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**JEE**  
**(Main)**

**PAPER-1 (B.E./B. TECH.)**

**2023**

**COMPUTER BASED TEST (CBT)**  
**Questions & Solutions**

**Date: 13 April, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)**

**Duration: 3 Hours | Max. Marks: 300**


**SUBJECT: MATHEMATICS**

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**PART : MATHEMATICS**

1. Let  $y = y_1(x)$  and  $y = y_2(x)$  be the solution curves of the differential equation  $\frac{dy}{dx} = y + 7$  with initial conditions  $y_1(0) = 0$  and  $y_2(0) = 1$  respectively. Then the curves  $y = y_1(x)$  and  $y = y_2(x)$  intersect at  
 (1) one point      (2) no point      (3) infinite number of points      (4) two points

NTA Ans. (2)

Reso Ans. (2)

Sol.  $\frac{dy}{y+7} = dx$

$\ln(y+7) = x + C$

$\Rightarrow y + 7 = e^{x+C}$

$\Rightarrow y = \lambda e^x - 7$

Let  $y_1(x) = \lambda_1 e^x - 7$  &  $y_2(x) = \lambda_2 e^x - 7$

As  $y_1(0) = 0 \Rightarrow \lambda = 7$  &  $y_2(0) = 1 \Rightarrow \lambda_2 = 8$

$\Rightarrow y_1(x) = 7e^x - 7$  &  $y_2(x) = 8e^x - 7$

Hence point of intersection of both curves, by solving

$7e^x - 7 = 8e^x - 7$

$\Rightarrow e^x = 0 \quad \Rightarrow$  No solution

2. Let the tangent and normal at the point  $(3\sqrt{3}, 1)$  on the ellipse  $\frac{x^2}{36} + \frac{y^2}{4} = 1$  meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line  $x = 2\sqrt{5}$  intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point  $(\alpha, \beta)$ , then  $\alpha^2 - \beta^2$  is equal to

- (1) 60      (2)  $\frac{304}{5}$       (3)  $\frac{314}{5}$       (4) 61

NTA Ans. (2)

Reso Ans. (2)

Sol.  $\frac{x^2}{36} + \frac{y^2}{4} = 1$

equation of tangent at point  $(3\sqrt{3}, 1)$  is

$\frac{x \cdot 3\sqrt{3}}{36} + \frac{y \cdot 1}{4} = 1$

Point of intersection with y-axis is A (0, 4)

Equation of normal at point  $(3\sqrt{3}, 1)$  is

$\frac{x \cdot 36}{3\sqrt{3}} - \frac{y \cdot 4}{1} = 36 - 4 = 32$

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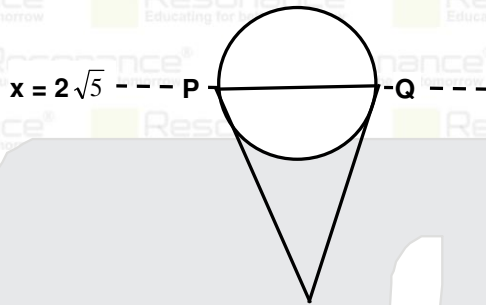
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point of intersection with y-axis is B (0, -8)

So, A(0, 4) and B(0, -8) are end points of diameter of circle C. Equation of circle C is

$$(x - 0)(x - 0) + (y - 4)(y + 8) = 0$$

$$\Rightarrow x^2 + y^2 + 4y - 32 = 0$$



$(\alpha, \beta)$

Equation of chord of contact of circle C with respect to point  $(\alpha, \beta)$  is

$$\alpha x + \beta y + 2(y + \beta) - 32 = 0$$

$$\Rightarrow \alpha x + (\beta + 2)y + 2\beta - 32 = 0$$

it is same as  $x = 2\sqrt{5}$

$$\text{So } \beta + 2 = 0 \quad \& \quad \frac{32 - 2\beta}{\alpha} = 2\sqrt{5} \Rightarrow \alpha = \frac{18}{\sqrt{5}}$$

$$\text{So, } (\alpha^2 - \beta^2) = \left( \frac{324}{5} - 4 \right) = \frac{304}{5}$$

3. The negation of the statement  $((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$  is

(1) a fallacy

(2) equivalent to  $B \vee \sim C$

(3) equivalent to  $\sim C$

(4) equivalent to  $\sim A$

NTA Ans. (4)

Reso Ans. (4)

Sol. Given statement

$$p : ((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$$

$$\equiv (\sim (A \wedge (B \vee C)) \vee (A \vee B)) \Rightarrow A$$

$$\equiv ((A \wedge (B \vee C)) \wedge \sim (A \vee B)) \vee A$$

$$\equiv (f \vee A)$$

$$\equiv A$$

$$\text{Now } \sim p \equiv \sim A$$

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4. For the system of linear equations

$$2x + 4y + 2az = b$$

$$x + 2y + 3z = 4$$

$$2x - 5y + 2z = 8$$

Which of following is NOT correct?

