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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 15 April, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. Let x = x(y) be the solution of the differential equation $2(y + 2)\log_e(y + 2)dx + (x + 4 - 2\log_e(y+ 2))dy = 0$, $y > -1$ with $x(e^4 - 2) = 1$. Then $x(e^9 - 2)$ is equal to

(1) $\frac{10}{3}$ (2) $\frac{32}{9}$ (3) $\frac{4}{9}$ (4) 3

NTA Ans. (2)

Reso Ans. (2)

Sol. $2(y + 2)\ln(y + 2)dx + (x + 4 - 2\ln(y+ 2))dy = 0$

$$\frac{dx}{dy} + \frac{x}{2(y+2)\ln(y+2)} = \frac{1}{y+2} - \frac{4}{2(y+2)\ln(y+2)}$$

$$\text{I.F.} = e^{\frac{1}{2}\int\frac{1}{(y+2)\ln(y+2)}dy}$$

$$= \sqrt{\ln(y+2)}$$

$$\text{Solution } x\sqrt{\ln(y+2)} = \int\left(\frac{\sqrt{\ln(y+2)}}{y+2} - \frac{2\sqrt{\ln(y+2)}}{(y+2)\ln(y+2)}\right)dy$$

$$x\sqrt{\ln(y+2)} = \int\left(\frac{\sqrt{\ln(y+2)}}{y+2} - \frac{2}{(y+2)\sqrt{\ln(y+2)}}\right)dy$$

$$x\sqrt{\ln(y+2)} = \frac{2}{3}\left(\ln(y+2)\right)^{\frac{3}{2}} - 4\sqrt{\ln(y+2)} + C$$

$$x = \frac{2}{3}\ln(y + 2) - 4 + \frac{C}{\sqrt{\ln(y+2)}}$$

Now, $x = 1$, $y = e^4 - 2$

$$1 = \frac{2}{3}(4) - 4 + \frac{C}{2} \Rightarrow C = \frac{14}{3}$$

$$x = \frac{2}{3}\ln(y + 2) - 4 + \frac{14}{3\sqrt{\ln(y+2)}}$$

Now, at $y = e^9 - 2$

$$x = \frac{2}{3} \times 9 - 4 + \frac{14}{3\times 3}$$

$$x = \frac{32}{9}$$

2. If $\int_0^1 \frac{1}{(5+2x-2x^2)(1+e^{(2-4x)})}dx = \frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right)$, then $\alpha^4 - \beta^4$ is equal to

(1) 21 (2) 0 (3) 19 (4) –21

NTA Ans. (1)

Reso Ans. (1)

Sol. $I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ ....(1)

$$I = \int_0^1 \frac{dx}{\left[5+2(1-x)-2(1-x)^2\right]\left[1+e^{2-4(1-x)}\right]}$$

$$I = \int_0^1 \frac{dx}{(5-2x+4x-2x^2)\left(1+e^{-2+4x}\right)}$$

$$I = \int_0^1 \frac{dx}{(5+2x-2x^2)\left(1+\frac{1}{e^{2-4x}}\right)}$$

$$I = \int_0^1 \frac{e^{2-4x}}{(5+2x-2x^2)\left(1+e^{2-4x}\right)}dx \;......(2)$$

From (1) and (2)

$$2I = \int_0^1 \frac{dx}{(5+2x-2x^2)}$$

$$2I = \frac{1}{2}\int_0^1 \frac{dx}{\left(\frac{5}{2}+x-x^2\right)} = \frac{1}{2}\int_0^1 \frac{dx}{\frac{11}{4}-\left(x-\frac{1}{2}\right)^2}$$

$$2I = \frac{1}{2}\cdot\frac{1}{2\sqrt{\frac{11}{4}}}\left[\ell n\left|\frac{\frac{\sqrt{11}}{2}+x-\frac{1}{2}}{\frac{\sqrt{11}}{2}-x+\frac{1}{2}}\right|\right]_0^1$$

