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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 13 April, 2023 (SHIFT-2) | TIME : (03.00 p.m. to 06.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. Let (α,β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation

(1) $x^2 - 14x + 48 = 0$ (2) $x^2 - 10x + 25 = 0$

(3) $x^2 - 13x + 42 = 0$ (4) $x^2 - 7x + 12 = 0$

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. Solving pair wise given equations we get vertices of ΔABC as

A (6,8), B (1,2), C(5–7)

$\therefore\ \alpha = \frac{6+1+5}{3} = 4$

$\beta = \frac{8+2-7}{3} = 1$

$\therefore$ Centroid G (4,1)

sum of roots = $\alpha + 2\beta + 2\alpha - \beta$

$= 3\alpha + \beta = 13$

product of roots $(\alpha + 2\beta)(2\alpha - \beta)$

$= 6 \times 7 = 42$

$\therefore$ equation is $x^2 - 13x + 42 = 0$

2. Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|2 \text{ adj } (2 \text{ adj } (2A))| = 32^n$, then $3n + \alpha$ is equal to

(1) 11 (2) 9 (3) 10 (4) 12

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$

$\Rightarrow |A| = 1(6-1) - \alpha(4-3) + 1(2-9) = 2$

$\Rightarrow$ $5 - \alpha - 7 = 2 \Rightarrow \alpha = -4$

Now $|2 \text{ adj } (2\text{adj } 2A)| = 32^n$

$\Rightarrow$ $2^3\,|2\text{adj } 2A|^2 = 32^n$

$\Rightarrow$ $2^3 \cdot 2^6\,|2A|^4 = 2^{5n}$

$\Rightarrow$ $2^9 \cdot 2^{12}\,|A|^4 = 2^{5n}$

$\Rightarrow$ $2^{25} = 2^{5n} \Rightarrow n = 5$

$\therefore$ $3n + \alpha = 15 - 4 = 11$ Ans.

3. The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to ______.

(1) $p \vee (\sim q)$ (2) $(\sim p) \vee q$ (3) $p \vee q$ (4) $(\sim p) \vee (\sim q)$

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$\equiv (p \wedge \sim q) \vee (\sim p \wedge (q \vee \sim q))$

$\equiv (p \wedge \sim q) \vee (\sim p \wedge t)$

$\equiv (p \wedge \sim q) \vee \sim p$

$\equiv (\sim q \vee p) \wedge (\sim p \vee \sim q)$

$\equiv t \wedge (\sim p \vee \sim q)$

$\equiv \sim p \vee \sim q$

4. The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right)$ is

(1) $[0, 2\pi)$ (2) $[0, \pi]$ (3) $[0, \pi)$ (4) $[0, 2\pi]$

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. $\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$

$\therefore 0 \le \frac{x^2}{1+x^2} < 1$

$\Rightarrow 0 \le \sin^{-1}\left(\frac{x^2}{1+x^2}\right) < \frac{\pi}{2}$

$\Rightarrow 0 \le 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right) < 2\pi$

5. The random variable X follows binomial distribution B (n, p), for which the difference of the mean and the variance is 1. If 2P(X = 2) = 3P(x = 1), then n^2 P(x > 1) is equal to

(1) 15 (2) 16 (3) 11 (4) 12

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. np – npq = 1 and

$2 \times {}^nC_2 P^2 (1-P)^{n-2} = 3 \times {}^nC_1 P(1-P)^{n-1}$

$\Rightarrow np^2 = 1$

$\Rightarrow p = \frac{1}{2}$, n = 4

$= n^2(P(X>1)) = 16\left(1 - {}^4C_0\left(\frac{1}{2}\right)^4 - {}^4C_1\left(\frac{1}{2}\right)^4\right) = 16\left(1 - \frac{5}{16}\right) = 11$

6. Let the centre of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then $(\alpha - \beta + r)$ is equal to

(1) 9 (2) 7 (3) 6 (4) 5

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. (α, β) is centre & $3x + 4y = 24$ & $3x - 4y = 32$ are tangents

$$\therefore \left|\frac{3\alpha+4\beta-24}{5}\right| = \left|\frac{3\alpha-4\beta-32}{5}\right|$$