- (1) It has unique solution if  $a = b = 6$
- (2) It has infinitely many solutions if  $a = 3, b = 8$
- (3) It has unique solution if  $a = b = 8$
- (4) It has infinitely many solutions if  $a = 3, b = 6$

NTA Ans. (4)

Reso Ans. (4)

Sol.  $D = \begin{vmatrix} 2 & 4 & 2a \\ 1 & 2 & 3 \\ 2 & -5 & 2 \end{vmatrix}$

$$= 2(4 + 15) - 4(2 - 6) + 2a(-5 - 4)$$

$$= 38 + 16 - 18a = 54 - 18a = 18(3 - a)$$

$$D_1 = \begin{vmatrix} b & 4 & 2a \\ 4 & 2 & 3 \\ 8 & -5 & 2 \end{vmatrix}$$

$$= b(4 + 15) - 4(8 - 24) + 2a(-20 - 16)$$

$$= 19b + 64 - 72a$$

$$= 19b - 72a + 64$$

$$D_2 = \begin{vmatrix} 2 & b & 2a \\ 1 & 4 & 3 \\ 2 & 8 & 2 \end{vmatrix}$$

$$= 2(8 - 24) - b(2 - 6) + 2a(8 - 8)$$

$$= -32 + 4b = 4(b - 8)$$

$$D_3 = \begin{vmatrix} 2 & 4 & b \\ 1 & 2 & 4 \\ 2 & -5 & 8 \end{vmatrix}$$

$$= 2(16 + 20) - 4(8 - 8) + b(-5 - 4)$$

$$= 72 - 9b = 9(8 - b)$$

Now when  $a \neq 3, b \in \mathbb{R}$  system have unique solution .

when  $a = 3, b \neq 8$ , system have no solution

when  $a = 3, b = 8$ , system have infinitely many solutions.

5. The distance of the point  $(-1, 2, 3)$  from the plane  $\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 10$  parallel to the line of the shortest distance between the lines  $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$  and  $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$  is

- (1)  $3\sqrt{6}$
- (2)  $3\sqrt{5}$
- (3)  $2\sqrt{5}$
- (4)  $2\sqrt{6}$

NTA Ans. (4)

Reso Ans. (4)

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**Sol.** Let DR's of line of shortest distance between given lines  $L_1$  and  $L_2$  are  $a, b, c$

$$\text{then } a\hat{i} + b\hat{j} + c\hat{k} = t \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix} = t(\hat{i} - \hat{j} - 2\hat{k})$$

Now equation of line passes through A  $(-1, 2, 3)$  and DR's are  $(1, -1, -2)$

$$\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k})$$

$$\vec{r} = (\lambda - 1)\hat{i} + (2 - \lambda)\hat{j} + (3 - 2\lambda)\hat{k} \quad \dots\dots(1)$$

point of intersection of line (1) and plane  $\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 10 \quad \dots\dots(2)$

$$(\lambda - 1) - 2(2 - \lambda) + 3(3 - 2\lambda) = 10$$

$$\Rightarrow 4 - 3\lambda = 10$$

$$\Rightarrow \lambda = -2$$

So, B  $(-3, 4, 7)$

$$\text{Distance AB} = \sqrt{4 + 4 + 16} = \sqrt{24} = 2\sqrt{6}$$

**6.** Among

$$(S1): \lim_{n \rightarrow \infty} \frac{1}{n^2} (2+4+6 + \dots + 2n) = 1$$

$$(S2): \lim_{n \rightarrow \infty} \frac{1}{n^{16}} (1^{15} + 2^{15} + 3^{15} + \dots + n^{15}) = \frac{1}{16}$$

- (1) Both (S1) and (S2) are false
- (2) Only (S2) is true
- (3) Both (S1) and (S2) are true
- (4) Only (S1) is true

**NTA Ans.** (3)

**Reso Ans.** (3)

**Sol.** S-1  $\lim_{n \rightarrow \infty} \frac{1}{n^2} (2 + 4 + 6 + \dots + 2n)$

$$\lim_{n \rightarrow \infty} \frac{n(n+1)}{n^2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) = 1$$

So Statement-1 is true

S-2  $\lim_{n \rightarrow \infty} \frac{1}{n} \left( \frac{1^{15} + 2^{15} + \dots + n^{15}}{n^{15}} \right)$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{r}{n}\right)^{15} \cdot \frac{1}{n}$$

$$= \int_0^1 x^{15} dx = \frac{x^{16}}{16} \Big|_0^1 = \frac{1}{16}$$

So Statement-2 is also true

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7. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is

(1)  $\frac{81}{64}$       (2)  $\frac{15}{16}$       (3)  $\frac{37}{16}$       (4)  $\frac{21}{16}$

NTA Ans. (4)

Reso Ans. (4)

Sol. Clearly  $P(H) = \frac{3}{4}$ ,  $P(T) = \frac{1}{4}$

Since coin is tossed until a head or three tails occur. So process will be end in the maximum 3 thrown so probability distribution is

$x_i$	1	2	3
$p_i$	$\frac{3}{4}$	$\frac{1}{4} \cdot \frac{3}{4}$	$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$

Hence mean =  $\sum p_i x_i = 1\left(\frac{3}{4}\right) + 2\left(\frac{3}{16}\right) + 3\left(\frac{1}{16}\right) = \frac{12+6+3}{16} = \frac{21}{16}$

8. Let  $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$ ,  $\alpha > 2$  be the adjoint of a matrix A and  $|A| = 2$ . Then  $[\alpha \ -2\alpha \ \alpha] B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$  is equal to

(1) 16      (2) 0      (3) -16      (4) 32

NTA Ans. (3)

Reso Ans. (3)

