{\pi}{2}$

$\Rightarrow 4 (4 + \lambda) - 1 (-1 + \lambda) + 1 (1 - \lambda) = 0$

$\Rightarrow 2\lambda + 18 = 0 \Rightarrow \lambda = -9$

Hence new plane i.e.

$x + 2y - 2z = \frac{26}{5}$

distance of P_1 from (2, 3, –4) is

$\alpha = \left|\frac{2+6+8-\frac{26}{5}}{3}\right| = \frac{18}{5} \Rightarrow 35\alpha = 126$

19. Let the lines $\ell_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and ℓ_2 : 3x+2y+z–2=0=x–3y+2z–13 be coplanar. If the point P(a, b, c) on l_1 is nearest to the point Q(-4, -3, 2), then $|a|+|b|+|c|$ is equal to

(1) 12 (2) 10 (3) 14 (4) 8

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $\ell_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$

And ℓ_2 : 3x + 2y + z – 2 = 0 = x – 3y + 2z – 13 are coplanar

⇒ D' ratio of ℓ_2 and a,b,c such that

$$\frac{a}{\begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 3 & 2 \\ 1 & -3 \end{vmatrix}} =$$

$$\frac{a}{7} = \frac{b}{-5} = \frac{c}{-11}$$

And ℓ_2 passes through $\left(\frac{32}{11}, -\frac{37}{11}, 0\right)$

ℓ_1 and ℓ_2 are coplanes so

$$\begin{vmatrix} \frac{32}{11}+5 & -\frac{37}{11}+4 & 0-\alpha \\ 3 & 1 & -2 \\ 7 & -5 & -11 \end{vmatrix} = 0$$

$$\Rightarrow \frac{87}{11}(-11-10) - \frac{7}{11}(-19) - \alpha(-22) = 0$$

$$\Rightarrow -\frac{21 \times 87}{11} + \frac{133}{11} + 22\alpha = 0$$

$$\Rightarrow 22\alpha = \frac{1694}{11} \Rightarrow \alpha = 7$$

Let point P(a,b,c) $\equiv (3\lambda-5, \lambda-4, -2\lambda+7)$

PQ^2 $(3\lambda-1)^2 + (\lambda-1)^2 + (-2\lambda+5)^2 = 14\lambda^2 - 28\lambda + 27$

$= 14(\lambda^2 - 2\lambda + 1) + 13$

PQ is minimum if $\lambda = +1$

⇒ p(a,b,c) $\equiv$ (–2, –3, 5)

⇒|a| + |b| + |c| = 10

20. The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to

(1) 96 (2) 72 (3) 132 (4) 120

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol.

$10{,}000^{th}$	1000^{th}	100^{th}	10^{th}	unit		
3	4	3	2	0	=	72
2	4	3	2	5	=	48

Total = 120

21. If $\int_{-0.15}^{0.15} |100x^2 - 1|\, dx = \frac{k}{3000}$, then k is equal to ______

NTA Ans. **(575)**
Reso Ans. **(575)**

Sol. $I = \int_{-0.15}^{0.15} |100x^2 - 1| dx = 2\int_0^{0.15} |100x^2 - 1| dx$

$= 2\left(\int_0^{0.1} (1 - 100x^2) dx + \int_{0.1}^{0.15} (100x^2 - 1) dx\right)$

$= 2\left(\left(x - \frac{100}{3}.x^3\right)_0^{0.1} + \left(\frac{100}{3}.x^3 - x\right)_{0.1}^{0.15}\right)$

$= 4\left(0.1 - \frac{1}{30}\right) + 2\left(\frac{100}{3}(0.15)^3 - 0.15\right) = \frac{8}{30} + 2\,(0.1125 - 0.15)$

$= \frac{8}{30} - 0.075 = \frac{8 - 2.25}{30} = \frac{5.75}{30} = \frac{575}{3000}$