$3\alpha + 4\beta - 24 = \pm (3\alpha - 4\beta - 32)$

(+) sign (–) sign

$8\beta + 8 = 0$ $6\alpha = 56$

$\beta = -1$ $\alpha = \frac{28}{3}$

$\therefore$ $4x + 3y = 1$ is normal $\therefore$ $4\alpha + 3\beta = 1$

$\beta = -1$ then $\alpha = 1$ $\therefore$ C(1, –1)

$\alpha = \frac{28}{3}$ then $\frac{112}{3} - 1 - 3\beta \Rightarrow \beta = -\frac{109}{9}$

$$C\left(\frac{28}{3}, \frac{-109}{9}\right)$$

C(1, –1) then $r = \left|\frac{3-4-24}{5}\right| = 5$

$C\left(\frac{28}{3}, \frac{-109}{9}\right)$ then $r = \left|\frac{\frac{84}{3} - \frac{436}{9} - 2}{5}\right| > 5$

$\therefore$ $C(\alpha, \beta) = C(1, -1), \gamma = 5$ $\therefore$ $\alpha - \beta + r = 7$ Ans.

7. Let a_1, a_2, a_3 …….. be a G.P of increasing positive numbers . Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to

(1) $3\sqrt{3}$ (2) 2 (3) $2\sqrt{2}$ (4) 3

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $a_6 + a_8 = 2 \Rightarrow ar^5 + ar^7 = 2$

& $a_3 . a_5 = \frac{1}{9} \Rightarrow a^2 . r^2 r^4 = \frac{1}{9}$

$\Rightarrow ar^3 = \frac{1}{3}$

by (1) $\frac{1}{3}r^2 + \frac{1}{3}r^4 = 2$

$r^2 + r^4 = 6 \Rightarrow r^4 + r^2 - 6 = 0$

$\Rightarrow (r^2 + 3)(r^2 - 2) = 0$

$\Rightarrow r^2 = 2 \therefore ar.2 = \frac{1}{3} \Rightarrow ar = \frac{1}{6}$

Now $6(a_2 + a_4)(a_4 + a_6) = 6(ar + ar^3)(ar^3 + ar^5)$

$$= 6\left(\frac{1}{6} + \frac{1}{3}\right)\left(\frac{1}{3} + \frac{1}{3}.2\right) = 6\frac{1+2}{6}.\frac{1+2}{3}$$

$= 3$

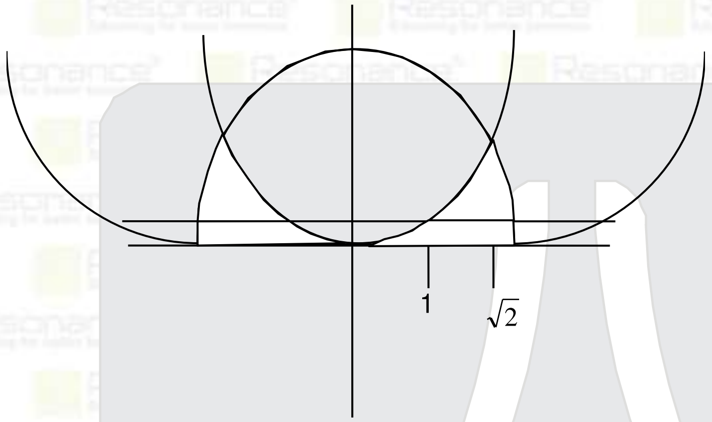
8. The area of the region {(x, y) : $x^2 \le y \le |x^2 - 4|$, $y \le 1$) is

(1) $\frac{3}{4}\left(4\sqrt{2}+1\right)$ (2) $\frac{4}{3}\left(4\sqrt{2}-1\right)$ (3) $\frac{3}{4}\left(4\sqrt{2}-1\right)$ (4) $\frac{4}{3}\left(4\sqrt{2}+1\right)$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol.



$$A = 2\left(\int_0^1 \left(3-x^2\right)dx + \int_1^{\sqrt{2}} \left(4-2x^2\right)dx\right) = 2\left(\left(3x - \frac{x^3}{3}\right)_0^1 + \left(4x - \frac{2x^3}{3}\right)_1^{\sqrt{2}}\right)$$

$$= 2\left(3 - \frac{1}{3} + \left(4\sqrt{2} - \frac{4\sqrt{2}}{3} - 4 + \frac{2}{3}\right)\right) = 2\left(\frac{-2}{3} + \frac{2}{3}\left(4\sqrt{2}\right)\right) = \frac{4\left(4\sqrt{2}-1\right)}{3}$$