Sol.  $|B| = |\text{adj } A| = |A|^{3-1} = |A|^2 = 4$

$1(8 - 3\alpha) - 3(4 - 3\alpha) + \alpha(\alpha - 2\alpha) = 4$

$\Rightarrow 8 - 3\alpha - 12 + 9\alpha - \alpha^2 = 4$

$\Rightarrow \alpha^2 - 6\alpha + 8 = 0$

$\Rightarrow \alpha = 2, 4$

$\Rightarrow \alpha = 4$  ( $\because \alpha > 2$ )

Now

$[4 \ -8 \ 4] \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix}$

$= 16[1 \ -2 \ 1] \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$= 16[1-2+4 \ 3-4+4 \ 4-6+4] \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$= 16[3-6+2]$

$= 16[-1] = [-16]$

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9. The set of all  $\alpha \in \mathbb{R}$  for which the equation  $x|x-1| + |x+2| + \alpha = 0$  has exactly one real root, is

- (1)  $(-\infty, \infty)$       (2)  $(-6, -3)$       (3)  $(-6, \infty)$       (4)  $(-\infty, -3)$

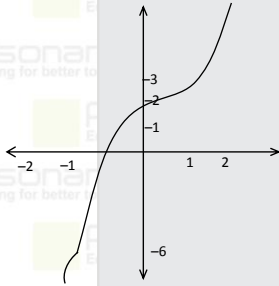
NTA Ans. (1)

Reso Ans. (1)

Sol.  $x|x-1| + |x-2| + \alpha = 0$

$$x|x-1| + |x-2| = -\alpha$$

$$y = x|x-1| + |x+2| = \begin{cases} -x^2 - 2 & x < -2 \\ -x^2 + 2x + 2 & -2 \leq x < 1 \\ x^2 + 2 & x \geq 1 \end{cases}$$



so by graph, for exactly one real root  $\alpha$  can be  $(-\infty, \infty)$

10.  $\int_0^{\infty} \frac{6}{e^{3x} + 6e^{2x} + 11e^x + 6} dx =$

- (1)  $\log_e \left( \frac{256}{81} \right)$       (2)  $\log_e \left( \frac{64}{27} \right)$       (3)  $\log_e \left( \frac{512}{81} \right)$       (4)  $\log_e \left( \frac{32}{27} \right)$

NTA Ans. (4)

Reso Ans. (4)

Sol. Let  $e^x = t \Rightarrow e^x dx = dt$

$$I = \int_1^{\infty} \frac{6}{t(t+1)(t+2)(t+3)} dt$$

$$\Rightarrow I = 6 \int_1^{\infty} \left\{ \frac{A}{t} + \frac{B}{t+1} + \frac{C}{t+2} + \frac{D}{t+3} \right\} dt$$

$$\Rightarrow I = 6 \int_1^{\infty} \left( \frac{1}{6t} + \frac{1}{(-2)(t+1)} + \frac{1}{2(t+2)} - \frac{1}{6(t+3)} \right) dt$$

$$\Rightarrow I = \left[ \ln t - 3 \ln(t+1) + 3 \ln(t+2) - \ln(t+3) \right]_1^{\infty}$$

$$\Rightarrow I = \left[ \ln \frac{t(t+2)^3}{(t+1)^3(t+3)} \right]_1^{\infty}$$

$$\Rightarrow I = 0 - \ln \frac{3^3 \cdot 4}{2^3 \cdot 6} = \ln \frac{32}{27}$$

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11. For  $x \in \mathbb{R}$ , two real valued functions  $f(x)$  and  $g(x)$  are such that,  $g(x) = \sqrt{x} + 1$  and  $fog(x) = x + 3 - \sqrt{x}$ . Then  $f(0)$  is equal  
 (1) 1 (2) 0 (3) 5 (4) -3

NTA Ans. (3)

Reso Ans. (Bonus)

Sol.  $f(g(x)) = x - \sqrt{x} + 3$

$$f(\sqrt{x} + 1) = x - \sqrt{x} + 3 \quad \dots (1)$$

Let  $\sqrt{x} + 1 = t$ ,  $x \in \mathbb{R}$ , so  $t \geq 1$

$$x = (t - 1)^2$$

$$\text{So } f(t) = (t - 1)^2 - \sqrt{(t - 1)^2} + 3$$

$$f(t) = (t - 1)^2 - (t - 1) + 3 \quad \dots (2)$$

$$f(0) = 1 + 1 + 3 = 5$$

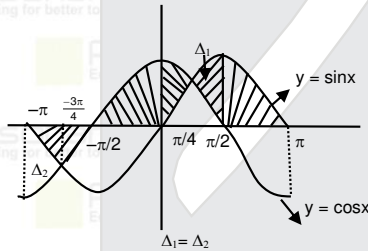
But here  $t \geq 1$ , So  $f(0)$  is not defined.

12. The area of the region enclosed by the curve  $f(x) = \max \{\sin x, \cos x\}$ ,  $-\pi \leq x \leq \pi$  and the x-axis is  
 (1)  $2(\sqrt{2} + 1)$  (2)  $4(\sqrt{2})$  (3) 4 (4)  $2\sqrt{2}(\sqrt{2} + 1)$

NTA Ans. (3)

Reso Ans. (3)

Sol.



$$\int_0^{\pi/2} \sin x \cdot dx = 1$$

$$\text{Required area} = 2 + 1 + 1 + \Delta_1 - \Delta_2 = 4$$

13. Let PQ be a focal chord of the parabola  $y^2 = 36x$  of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that  $PM:MQ = 3:1$ . Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ?