So $I = \frac{1}{\sqrt{11}}\ell n\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right)$

Hence a $= \sqrt{11}, \beta = \sqrt{10}$

$\Rightarrow \alpha^4 - \beta^4 = 121 - 100 = 21$

3. Let [x] denote the greatest integer function and f(x) = max {1 + x + [x], 2 + x, x + 2[x]}, $0 \le x \le 2$. Let m be the number of points in [0,2], where f is not continuous and n be the number of points in (0,2) where f is not differentiable. Then $(m + n)^2 + 2$ is equal to

(1) 3 (2) 6 (3) 2 (4) 11

NTA Ans. (1)

Reso Ans. (1)

Sol. f(x) maximum {1+x+[x], x + 2, x+ 2[x]}

$$= \begin{cases} x+2 & 0 \le x < 2 \\ 6 & x = 2 \end{cases}$$

f(x) is discontinuous at x = 2 only and f(x) is differentiable in $x \in (0,2)$

m = 1, n = 0, now $(m + n)^2 + 2 = 1 + 2 = 3$

4. The total number of three digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8 , if repetition of digits is allowed , is

(1) 20 (2) 22 (3) 21 (4) 18

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. Sum of three digits is 3 then digits are 1, 1, 1

Sum of three digits is 9 then digits are 1, 3, 5 or 3, 3, 3

Sum of three digits is 12 then digits are 1, 3, 8

Sum of three digits is 15 then digits are 5, 5, 5

Sum of three digits is 18 then digits are 5, 5, 8

Sum of three digits is 21 then digits are 5, 8, 8

Sum of three digits is 24 then digits are 8, 8, 8

Now possible number are $= 1 + 3! + 1 + 3! + 1 + \frac{3!}{2!} + \frac{3!}{2!} + 1$

$= 1 + 6 + 1 + 6 + 1 + 3 + 3 + 1$

$= 22$

5. Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$ is equal to

(1) $2(A_1 + A_2) G_1^2 G_3^2$ (2) $(A_1 + A_2)^2 G_1 G_3$

(3) $2(A_1 + A_2) G_1 G_3$ (4) $(A_1 + A_2) G_1^2 G_3^2$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $A_1 = \frac{2a+b}{3}$ and $A_2 = \frac{a+2b}{3} \Rightarrow A_1 + A_2 = (a+b)$

also $G_1 = (a^3b)^{1/4}, G_2 = \sqrt{ab}, G_3 = (ab^3)^{1/4}$

$G_1^4 = a^3b,\ G_2^4 = a^2b^2,\ G_3^4 = ab^3$

$\Rightarrow$ $G_1^4 + G_2^4 + G_3^4 + G_1^2G_3^2 = a^3b + a^2b^2 + ab^3 + a^2b^2$

$= ab(a^2 + 2ab + b^2)$

$= ab(a + b)^2$

$= G_1G_3(A_1 + A_2)^2$

6. A bag contains 6 white and 4 black balls. A die is rolled once and the number of balls equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is

(1) $\frac{11}{50}$ (2) $\frac{1}{4}$ (3) $\frac{9}{50}$ (4) $\frac{1}{5}$

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. Probability of all drawn balls white

$$= \frac{1}{6}\left[\frac{{}^6C_1}{{}^{10}C_1} + \frac{{}^6C_2}{{}^{10}C_2} + \frac{{}^6C_3}{{}^{10}C_3} + \frac{{}^6C_4}{{}^{10}C_4} + \frac{{}^6C_5}{{}^{10}C_5} + \frac{{}^6C_6}{{}^{10}C_6}\right]$$

$$= \frac{1}{6}\left[\frac{6}{10} + \frac{6\times5}{10\times9} + \frac{6\times5\times4}{10\times9\times8} + \frac{6\times5\times4\times3}{10\times9\times8\times7} + \frac{6\times120}{10\times9\times8\times7\times6} + \frac{24}{10\times9\times8\times7}\right]$$