9. The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ is

(1) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$ (2) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(3) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ (4) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. Check by option

option (2) $\Rightarrow \begin{vmatrix} -3+1 & 1-2 & 5-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$

$= \begin{vmatrix} -2 & -1 & 0 \\ -2 & -1 & 0 \\ -1 & 2 & 5 \end{vmatrix} = 0$

$\therefore$ option (2) is correct

10. The plane, passing through the points (0, -1, 2) and (-1, 2, 1) and parallel to the line passing through (5,1,-7) and (1,-1,-1), also passes through the point

(1) (2, 0, 1) (2) (1, -2, 1) (3) (0, 5, -2) (4) (-2, 5, 0)

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. Any plane passing through (0,–1,2) is a(x – 0) + b (y + 1) + c (z –2) = 0 ... (1)

It also passes through (–1, 2,1)

$\therefore$ $-a + 3b - c = 0$

$a - 3b + c = 0$ ... (2)

Dr's of line joining points A (5,1,–7) & B (1,–1,–1) are 4, 2, –6

Plane (1) || to AB $\therefore$ $4a + 2b - 6c = 0$

$\Rightarrow$ $2a + b - 3c = 0$ ... (3)

$a - 3b + c = 0$ ... (2)

$\Rightarrow$ $\frac{a}{1-9} = \frac{b}{-3-2} = \frac{c}{-6-1}$

$\Rightarrow$ $\frac{a}{8} = \frac{b}{5} = \frac{c}{7}$

$\therefore$ Required plane is $8x + 5y + 7z = 9$

Clearly (–2, 5, 0) satisfying the plane so Ans. (4)

11. If $\lim\limits_{x\to 0}\frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1-\cos(2x)} = 17$, then $5a^2 + b^2$ is equal to

(1) 72 (2) 64 (3) 76 (4) 68

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $\lim\limits_{x\to 0}\frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1-\cos(2x)} = 17,$

by L-H rule

$$\lim_{x\to 0}\frac{ae^{ax} + b\sin bx - \frac{c}{2}\left(e^{-cx} - xe^{-cx}.c\right)}{2\sin 2x} = 17$$

$$a - \frac{c}{2} = 0$$

$$\lim_{x\to 0}\frac{a^2e^{ax} + b^2\cos bx + \frac{c^2}{2}e^{-cx} + \frac{c^2}{2}\left(e^{-cx} - xce^{-cx}\right)}{4\cos 2x} = 17$$

$$\frac{a^2 + b^2 + \frac{c^2}{2} + \frac{c^2}{2}}{4} = 17$$

$\Rightarrow$ $a^2 + b^2 + 4a^2 = 68$

$\Rightarrow$ $5a^2 + b^2 = 68$

12. Let for a triangle ABC,

$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

if $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB}.\overrightarrow{CA}$ is equal to

(1) 120 (2) 54 (3) 60 (4) 108

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. Ar of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{CB} \times \overrightarrow{CA}\right| = 5\sqrt{6}$

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{O}$

$\Rightarrow$ $-2i + j + 3k - \alpha i - \beta j - \gamma k + 4i + 3j + \delta k = \overrightarrow{O}$

$\Rightarrow$ $-2 - \alpha + 4 = 0 \Rightarrow \alpha = 2$

$1 - \beta + 3 = 0 \Rightarrow \beta = 4$

$3 - \gamma + \delta = 0 \Rightarrow \gamma = 3 + \delta$

Now $\overrightarrow{CB} \times \overrightarrow{CA} = \begin{vmatrix} i & j & k \\ 2 & 4 & 3+\delta \\ 4 & 3 & \delta \end{vmatrix}$

$= (\delta - 9)i + (2\delta + 12)j - 10k$

$(\delta - 9)^2 + (2\delta + 12)^2 + 100 = 600$

$\therefore$ $5\delta^2 + 30\delta - 275 = 0$

$\delta^2 + 6\delta - 55 = 0$

$(\delta + 11)(\delta - 5) = 0 \quad \Rightarrow \quad \delta = 5$

Now

$\overrightarrow{CB} \times \overrightarrow{CA} = (2i + 4j + 8k)\cdot(4i + 3j + 5k)$

$= 8 + 12 + 40 = 60$ Ans.