22. Let the digits a, b, c be in A. P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

NTA Ans. **(1260)**
Reso Ans. **(1260)**

Sol. Three consecutive digits can be abc or cba → 2 ways

and these digits can placed at (1,2,3), (2,3,4), (3,4,5) ……… (7,8,9) → 7 ways

so number of such numbers = $2 \times 7 \times \frac{\lfloor 6}{\lfloor 2 \lfloor 2 \lfloor 2} = 1260$

23. A fair n (n> 1) faces die is rolled repeatedly until a number less than appears. If the mean of the number of tosses required is $\frac{n}{9}$, then n is equal to______

NTA Ans. **(10)**
Reso Ans. **(10)**

Sol.. Let in r^{th} thrown, L^{st} number appears which is smaller than n. (where r ≥ 1)

hence mean = $\sum_{r=1}^{\infty} r.\left(\frac{n-1}{n}\right)\left(\frac{1}{n}\right)^{r-1} = \frac{n-1}{n}\sum_{r=1}^{\infty} r.\left(\frac{1}{n}\right)^{r-1}$

$= \frac{n-1}{n}\left[1 + 2x + 3x^2 + 4x^3 + ...\right]$ where $x = \frac{1}{n}$

$= \frac{n-1}{n}\frac{1}{(1-x)^2} = \frac{n-1}{n}\left(\frac{n^2}{(n-1)^2}\right) = \frac{n}{n-1}$

But given mean = $\frac{n}{9} \Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

24. The number of relations, on the set {1, 2, 3} containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is______

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. R is reflexive $\Rightarrow$ R have (1, 1), (2, 2), (3, 3)

R is transitive

$\because$ (1, 2), (2, 3) $\in R$ $\quad\therefore$ (1, 3) $\in R$

$\therefore R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$

Clearly R_1 is reflexive & transitive but not symmetric

Similarly

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (3, 2)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)\}$

$R_4 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (3, 1), (2, 1)\}$

So four relations

25. Let the plane x + 3y–2z+6=0 meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is $\left(\alpha,\beta,\frac{6}{7}\right)$ then 98 $(\alpha + \beta)^2$ is equal to______

NTA Ans. **(288)**

Reso Ans. **(288)**

Sol. Coordinates of A(–6, 0, 0) ; B(0, –2, 0) & C(0, 0, 3) orthocentre of ABC is H$(\alpha,\beta,\frac{6}{7})$

Now $\overrightarrow{AH} \perp \overrightarrow{BC}$: $\overrightarrow{BH} \perp \overrightarrow{AC}$ & $\overrightarrow{CH} \perp \overrightarrow{AB}$

$\Rightarrow 0(\alpha+6) + 2(\beta - 0) + 3\left(\frac{6}{7}\right) = 0$

& $6(\alpha) + 0(\beta + 2) + 3\left(\frac{6}{7}\right) = 0$

& $6(\alpha) - 2(\beta) + 3\left(\frac{6}{7} - 3\right) = 0$

$\beta = -\frac{9}{7}$ & $\alpha = -\frac{3}{7}$

Hence $98(\alpha+\beta)^2 = 98\left(-\frac{12}{7}\right)^2 = 2 \times 144 = 288$

26. Let $I(x) = \int\sqrt{\frac{x+7}{x}}dx$ and $I(9) = 12 + 7\log_e 7$. If $I(1) = \alpha + 7\log_e\left(1+2\sqrt{2}\right)$ then α^4 is equal to ______

NTA Ans. **(64)**

Reso Ans. **(64)**

Sol. $I(x) = \int\sqrt{\frac{x+7}{x}}\ dx$

$= \int\frac{\sqrt{\left(\sqrt{x}\right)^2+\left(\sqrt{7}\right)^2}}{\sqrt{x}}\ dx$

Let $\sqrt{x} = t \therefore \frac{1}{2\sqrt{x}} dx = dt$

$= 2\int\sqrt{t^2+(\sqrt{7})^2}\,dt = 2\left(\frac{t}{2}\sqrt{t^2+7}+\frac{7}{2}\ell n\left(t+\sqrt{t^2+7}\right)\right)+c$