$$= \frac{1}{10} + \frac{1}{18} + \frac{1}{36} + \frac{6}{9 \times 8 \times 7} + \frac{1}{36 \times 7} + \frac{4}{90 \times 56}$$

$$= \frac{1}{10} + \frac{1}{18} + \frac{1}{36} + \frac{1}{84} + \frac{1}{252} + \frac{1}{90 \times 14}$$

$$= \frac{1}{10} + \frac{1}{12} + \frac{1}{84} + \frac{1}{252} + \frac{1}{1260} = \frac{127}{1260} + \frac{8}{84} + \frac{1}{252}$$

$$= \frac{127}{1260} + \frac{2}{21} + \frac{1}{252} = \frac{254 + 240 + 10}{2520} = \frac{504}{2520} = \frac{1}{5}$$

7. If the domain of the function $f(x) = \log_e(4x^2 + 11x + 6) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha, \beta]$, then $36|\alpha + \beta|$ is equal to

(1) 45 (2) 54 (3) 63 (4) 72

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. $4x^2 + 11x + 6 > 0$ and $-1 \le 4x + 3 \le 1$ and $-1 \le \frac{10x+6}{3} \le 1$

$(4x + 3)(x + 2) > 0$ $\quad -4 \le 4x \le -2$ $\quad -3 \le 10x + 6 \le 3$

$x \in (-\infty, -2) \cup \left(-\frac{3}{4}, \infty\right)$ ....(i) $\quad -1 \le x \le -\frac{1}{2}$ ......(ii) $\quad -\frac{9}{10} \le x \le -\frac{3}{10}$ ......(iii)

So Domain of f(x) is $x \in \left(-\frac{3}{4}, -\frac{1}{2}\right]$

$36|\alpha + \beta| = 36\left|-\frac{3}{4} - \frac{1}{2}\right| = 45$

8. Let ABCD be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $(\overrightarrow{AB} - \overrightarrow{BC}) + (\overrightarrow{AD} - \overrightarrow{DC}) = k\overrightarrow{FE}$, then k is equal to

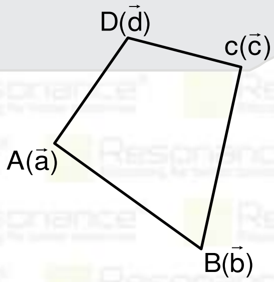
(1) 4 (2) – 4 (3) – 2 (4) 2

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. Let P.V. & A, B, C, D are $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ resp. mid-point of AC is E

$\Rightarrow$ P.V. of E are $\left(\frac{\vec{a} + \vec{c}}{2}\right)$



Mid point of BD is F

$\Rightarrow$ P.V of F are $\left(\frac{\vec{b} + \vec{d}}{2}\right)$

$$(\overrightarrow{AB} - \overrightarrow{BC}) + (\overrightarrow{AD} - \overrightarrow{DC}) = (\vec{b} - \vec{a}) - (\vec{c} - \vec{b}) + (\vec{d} - \vec{a}) - (\vec{c} - \vec{d})$$

$= 2\vec{b} - 2\vec{a} - 2\vec{c} + 2\vec{d}$

$= 2(\vec{b} + \vec{d}) - 2(\vec{a} + \vec{c})$

$= 4\left[\left(\frac{\vec{b} + \vec{d}}{2}\right) - \left(\frac{\vec{a} + \vec{c}}{2}\right)\right] = 4\,\overrightarrow{EF} = -\,4\overrightarrow{FE}$

9. Let $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$, a, b, c ∈ N. If $p_1 = 20$ and $p_2 = 210$, then 2(a + b + c) is equal to

(1) 6 (2) 12 (3) 15 (4) 8

NTA Ans. (2)

Reso Ans. (2)