- (1) (3, 33) (2) (6, 29) (3) (-3, 43) (4) (-6, 45)

NTA Ans. (3)

Reso Ans. (3)

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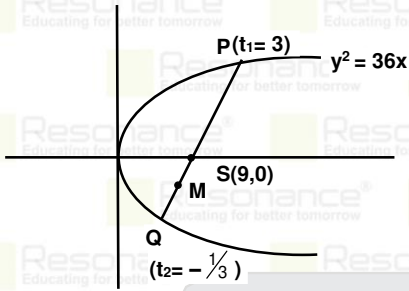
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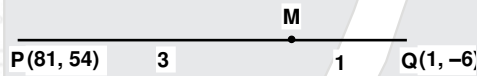
Sol.



$$\text{Length of focal chord PQ} = 9 \left( t_1 + \frac{1}{t_1} \right)^2 = 100$$

$$t_1 + \frac{1}{t_1} = \pm \frac{10}{3}$$

$$t_1 = 3 \text{ or } \frac{1}{3}$$



$$M = (21, 9)$$

Equation of line passing through M and perpendicular to line PQ is

$$y - 9 = -\frac{80}{60}(x - 21) \Rightarrow 4x + 3y = 111$$

point  $(-3, 43)$  does not satisfy this line

14. Fractional part of the number  $\frac{4^{2022}}{15}$  is equal to

(1)  $\frac{4}{15}$

(2)  $\frac{1}{15}$

(3)  $\frac{14}{15}$

(4)  $\frac{8}{15}$

NTA Ans. (2)

Reso Ans. (2)

Sol.  $\frac{4^{2022}}{15} = \frac{16^{1011}}{15} = \frac{(15+1)^{1011}}{15} = \left( \frac{15\lambda + 1}{15} \right)$

Hence fractional part is  $\frac{1}{15}$

15. Let the equation of plane passing through the line of intersection of the planes  $x+2y+az=2$  and  $x-y+z=3$  be  $5x-11y+bz=6a-1$ . For  $c \in \mathbb{Z}$ , if the distance of this plane from the point  $(a, -c, c)$  is  $\frac{2}{\sqrt{a}}$  then  $\frac{a+b}{c}$

is equal to

(1) 4

(2) -4

(3) 2

(4) -2

NTA Ans. (2)

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Reso Ans. (2)

Sol. Equation of plane passing through line of intersection of the planes 1 and 2 is

$$x + 2y + az - 2 + \lambda(x - y + z - 3) = 0$$

$$(1+\lambda)x + (2-\lambda)y + (a+\lambda)z - (2+3\lambda) = 0 \dots(1)$$

$$5x - 11y + bz - (6a-1) = 0 \dots(2)$$

comparing (1) and (2)

$$\frac{1+\lambda}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$$

$$-11 - 11\lambda = 10 - 5\lambda$$

$$6\lambda = -21$$

$$\lambda = -\frac{7}{2}$$

$$1 - \frac{7}{2} = \frac{a - \frac{7}{2}}{b} = \frac{2 - \frac{21}{2}}{6a - 1}$$

$$-\frac{1}{2} = \frac{2a - 7}{2b} = \frac{-17}{2(6a - 1)}$$

$$\therefore a = 3, b = 1$$

equation of plane is  $5x - 11y + z - 17 = 0$

Distance of point  $(3, -c, c)$  from above plane is

$$\frac{|15 + 11c + c - 17|}{\sqrt{25 + 121 + 1}} = \frac{2}{\sqrt{3}}$$

$$\frac{|12c - 2|}{\sqrt{147}} = \frac{2}{\sqrt{3}}$$

$$(6c - 1)^2 = \frac{147}{3}$$

$$(6c - 1)^2 = 49$$

$$6c - 1 = \pm 7$$

$$c = \frac{4}{3}, -1 \quad (\because c \in \mathbb{Z})$$

$$\text{So } c = -1$$

$$\text{Now } \frac{a+b}{c} = \frac{3+1}{-1} = -4$$

16. For the differentiable function  $f: \mathbb{R} - \{0\} \rightarrow \mathbb{R}$ , let  $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$  then  $\left|f(3) + f'\left(\frac{1}{4}\right)\right|$  is equal to

- (1) 13      (2) 7      (3)  $\frac{29}{5}$       (4)  $\frac{33}{5}$

NTA Ans. (1)

Reso Ans. (1)

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Sol.  $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$

Replace  $x$  by  $\frac{1}{x}$

$$3f\left(\frac{1}{x}\right) + 2f(x) = x - 10$$

Now  $9f(x) + 6f\left(\frac{1}{x}\right) = \frac{3}{x} - 30$

$$4f(x) + 6f\left(\frac{1}{x}\right) = 2x - 20$$

$$5f(x) = \frac{3}{x} - 2x - 10$$

$$f(x) = \frac{3}{5x} - \frac{2x}{5} - 2$$

$$f'(x) = -\frac{3}{5x^2} - \frac{2}{5}$$

$$f(3) = \frac{1}{5} - \frac{6}{5} - 2 = -3$$

$$f'\left(\frac{1}{4}\right) = \frac{-48}{5} - \frac{2}{5} = -10$$

$$\left|f(3) + f'\left(\frac{1}{4}\right)\right| = 13$$

17. Let  $s_1, s_2, s_3, \dots, s_{10}$  respectively be the sum to 12 terms of 10 A.P. s whose first terms are 1, 2, 3, ..... 10 and the common differences are 1, 3, 5, ....., 19 respectively. Then  $\sum_{i=1}^{10} s_i$  is equal to
- (1) 7360                      (2) 7380                      (3) 7260                      (4) 7220