13. If the system of equations

$2x + y - z = 5$

$2x - 5y + \lambda z = \mu$

$x + 2y - 5z = 7$

has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to

(1) 912 (2) 916 (3) 904 (4) 920

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $2x + y - z = 5$

$2x - 5y + \lambda z = \mu$

$x + 2y - 5z = 7$

for infinite solution $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$2(25 - 2\lambda) - (-10 - \lambda) - 1(4 + 5) = 0$

$50 - 4\lambda + 10 + \lambda - 9 = 0 \quad \Rightarrow \quad \lambda = 17$

$$D_1 = \begin{vmatrix} 5 & 1 & -1 \\ \mu & -5 & 17 \\ 7 & 2 & -5 \end{vmatrix} = 0$$

$5(25 - 34) - \mu(-5 + 2) + 7(17 - 5) = 0$

$-45 + 3\mu + 84 = 0 \Rightarrow \mu = -13$

Now $(\lambda + \mu)^2 + (\lambda - \mu)^2 = (17 - 13)^2 + (17 + 13)^2$

$= 16 + 900 = 916$

14. let $S = \{z \in C : \bar{z} = i(z^2 + \text{Re}(\bar{z}))\}$. Then $\sum_{z \in S} |z|^2$ is equal to

(1) 4 (2) $\frac{7}{2}$ (3) $\frac{5}{2}$ (4) 3

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. Let $Z = x + iy$

$x - iy = i(x^2 - y^2 + (2xy)i + x)$

$= (-2xy) + i(x^2 + x - y^2)$

$x = -2xy$--------(1)

$x(1 + 2y) = 0$

$x = 0,\ y = -\frac{1}{2}$

$-y = x^2 + x - y^2$ ---------(2)

If $x = 0$, $y^2 - y = x^2 + x \Rightarrow y = 0, 1$

If $y = -\frac{1}{2}$, $\frac{1}{4} + \frac{1}{2} = x^2 \times x \Rightarrow x^2 \times x - \frac{3}{4} = 0$

$X = \frac{1}{2}, -\frac{3}{2}$

Possible $Z = 0 + i\,0,\ 0 + i\ \frac{1}{2} - \frac{i}{2}, -\frac{3}{2} - \frac{i}{2}$

$|z|^2 = 0, 1, \frac{1}{2}, \frac{10}{4}$

Sum of values of $|z|^2 = 1 + \frac{1}{2} + \frac{10}{4} = 1 + 3 = 4$

15. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is .

(1) $\frac{26}{3}$ (2) $\frac{80}{9}$ (3) 8 (4) 9

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $T_{r+1} = {}^5c_r (2x^3)^{5-r} \left(\frac{-1}{3x^2}\right)^r$

Power of $x = 15 - 3r - 2r = 5 \quad \Rightarrow r = 2$

coefficient $= {}^5c_2 (2^3)\left(\frac{1}{9}\right) = \frac{80}{9}$

16. The value of $\dfrac{e^{-\frac{\pi}{4}}+\int_0^{\frac{\pi}{4}} e^{-x}(\tan^{50}x)dx}{\int_0^{\frac{\pi}{4}} e^{-x}\left(\tan^{49}x+\tan^{51}x\right)dx}$ is

(1) 25 (2) 49 (3) 50 (4) 51

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $$\frac{e^{-\pi/4}+\int_0^{\pi/4} e^{-x}(\tan^{50}x)dx}{\int_0^{\pi/4} e^{-x}\left(\tan^{49}x\sec^2x\right)dx}=\frac{e^{-\pi/4}+\int_0^{\pi/4} e^{-x}\left(\tan^{50}x\right)dx}{\left(\left(\frac{e^{-x}\tan^{50}x}{50}\right)_0^{\pi/4}+\int_0^{\pi/4}\frac{e^{-x}\tan^{50}x}{50}dx\right)}=50$$

17. Let N be the foot of perpendicular from the point P(1, –2, 3) on the line passing through the point (4,5,8) and (1, –7, 5). Then the distance of N from the plane 2x –2y + z + 5 = 0 is

(1) 6 (2) 9 (3) 8 (4) 7

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $N\equiv\left(\frac{\lambda+4}{\lambda+1},\frac{-7\lambda+5}{\lambda+1},\frac{5\lambda+8}{\lambda+1}\right)$