$I(x) = \sqrt{x}\sqrt{x+7} + 7\,\ell n\left(\sqrt{x}+\sqrt{x+7}\right) + c$

$I(9) = 12 + 7\ell n7 = 12 + 7\ell n(3 + 4) + c$

$\Rightarrow c = 0$

$\therefore I(x) = \sqrt{x}\sqrt{x+7} + 7\ell n\left(\sqrt{x}+\sqrt{x+7}\right)$

at x = 1

$I(1) = \sqrt{8} + 7\ell n(1+\sqrt{8}) = 7\ell n(1+2\sqrt{2}) + \alpha$

$\alpha = \sqrt{8}$

$\alpha^4 = 64$

27. Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n} D_k = 96$, then n is equal to

NTA Ans. (6)

Reso Ans. (6)

Sol. $\sum_{k=1}^{n} D_k = \begin{vmatrix} \Sigma 1 & \Sigma 2k & \Sigma(2k-1) \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$

By $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

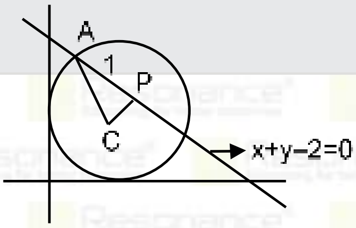
$= \begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 2n(n+2) = 96 \Rightarrow n = 6, -8$ but $n \in N \Rightarrow n = 6$

28. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line x + y = 2. Then $r_1^2 + r_2^2 - r_1r_2$ is equal to

NTA Ans. (7)

Reso Ans. (7)

Sol.



Let equation of circle is

$(x-r)^2 + (y-r)^2 = r^2$

$CP = \frac{|r+r-2|}{\sqrt{2}} = \sqrt{r^2-1}$

$\left(\sqrt{2}(r-1)\right)^2 = r^2-1$

$r^2 - 4r + 3 = 0$

$r_1 = 1, r_2 = 3$

$r_1^2 + r_2^2 - r_1r_2 = 1+9-3 = 7$

29. Let [x] be the greatest integer ≤ x. Then the number of points in the interval (–2, 1) , where the function $f(x) = |[x]| + \sqrt{x - [x]}$ is discontinuous is

NTA Ans. (2)

Reso Ans. (2)

Sol. $f(x) = |[x]| + \sqrt{x - [x]}$, $x \in [-2, 1]$

$$f(x) = \begin{cases} 2 + \sqrt{x+2} & -2 \le x < -1 \\ 1 + \sqrt{x+1} & -1 \le x < 0 \\ \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \end{cases}$$

f(x) is dc at x = –1, 0,

Clearly number of discontinuity points = 2

30. Let the positive numbers a_1 , a_2, a_3, a_4 and a_5 be in G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively where m and n are co-prime. If the mean of their reciprocals is $\frac{31}{40}$ and $a_3 + a_4 + a_5 = 14$, then m + n is equal to

NTA Ans. (211)

Reso. Ans. (211)

Sol. Let a, ar, ar^2, ar^3, ar^4

gives $\frac{a + ar + ar^2 + ar^3 + ar^4}{5} = \frac{31}{10}$ —— (1)

$\Rightarrow a(1 + r + r^2 + r^3 + r^4) = \frac{31}{2}$ —— (1)

also $\frac{\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4}}{5} = \frac{31}{40}$ —— (2)

$\therefore$ by (1) & (2) $a^2r^4 = 4 \Rightarrow ar^2 = 2$

$ar^2 = 2$ by (1) $r = 2, a = \frac{1}{2}$ or $r = \frac{1}{2}$ and $a = 8$

$\therefore$ Number are $\frac{1}{2}$, 1, 2, 4, 8 or 8, 4, 2, 1 , $\frac{1}{2}$ (rejected $a_3 + a_4 + a_5 = 14$)

$$\sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2 = \frac{\frac{1}{4} + 1 + 4 + 16 + 64}{5} - \left(\frac{31}{10}\right)^2$$

$= \frac{341}{20} - \frac{961}{100}$

$= \frac{186}{25}$

$\therefore m + n = 211$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