NTA Ans. (3)

Reso Ans. (3)

Sol.  $1 \leq r \leq 10$

$$S_r = \frac{12}{2} [2r + (12-1)(2r-1)] = 6(2r + 22r - 11)$$

$$= 6(24r - 11)$$

$$\sum_{r=1}^{10} S_r = 6 \sum_{r=1}^{10} (24r - 11) = 6 \times \frac{10}{2} [24 - 11 + 240 - 11]$$

$$= 30 [242] = 7260$$

18. Let  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ . If a vector  $\vec{d}$  satisfies

$\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$  and  $\vec{d} \cdot \vec{a} = 24$  then  $|\vec{d}|^2$  is equal to

- (1) 323                      (2) 423                      (3) 313                      (4) 413

NTA Ans. (4)

Reso Ans. (4)

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**Sol.**  $\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b} = 0 \Rightarrow (\vec{d} - \vec{c}) \cdot \vec{b} = 0$

So  $\vec{d} - \vec{c} = \lambda \vec{b} \Rightarrow \vec{d} = \vec{c} + \lambda \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{d} = \vec{a} \cdot \vec{c} + \lambda \vec{a} \cdot \vec{b}$

$\Rightarrow 24 = (2 - 4 + 8) + \lambda (3 - 8 + 14)$

$\Rightarrow 24 = 6 + 9\lambda \Rightarrow 9\lambda = 18 \Rightarrow \lambda = 2$

So  $\vec{d} = \vec{c} + 2\vec{b}$

$\Rightarrow \vec{d} = (2\hat{i} - 4\hat{j} + 4\hat{k}) + 6\hat{i} - 4\hat{j} + 14\hat{k}$

$\Rightarrow \vec{d} = 8\hat{i} - 5\hat{j} + 18\hat{k}$

So  $|\vec{d}|^2 = 64 + 25 + 324 = 413$

19. The number of symmetric matrices of order 3, with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, is

(1)  $6^{10}$

(2)  $10^9$

(3)  $10^6$

(4)  $9^{10}$

**NTA Ans.** (3)

**Reso Ans.** (3)

**Sol.**  $A = \begin{bmatrix} a & \alpha & \beta \\ \alpha & b & \gamma \\ \beta & \gamma & c \end{bmatrix}$

a, b, c,  $\alpha, \beta, \gamma$  each has 10 options

So number of symmetric matrices =  $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$

20.  $\max_{0 \leq x \leq \pi} \left\{ x - 2\sin x \cos x + \frac{1}{3} \sin 3x \right\} =$

(1)  $\pi$

(2)  $\frac{5\pi + 2 + 3\sqrt{3}}{6}$

(3)  $\frac{\pi + 2 - 3\sqrt{3}}{6}$

(4) 0

**NTA Ans.** (2)

**Reso Ans.** (2)

**Sol.**  $f(x) = x - 2\sin x \cos x + \frac{1}{3} \sin 3x$

$f'(x) = 1 - 2\cos 2x + \cos 3x$

$f''(x) = 4\sin 2x - 3\sin 3x$

$f'(x) = 0$

$1 - 2\cos 2x + \cos 3x = 0$

$1 - 2(2\cos^2 x - 1) + 4\cos^3 x - 3\cos x = 0$

$4\cos^3 x - 4\cos^3 x - 3\cos x + 3 = 0$

$4\cos^3 x (\cos x - 1) - 3(\cos x - 1) = 0$

$(2\cos x + \sqrt{3})(2\cos x - \sqrt{3})(\cos x - 1) = 0$

$\cos x = -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 1$

$x = \frac{5\pi}{6}, \frac{\pi}{6}, 0$

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$$f''\left(\frac{5\pi}{6}\right) = 4\sin\frac{5\pi}{3} - 3\sin\frac{5\pi}{2} = -2\sqrt{3} - 3 < 0$$

$$f''\left(\frac{\pi}{6}\right) = 4\sin\frac{\pi}{3} - 3\sin\frac{\pi}{2} = 2\sqrt{3} - 3 > 0$$

$$f''(0) = 0$$

So  $x = \frac{5\pi}{6}$  is a local maxima point

$$\text{Maximum value of } f(x) \text{ is } f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3} = \frac{5\pi + 2 + 3\sqrt{3}}{6}$$

21. Let for  $x \in \mathbb{R}$ ,  $S_0(x) = x$ ,  $S_k(x) = C_k x + k \int_0^x S_{k-1}(t) dt$ , where  $C_0 = 1$ ,  $C_k = 1 - \int_0^1 S_{k-1}(x) dx$ ,  $k = 1, 2, 3$ ,

.....then  $S_2(3) + 6C_3$  is equal to \_\_\_\_\_.