Dr's of $AN=\frac{3}{\lambda+1},\frac{-5\lambda+7}{\lambda+1},\frac{2\lambda+5}{\lambda+1}$

Dr's of BC = 3,12,3

$\because$ AN $\perp$ BC

$\frac{9}{\lambda+1}+\frac{12(-5\lambda+7)}{\lambda+1}+\frac{3(2\lambda+5)}{\lambda+1}=0$

$-54\lambda+108=0$

N (2,–3, 6)

Perpendicular distance of N form 2x – 2y + z + 5 = 0 is

$\left|\frac{4+6+6+5}{\sqrt{4+4+1}}\right|=\frac{21}{3}=7$

18. Let $|\vec{a}|=2$, $|\vec{b}|=3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $\left|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})\right|^2$ is equal to

(1) 441 (2) 882 (3) 841 (4) 482

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $|\vec{a}|=2$, $|\vec{b}|=3$. $\vec{a}\,\hat{}\,\vec{b}=\frac{\pi}{4}$

Now $\left|(\vec{a}+2\vec{b})\times(2\vec{a}-3\vec{b})\right|^2=\left|2\vec{a}\times\vec{a}-3\vec{a}\times\vec{b}+4\vec{b}\times\vec{a}-6\vec{b}\times\vec{b}\right|^2$

$\left|7(\vec{a}\times\vec{b})\right|^2=49|\vec{a}|^2|\vec{b}|^2\sin^2\frac{\pi}{4}=49.4.\,9.\,\frac{1}{2}=882$

19. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to

(1) – 64 (2) – 128 (3) $-128\sqrt{2}$ (4) $-64\sqrt{2}$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $\therefore \alpha + \beta = \sqrt{2}$ $\alpha\beta = 2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = -2$

$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = 4 - 8 = -4$

$\alpha^8 + \beta^8 = (\alpha^4 + \beta^4)^2 - 2(\alpha\beta)^4 = 16 - 2 = -16$

$\alpha^6 + \beta^6 = (\alpha^2 + \beta^2)(\alpha^4 + \beta^4) - (\alpha\beta)^2(\alpha^2 + \beta^2) = -2(-4-4) = 16$

$\alpha^{14} + \beta^{14} = (\alpha^8 + \beta^8)(\alpha^6 + \beta^6) - (\alpha\beta)^6(\alpha^2 + \beta^2)$

$= (-16)(16) - 2^6 . (-2)$

$= -256 + 8.16$

$= 8 \times 16 = -128$

20. All words, with or without meaning, are made using all the letters of the word "MONDAY" . These words are written as in a dictionary with serial numbers. The serial number of the word "MONDAY" is

(1) 324 (2) 326 (3) 327 (4) 328

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. given :- MONDAY

A, D, M, N, O, Y

A – 5! –––––– = 120

D 5! = 120

MA 4! = 24

MD 4! = 24

MN 4! = 24

MOA 3! = 6

MOD 3! = 6

MONA 2! = 2

MONDAY = 1

327

21. The remainder, when 7^{103} is divided by 17, is

NTA Ans. **(12)**

Reso Ans. **(12)**

Sol. $7^{103} = 7.(49)^{51} = 7.(51-2)^{51}$

$= 7(51\lambda - 2^{51}) = 17k - 7.5^{51}$

$= 17k - 56(17-1)^{12}$

$= 17k - 56(17\lambda + 1)$

$= 17m - 51 - 17 + 12$

So Remainder 12

22. Let [α] denote the greatest integer ≤ α. Then $\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+---+\left[\sqrt{120}\right]$ is equal to ______ .

NTA Ans. **(825)**

Reso Ans. **(825)**

Sol. If $x \in [n^2,(n + 1)^2)$ then $\left[\sqrt{x}\right] = n$

required value = $\sum_{r=1}^{10}\left((r+1)^2-r^2\right)r = \sum_{r=1}^{10}\left(2r^2+r\right)$