Sol. General term = $\frac{10!}{r_1!r_2!r_3!}.a^{r_1}(bx)^{r_2}(cx^2)^{r_3}$

Coefficient of x' = $p_1 = \frac{10!}{9!\,1!\,\,0!}.a^9b = 20$

$a^9b = 2$ ....(1)

Coefficient of x^2 = $p_2 = \frac{10!}{8!\,2!\,\,0!}.a^8b^2 + \frac{10!}{9!\,0!\,1!}a^9c = 210$

$45\,a^8b^2 + 10a^9c = 210$

$9\,a^8b^2 + 2\,a^9c = 42$.......(2)

By (1) a, b, c ∈ N ⇒ a = 1, b = 2 , c = 3

Hence 2(a + b + c) = 12

10. If the set $\left\{\operatorname{Re}\left(\frac{z - \bar{z} + z\bar{z}}{2 - 3z + 5\bar{z}}\right) : z \in C, \operatorname{Re}(z) = 3\right\}$ is equal to the interval (α,β] , then 24 (β – α) is equal to

(1) 27 (2) 36 (3) 42 (4) 30

NTA Ans. (4)

Reso Ans. (4)

Sol. z = 3 + iy

$\operatorname{Re}\left(\frac{2iy + 9 + y^2}{8(1 - iy)}\right)$

$\operatorname{Re}\left(\frac{2iy + 9 + y^2}{8(1 - iy)} \times \frac{1 + iy}{1 + iy}\right)$

$\frac{9 + y^2 - 2y^2}{8(1 + y^2)} \in (\alpha, \beta]$

Let $t = \frac{9 - y^2}{8(1 + y^2)}$

$8t + 8ty^2 = 9 - y^2$

$y^2(8t + 1) = 9 - 8t$

$y^2 = \frac{9 - 8t}{8t + 1} \geq 0$

$t \in \left(-\frac{1}{8}, \frac{9}{8}\right]$

$\alpha = -\frac{1}{8}, \beta = \frac{9}{8}$

Now, $24\left(\frac{9}{8} + \frac{1}{8}\right) = 24 \times \frac{5}{4} = 30$

11. Let the system of linear equations

$-x + 2y - 9z = 7$

$-x + 3y + 7z = 9$

$-2x + y + 5z = 8$

$-3x + y + 13z = \lambda$

has a unique solutions $x = \alpha$, $y = \beta$, $z = \gamma$. Then the distance of the point (α, β, γ) from the plane]

$2x - 2y + z = \lambda$ is

(1) 9 (2) 11 (3) 13 (4) 7

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $-x + 2y - 9z = 7$ ....(i)

$-x + 3y + 7z = 9$ ....(ii)

$-2x + y + 5z = 8$ ....(iii)

Solve (i), (ii) & (iii) for x, y, z

$x = -3, y = 2, z = 0$

$\Rightarrow \alpha = -3, \beta = 2, z = 0$

$\Rightarrow \lambda = 11$

Now, Distance of point (–3, 2, 0) from plane:

$$= \frac{|-6-4-11|}{\sqrt{4+4+1}} = \frac{21}{3} = 7$$

12. The mean and standard deviation of 10 observation are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is

(1) 12 (2) 13 (3) 11 (4) 14

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $$\frac{\sum_{i=1}^{10} x_i}{10} = 20 \Rightarrow \sum x_i = 200$$

Correct $\sum x_i = 200 - 50 + 40 = 190$

$$\sigma^2 = \frac{\sum_{i=1}^{10} x_i^2}{10} - (20)^2 = 64$$

$\sum x_i^2 = 4640$

Correct $\sum x_i^2 = 4640 - 2500 + 1600 = 3740$

$$\text{Correct } \sigma^2 = \frac{3740}{10} - \left(\frac{190}{10}\right)^2$$

$= 374 - 361 = 13.$

13. Let S be the set of all values of λ, for which the shortest distance between the line $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13. Then $8\left|\sum_{\lambda\in S}\lambda\right|$ is equal to