NTA Ans. (18)

Reso Ans. (18)

Sol.  $C_0 = 1$

$$C_1 = 1 - \int_0^1 S_0(x) dx = 1 - \int_0^1 x dx = 1 - \frac{1}{2} = \frac{1}{2}$$

$$S_1(x) = C_1 x + \int_0^x t dt$$

$$= \frac{1}{2}x + \frac{x^2}{2} = \frac{x + x^2}{2}$$

$$C_2 = 1 - \int_0^1 \left(\frac{x + x^2}{2}\right) dx = 1 - \frac{1}{2} \left(\frac{1}{2} + \frac{1}{3}\right) = 1 - \frac{5}{12} = \frac{7}{12}$$

$$S_2(x) = C_2 x + 2 \int_0^x \left(\frac{t + t^2}{2}\right) dt = \frac{7}{12}x + \left(\frac{x^2}{2} + \frac{x^3}{3}\right) = \frac{7x + 6x^2 + 4x^3}{12}$$

$$S_2(3) = \frac{21 + 54 + 108}{12} = \frac{183}{12} = \frac{61}{4}$$

$$C_3 = 1 - \int_0^1 \left(\frac{7x + 6x^2 + 4x^3}{12}\right) dx$$

$$= 1 - \frac{1}{12} \left[ \frac{7}{2} + \frac{6}{3} + \frac{4}{4} \right]$$

$$= 1 - \frac{1}{12} \left( \frac{7}{2} + 3 \right) = \frac{24 - 7 - 6}{24} = \frac{11}{24}$$

$$\text{Now } S_2(3) + 6C_3 = \frac{61}{4} + \frac{11}{4} = \frac{72}{4} = 18$$

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22. Let  $m_1$  and  $m_2$  be the slopes of the tangents drawn from the point  $P(4, 1)$  to the hyperbola

$$H: \frac{y^2}{25} - \frac{x^2}{16} = 1. \text{ If } Q \text{ is the point from which the tangents drawn to } H \text{ have slopes } |m_1| \text{ and } |m_2| \text{ and}$$

they make positive intercepts  $\alpha$  and  $\beta$  on the  $x$ -axis, then  $\frac{(PQ)^2}{\alpha\beta}$  is equal to

NTA Ans. (8)

Reso Ans. (8)

Sol. Let equation of tangent to the hyperbola  $\frac{y^2}{25} - \frac{x^2}{16} = 1$  is

$$y = mx \pm \sqrt{25 - 16m^2} \dots\dots (1)$$

It passes through  $P(4, 1)$ , so

$$1 = 4m \pm \sqrt{25 - 16m^2}$$

$$\Rightarrow (1 - 4m)^2 = 25 - 16m^2$$

$$\Rightarrow 32m^2 - 8m - 24 = 0$$

$$\Rightarrow 4m^2 - m - 3 = 0$$

$$\Rightarrow m = 1, -\frac{3}{4}$$

Now equation of tangents with slope 1 and  $\frac{3}{4}$  with positive intercepts on  $x$ -axis, are

$$y = x - \sqrt{25 - 16}$$

$$\Rightarrow y = x - 3 \dots\dots\dots (2)$$

$$y = \frac{3}{4}x - \sqrt{25 - 9}$$

$$\Rightarrow 4y = 3x - 16 \dots\dots\dots (3)$$

solving (2) and (3),  $Q(-4, -7)$

$$PQ^2 = (4 + 4)^2 + (1 + 7)^2 = 64 + 64 = 128$$

$$\text{now } \alpha = 3 \text{ and } \beta = \frac{16}{3}$$

$$\text{So } \frac{PQ^2}{\alpha\beta} = \frac{128}{3 \times \frac{16}{3}} = 8$$

23. The sum to 20 terms of the series  $2 \cdot 2^2 - 3^2 + 2 \cdot 4^2 - 5^2 + 2 \cdot 6^2 - \dots\dots$  is equal \_\_\_\_\_.

NTA Ans. (1310)

Reso Ans. (1310)

Sol.  $2 \cdot 2^2 - 3^2 + 2 \cdot 4^2 - 5^2 + 2 \cdot 6^2 - 7^2 + \dots\dots + 2 \cdot (20)^2 - (21)^2$

$$3(2^2 + 4^2 + 6^2 + \dots\dots + (20)^2) - (2^2 + 3^2 + 4^2 + \dots\dots + (21)^2)$$

$$12(1^2 + 2^2 + 3^2 + \dots\dots + (10)^2) - (2^2 + 3^2 + 4^2 + \dots\dots + 21^2)$$

$$= 12 \left( \frac{10 \times 11 \times 21}{6} \right) - \left( \frac{21 \times 22 \times 43}{6} - 1 \right) = 2(2310) - (7 \times 11 \times 43 - 1) = 4620 - 3310 = 1310$$

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24. If  $S = \left\{ x \in \mathbb{R} : \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4} \right\}$ , then  $\sum_{x \in S} \left( \sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right) - \cos\left(\left(x^2+x+5\right)\pi\right) \right)$  is equal to \_\_\_\_\_.