$= 2\frac{10.11.21}{6}+\frac{10.11}{2} = 770 + 55 = 825$

23 The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _______ .

NTA Ans. **(269)**

Reso Ans. **(269)**

Sol. $\bar{x} = \frac{\sum_{i=1}^{5} x_i}{10} = 50$

$\therefore \sum x_i = 500$

correct sum = $\sum x_i - (45 + 50) + (20 + 25)$

$= 4450$

$\sigma^2 = \frac{\sum x^2}{50} - \bar{x}^2$

$\Rightarrow 144 = \frac{\sum x^2}{10} - 2500$

$\Rightarrow \sum x^2 = 10 \times 2644 = 26440$

$\therefore$ correct $\sum x^2 = \sum x^2 - (45^2 + 50^2) + (20^2 + 25^2)$

$= 26440 - (2025 + 2500) + (400 + 625)$

$= 22940$

correct variance = $\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$= \frac{22940}{10} - \left(\frac{450}{10}\right)^2$

$= 2294 - 2025$

$= 269$

24. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}, x>1$ such that

$y(2)=\frac{2}{9}\log_e(2+\sqrt{3})$ and $y(\sqrt{2})=\alpha\log_e(\sqrt{\alpha}+\beta)+\beta-\sqrt{\gamma}, \alpha,\beta,\gamma\in N$, then $\alpha\beta\gamma$ is equal to _____.

NTA Ans. **(6)**
Reso Ans. **(6)**

Sol. $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}, x>1$ which is linear in y

I.f. $e^{\int\frac{4x}{x^2-1}dx}=e^{2\ln(x^2-1)}=(x^2-1)^2$

Solution is $y.(x^2-1)^2=\int\frac{x+2}{(x^2-1)^{\frac{5}{2}}}(x^2-1)^2dx+c$

$\Rightarrow\quad y(x^2-1)^2=\int\frac{x+2}{\sqrt{x^2-1}}dx+c$

$\Rightarrow\quad y(x^2-1)^2=\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)+c$

$\therefore\quad y(2)=\frac{2}{9}\ln(2+\sqrt{3})$

$\therefore\quad \frac{2}{9}\ln(2+\sqrt{3}).9=\sqrt{3}+2\ln(2+\sqrt{3})+c$

$c=-\sqrt{3}$

$\therefore\quad y(x^2-1)^2=\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{3}$

Put $x=\sqrt{2}$

$\therefore\quad y(\sqrt{2}).1=1+2\ln(\sqrt{2}+1)-\sqrt{3}$

$y(\sqrt{2})=2\ln(\sqrt{2}+1)+1-\sqrt{3}$

$=\alpha\ln(\sqrt{\alpha}+\beta)+\beta-\sqrt{\gamma}$

$\alpha=2, \beta=1, \gamma=3 \quad \therefore \quad \alpha.\beta.\gamma=6$ Ans,

25. Let $f_n=\int_0^{\frac{\pi}{2}}\left(\sum_{k=1}^{n}\sin^{k-1}x\right)\left(\sum_{k=1}^{n}(2k-1)\sin^{k-1}x\right)\cos x\,dx$, $n\in N$. Then $f_{21}-f_{20}$ is equal to

NTA Ans. **(41)**
Reso Ans. **(41)**

Sol. $f_n=\int_0^{\frac{\pi}{2}}\left(\sum_{k=1}^{n}\sin^{k-1}x\right)\left(\sum_{k=1}^{n}(2k-1)\sin^{k-1}x\right)\cos x\,dx$

Let $\sin x=t \quad \therefore \quad \cos x dx=dt$

$f_n=\int_0^1\left(\sum_{k=1}^{n}t^{k-1}\right)\left(\sum_{k=1}^{n}(2k-1)t^{k-1}\right)dt$

$$= 2\int_0^1\left(\sum_{k=1}^{n} t^{\frac{2k-1}{2}}\right)\left(\sum_{k=1}^{n}\frac{(2k-1)}{2}t^{\frac{2k-3}{2}}\right)dt$$

Let $\sum_{k=1}^{n} t^{\frac{2k-1}{2}} = y$ $\therefore$ $\left(\sum_{k=1}^{n}\frac{(2k-1)}{2}t^{\frac{2k-3}{2}}\right)dt = dy$

$$= 2\int_0^n y\,dy = 2.\frac{n^2}{2} = n^2$$

$\therefore$ $I_{21} - I_{20} = 21^2 - 20^2 = 21 + 20 = 41$ Ans.