(1) 306 (2) 304 (3) 302 (4) 308

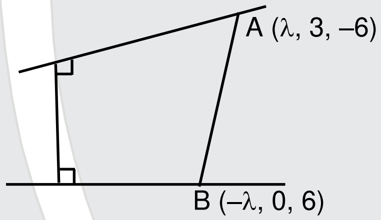
NTA Ans. **(1)**
Reso Ans. **(1)**

Sol. Given lines: $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ ......(i)

$\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ ....(ii)

DR of $\overrightarrow{AB}$: $2\lambda, 3, -12$

$\vec{p}\times\vec{q}$ is perpendicular both the lines



$$\vec{p}\times\vec{q}=\begin{vmatrix} i & j & k \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}$$

SD = 13

$$\left|\frac{\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = 13$$

$$\Rightarrow \left|\frac{8\lambda+153}{13}\right| = 13$$

$\Rightarrow 8\lambda = 16$, or $8\lambda = -322$

Now, $8\left|\sum_{\lambda\in S}\lambda\right| = |16-322| = 306$

14. The number of common tangent, to the circle $x^2+y^2-18x-15y+131=0$ and $x^2+y^2-6x-6y-7=0$, is

(1) 2 (2) 1 (3) 4 (4) 3

NTA Ans. **(4)**
Reso Ans. **(4)**

Sol. $C_1=\left(9,\frac{15}{2}\right)$, $r_1=\sqrt{81+\frac{225}{4}-131}=\sqrt{\frac{25}{4}}=\frac{5}{2}$

$C_2=(3,3)$, $r_2=\sqrt{9+9+7}=5$

$C_1C_2=\sqrt{6^2+\frac{81}{4}}=\frac{15}{2}$

$\Rightarrow C_1C_2=r_1+r_2$

$\Rightarrow$ No. of common tangents = 3

15. Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i}-\hat{j}+\hat{k},\ \hat{i}+2\hat{j}+\mu\hat{k}$ and $3\hat{i}-4\hat{j}+5\hat{k}$, where $\lambda-\mu=5$, are coplanar, then $\sum_{(\lambda,\mu)\in S}80(\lambda^2+\mu^2)$ is equal to

(1) 2210 (2) 2370 (3) 2130 (4) 2290

NTA Ans. (4)

Reso Ans. (4)

Sol. $\begin{vmatrix}\lambda & -1 & 1\\ 1 & 2 & \mu\\ 3 & -4 & 5\end{vmatrix}=0$

$\lambda(10+4\mu)+1(5-3\mu)+1(-4-6)=0$

$10\lambda+4\lambda\mu+5-3\mu-10=0$

$10\lambda+4\lambda\mu-3\mu=5$ ....(i)

And $\lambda=5+\mu$ ....(ii)

$10(5+\mu)+4\mu(5+\mu)-3\mu=5$

$10\mu+20\mu-3\mu+4\mu^2+50=5$

$4\mu^2+27\mu+45=0$

$4\mu^2+12\mu+15\mu+45=0$

$4\mu(\mu+3)+15(\mu+3)=0$

$\mu=-3$ or $-\frac{15}{4}$

$\lambda=5-3=2$ or $\lambda=5-\frac{15}{4}=\frac{5}{4}$

$\sum_{\lambda,\mu}80(\lambda^2+\mu^2)=\left(9+4+\frac{225}{16}+\frac{25}{16}\right)80=\left(13+\frac{250}{16}\right)80=1040+1250=2290$

16. Let the foot of perpendicular of the point P(3, – 2, – 9) on the plane passing through the points (–1, –2, –3), (9, 3, 4), (9, –2, 1) be Q (α, β, γ) . Then the distance of Q from the origin is

(1) $\sqrt{42}$ (2) $\sqrt{29}$ (3) $\sqrt{35}$ (4) $\sqrt{38}$

NTA Ans. (1)

Reso Ans. (1)