NTA Ans. (4)

Reso Ans. (4)

Sol.  $\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}$

$$\tan^{-1}(x+1) - \tan^{-1}x = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{(x+1)-x}{1+(x+1)x}\right) = \frac{\pi}{4}$$

$$\left(\frac{1}{1+x^2+x}\right) = \frac{\pi}{4} \Rightarrow \frac{1}{1+x^2+x} = 1$$

$$x^2+x+1=1 \Rightarrow x^2+x=0$$

$$\Rightarrow x=0, -1 \Rightarrow S = \{0, -1\}$$

Now  $\sum_{x \in S} \left( \sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right) - \cos\left(\left(x^2+x+5\right)\pi\right) \right)$

$$= \left( \sin\frac{5\pi}{2} - \cos 5\pi \right) + \left( \sin\frac{5\pi}{2} - \cos 5\pi \right) = (1+1) + (1+1) = 4$$

25. Let  $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$  and  $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$ . If  $\vec{b}$  is a vector such that  $\vec{a} = \vec{b} \times \vec{c}$  and  $|\vec{b}|^2 = 50$ , then  $\left| 72 - |\vec{b} + \vec{c}|^2 \right|$  is equal to \_\_\_\_\_.

NTA Ans. (66)

Reso Ans. (66)

Sol.  $\vec{a} = \vec{b} \times \vec{c}$

$$\Rightarrow |\vec{a}|^2 = |\vec{b} \times \vec{c}|^2$$

$$\Rightarrow |\vec{a}|^2 = |\vec{b}|^2 |\vec{c}|^2 \sin^2 \theta$$

$$\Rightarrow 11 = (50)(22) \sin^2 \theta$$

$$\Rightarrow \sin^2 \theta = \frac{1}{100} \Rightarrow \cos^2 \theta = 1 - \frac{1}{100} = \frac{99}{100}$$

Now  $\left| 72 - |\vec{b} + \vec{c}|^2 \right| = \left| 72 - (50 + 22 + 2\sqrt{50} \cdot \sqrt{22} \cos \theta) \right|$

$$= \left| 72 - 72 - 2\sqrt{50} \cdot \sqrt{22} \cdot \frac{\sqrt{99}}{10} \right|$$

$$= \left| \frac{-2 \times 5 \sqrt{2} \times 3 \times 11 \sqrt{2}}{10} \right|$$

$$= 66$$

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26. The number of seven digit positive integers formed using the digits 1,2,3 and 4 only and sum of the digits equal to 12 is \_\_\_\_\_ .

NTA Ans. (413)

Reso Ans. (413)

Sol.  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12$  where  $x_i \in \{1, 2, 3, 4\}$

Number of solution = coefficient of  $x^{12}$  in  $(x^1 + x^2 + x^3 + x^4)^7$

= coefficient of  $x^5$  in  $(1 + x + x^2 + x^3)^7$

= coefficient of  $x^5$  in  $\frac{(1-x^4)^7}{(1-x)^7}$

= coefficient of  $x^5$  in  $(1 - 7x^4)(1-x)^{-7}$

= coefficient of  $x^5$  in  $(1 - 7x^4) \sum_{r=0}^{\infty} {}^{7+r-1}C_r x^r$

=  ${}^{11}C_5 - 7 \times {}^7C_1$

=  $462 - 49 = 413$

Alternate :  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12$

Case-I 1 1 1 1 1 3 4

Case-II 1 1 1 1 2 2 4

Case-III 1 1 2 2 2 2 2

Case-IV 1 1 1 2 2 2 3

Case-V 1 1 1 1 2 3 3

$$\frac{7!}{5!} + \frac{7!}{4!2!} + \frac{7!}{2!5!} + \frac{7!}{3!3!} + \frac{7!}{4!2!}$$

$$= 42 + 105 \times 2 + \frac{7 \times 6}{2} + \frac{7 \times 6 \times 5 \times 4}{3 \times 2}$$

$$= 42 + 210 + 21 + 140$$

$$= 413$$

27. Let the image of the point  $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$  in the plane  $x - 2y + z - 2 = 0$  be P. If the distance of the point

Q(6, -2,  $\alpha$ ),  $\alpha > 0$ , from P is 13, then  $\alpha$  is equal to \_\_\_\_\_ .

NTA Ans. (15)

Reso Ans. (15)

Sol. Image of  $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$  in plane  $x - 2y + z - 2 = 0$

$$\frac{x - \frac{5}{3}}{1} = \frac{y - \frac{5}{3}}{-2} = \frac{z - \frac{8}{3}}{1} = -2 \left( \frac{\frac{5}{3} - \frac{10}{3} + \frac{8}{3} - 2}{1 + 4 + 1} \right)$$

$$\frac{x - \frac{5}{3}}{1} = \frac{y - \frac{5}{3}}{-2} = \frac{z - \frac{8}{3}}{1} = \frac{1}{3}$$

$\therefore P(2, 1, 3)$

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$$PQ = \sqrt{16 + 9 + (\alpha - 3)^2} = 13$$

$$\Rightarrow 25 + (\alpha - 3)^2 = 169$$

$$\Rightarrow (\alpha - 3)^2 = 144$$

$$\Rightarrow \alpha - 3 = \pm 12$$

$$\Rightarrow \alpha = 15, -9 \quad (\alpha > 0)$$

$$\Rightarrow \alpha = 15$$

28. Let  $w = z\bar{z} + k_1z + k_2iz + \lambda(1+i)$ ,  $k_1, k_2 \in \mathbb{R}$ . Let  $\text{Re}(w) = 0$  be the circle C of radius 1 in the first quadrant touching the line  $y=1$  and the  $y$ -axis. If the curve  $\text{Im}(w) = 0$  intersects C at A and B, then  $30(AB)^2$  is equal to \_\_\_\_\_.