26. Let A = {–4, –3, –2, 0, 1, 3, 4} and R = {(a, b) ∈ A × A : b = |a| or $b^2 = a + 1$ } be a relation on A. then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is

NTA Ans. **(7)**

Reso Ans. **(7)**

Sol. A = {–4, –3, –2, 0, 1, 3, 4}

R = {(a, b) ∈ A × A : b = |a| or $b^2 = a + 1$}

= {(–4, 4), (–3, 3),(0,0), (1,1), (3,3), (4,4), (0,1) (3,–2)}

For reflexive relation (–4, –4), (–3, –3) (–2, –2) must be added

For symmetric relation (4, –4) , (3, –3) (1,0), (–2, 3) must be added

$\therefore$ Total number of elements = 7

27. For x ∈ (–1, 1], then number of solution of the equation $\sin^{-1}x = 2\tan^{-1}x$ is equal to

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. Number of solution = 2

Alternate method

$\text{Sin}^{-1} x = \sin^{-1}\frac{2x}{1+x^2}$ $\forall x \in (-1,1]$

$x = \frac{2x}{1+x^2}$

x = 0, 1

28. Let $f(x) = \sum_{k=1}^{10} k\,x^k, x \in R$. If $2f(2) + f'(2) = 119\,(2)^n + 1$ then n is equal to

NTA Ans. **(10)**

Reso Ans. **(10)**

Sol. $f(x) = \sum_{k=1}^{10} k\,x^k, x \in R.$ $\qquad 2f(2) + f'(2) = 119\,(2)^n + 1$

$f(x) = x + 2x^2 + 3x^3 + ... + 10x^{10}$ (A.G.P.)

$\Rightarrow \quad f(x) = \frac{x(1-x^{10})}{(1-x)^2} - \frac{10x^{11}}{1-x}$ ... (1)

$f'(x) = \frac{(1-x)^2(1-11x^{10}) + (x-x^{11})\,2(1-x)}{(1-x)^4} - \frac{10((1-x)11x^{10} + x^{11})}{(1-x)^2}$ ... (2)

$\therefore$ $f(2) = 2\,(1 - 2^{10}) + 20.2^{10}$

$f'(2) = 83\cdot2^{10} - 3$

$\therefore$ $2f(2) + f'(2) = 119.2^{10} + 1$

$\therefore$ n = 10 Ans.

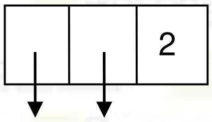
29. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1,2,3,4, 5 with repetition, is

NTA Ans. **(16)**

Reso Ans. **(16)**

Sol.

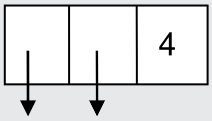
Case-1 Unit digit is 2



(i) 2 2 = 4

(ii) 2 2 = 4

Case-2 Unit digit is 4



(i) 2 2 = 4

(ii) 2 2 = 4

Total number = 16

30. The foci of hyperbola are (± 2, 0) and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line 2x + 3y = 6, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x – and y – axes are a and be respectively, then |6a| + |5b| is equal to ________.

NTA Ans. **(12)**

Reso Ans. **(12)**

Sol. s(± 2, 0) $e = \frac{3}{2}$

ae = 2, $\therefore\ a = \frac{4}{3}$, $b^2 = a^2 (e^2 - 1)$

$$b^2 = \frac{16}{9}\left(\frac{9}{4} - 1\right) = \frac{16}{9} \times \frac{5}{4}$$

$$b^2 = \frac{20}{9}$$

$\therefore$ Hyperbola is $\frac{9x^2}{16} - \frac{9y^2}{20} = 1$

Line ⊥ to 2x + 3y = 6 is

$3x - 2y + \lambda = 0$

$2y = 3x + \lambda \Rightarrow y = \frac{3}{2}x + \frac{\lambda}{2}$

Condition of tangency

$$\frac{\lambda^2}{4} = \frac{16}{9} \cdot \frac{9}{4} - \frac{20}{9} = \frac{16}{9}$$

$$\lambda^2 = \frac{64}{9} = \lambda = \pm\frac{8}{3}$$

∴ Tangent line is $3x - 2y \pm \frac{8}{3} = 0$

$y = 0$ then $a = \pm\frac{8}{9}$

$x = 0$ then $-b = \pm\frac{4}{3}$

Now $|6a| + |5b| = \frac{16}{3} + \frac{20}{3} = \frac{36}{3} = 12$