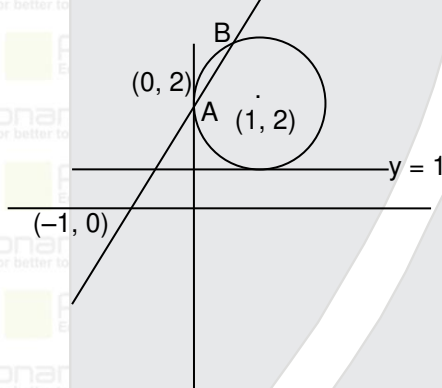
NTA Ans. (24)

Reso Ans. (24)

Sol.  $w = x^2 + y^2 + k_1(x + iy) + k_2(ix - y) + \lambda(1 + i)$

$$\text{Re}(w) = 0$$

$$x^2 + y^2 + k_1x - k_2y + \lambda = 0 \quad \dots\dots\dots (i)$$



$$\text{Circle C : } (x - 1)^2 + (y - 2)^2 = 1$$

$$x^2 + y^2 - 2x - 4y + 4 = 0 \quad \dots\dots\dots (ii)$$

compare (i) & (ii),  $k_1 = -2$ ,  $k_2 = 4$  and  $\lambda = 4$

also  $\text{Im}(w) = 0$

$$\Rightarrow k_1y + k_2x + \lambda = 0$$

$$\Rightarrow 4x - 2y + 4 = 0$$

$$\Rightarrow 2x - y + 2 = 0 \quad \dots\dots\dots (iii)$$

$$\text{length of chord AB} = 2\sqrt{r^2 - p^2}$$

$$\Rightarrow AB = 2\sqrt{1 - \left(\frac{2-2+2}{\sqrt{4+1}}\right)^2}$$

$$\Rightarrow AB = 2\sqrt{1 - \frac{4}{5}}$$

$$\Rightarrow AB^2 = \frac{4}{5}$$

$$\Rightarrow 30AB^2 = 24$$

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29. Let the mean of the data

x	1	3	5	7	9
Frequency(f)	4	24	28	$\alpha$	8

be 5. If  $m$  and  $\sigma^2$  are respectively the mean deviation about the mean and the variance of the data, then

$\frac{3\alpha}{m + \sigma^2}$  is equal to \_\_\_\_\_.

NTA Ans. (8)

Reso Ans. (8)

Sol.  $\frac{4 + 72 + 28 \times 5 + 7\alpha + 72}{56 + 8 + \alpha} = 5 \Rightarrow \alpha = 16$

$\sum f_i = 80$

M.D. =  $\frac{\sum f_i |x_i - 5|}{\sum f_i} = \frac{4 + 4 + 24 \times 2 + 0 + 16 \times 2 + 8 \times 4}{80} = \frac{16}{10}$

$\sigma^2 = \frac{\sum f_i (x_i - 5)^2}{\sum f_i} = \frac{4 + 16 + 24 \times 4 + 0 + 16 \times 4 + 8 \times 16}{80} = \frac{44}{10}$

$\frac{3\alpha}{m + \sigma^2} = \frac{3 \times 16}{\frac{16}{10} + \frac{44}{10}} = \frac{3 \times 16 \times 10}{60} = 8$

30. Let  $\alpha$  be the constant term in the binomial expansion of  $\left(\sqrt{x} - \frac{6}{x^2}\right)^n$ ,  $n \leq 15$ . If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of  $x^{-n}$  is  $\lambda\alpha$ , then  $\lambda$  is equal to \_\_\_\_\_

NTA Ans. (36)

Reso Ans. (36)

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Sol. General term =  $T_{r+1} = {}^n C_r (\sqrt{x})^{n-r} \left(-\frac{6}{x^{3/2}}\right)^r = {}^n C_r (-6)^r \cdot x^{\frac{n-r}{2} - \frac{3r}{2}} = {}^n C_r (-6)^r \cdot x^{\frac{n-2r}{2}}$

For constant term  $\frac{n}{2} - 2r = 0$

$$r = \frac{n}{4}$$

Now  ${}^n C_{\frac{n}{4}} (-6)^{\frac{n}{4}} = \alpha \dots(1)$

Sum of coefficients of remaining terms in the expansion = 649

$$\left(1 - \frac{6}{1}\right)^n - \alpha = 649$$

$$(-5)^n - \alpha = 649$$

$$\alpha = (-5)^n - 649 \dots(2)$$

$${}^n C_{\frac{n}{4}} (-6)^{\frac{n}{4}} = (-5)^n - 649$$

here n will be multiple of 4

Let  $n = 4 \Rightarrow {}^n C_1 (-6)^1 = (-5)^4 - 649$  which is true

So value of n is 4

for coefficient of  $x^{-4}$ ,  $\frac{4}{2} - 2r = -4 \Rightarrow 2r = 6 \Rightarrow r = 3$

coefficient of  $x^{-4} = {}^4 C_3 (-6)^3 = 4(-216) = \lambda\alpha \Rightarrow \lambda = -\frac{864}{\alpha}$

and  $\alpha = (-5)^4 - 649 = 625 - 649 = -24$

$$\therefore \lambda = \frac{864}{24} = 36$$

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