Sol. Equation of plane passing through A(–1, –2, –3), B(9, 3, 4) and C(9, –2, 1) is

$\begin{vmatrix}x+1 & y+2 & z+3\\ 10 & 5 & 7\\ 10 & 0 & 4\end{vmatrix}=0$

$\Rightarrow 2x+3y-5z=7$

Now for foot of point P(3, –2, –9)

$\frac{\alpha-3}{2}=\frac{\beta+2}{3}=\frac{\gamma+9}{-5}=-\left(\frac{6-6+45-7}{4+9+25}\right)=-1$

$\Rightarrow \alpha=1,\ \beta=-5,\ \gamma=-4$

$OQ=\sqrt{1+25+16}=\sqrt{42}$

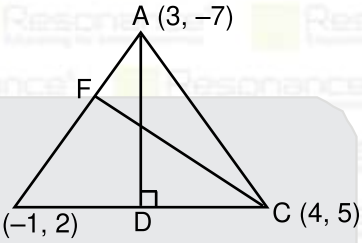
17. If (α, β) is the orthocentre of the triangle ABC with vertices A(3,–7), B (–1,2) and C(4,5) , then $9\alpha - 6\beta + 60$ is equal to

(1) 35 (2) 30 (3) 25 (4) 40

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $M_{BC} = \frac{5-2}{4+1} = \frac{3}{5}$



Equation of AD:

$y + 7 = -\frac{5}{3}(x - 3)$

$5x + 3y + 6 = 0$ ....(i)

$M_{AB} = \frac{2+7}{-1-3} = \frac{9}{-4}$

Equation of CF:

$y - 5 = \frac{4}{9}(x - 4)$

$4x - 9y + 29 = 0$ ......(ii)

from (i) & (ii)

$x = -\frac{47}{19}, y = \frac{121}{57}$

Now, $9\alpha - 6\beta + 60$

$= 9\left(-\frac{49}{19}\right) - 6\left(\frac{121}{57}\right) + 60$

$= \frac{475}{19} = 25$

18. Negation of $p \wedge (q \wedge \sim (p \wedge q))$ is

(1) $(\sim (p \wedge q)) \vee p$ (2) $(\sim (p \wedge q)) \wedge q$ (3) $\sim (p \vee q)$ (4) $p \vee q$

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. $= \sim\{P \wedge (q \wedge (\sim p \vee \sim q))\}$

$= \sim \{P \wedge \{(q \wedge \sim p) \vee (q \wedge \sim q)\}\}$

$= \sim\{p \wedge ((q \wedge \sim p) \vee c)\}$

$= \sim\{p \wedge (q \wedge \sim p)\}$

$= \sim\{(p \wedge q) \wedge (p \wedge \sim p)\}$

$= \sim\{(p \wedge q) \wedge c\}$

$= \sim(p \wedge q) \vee t$

$= t$

Now option 1:

$(\sim(p \wedge q)) \vee p$

$= (\sim p \vee \sim q) \vee p$

$= (\sim p \vee p) \vee (\sim q \vee p)$

$= t$

19. The number of real roots of the equation $x|x| - 5|x + 2| + 6 = 0$, is

(1) 4 (2) 3 (3) 6 (4) 5

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $x|x| - 5|x+2| + 6 = 0.$

Case-I $x \le -2,\ -x^2 + 5(x+2) + 6 = 0$

$x^2 - 5x - 16 = 0$

$x^2 - 5x + \frac{25}{4} = 16 + \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{89}{4}$

$x = \frac{5}{2} \pm \frac{\sqrt{89}}{2}$

$x = \frac{5-\sqrt{89}}{2}$ $\qquad x \ne \frac{5+\sqrt{89}}{2}$

Case-II $-2 < x < 0$

$-x^2 - 5x - 10 + 6 = 0$

$x^2 + 5x + 4 = 0 \Rightarrow x -1, -4$

$x = -1$ $\qquad x \ne -4$

Case-III $x \ge 0$

$x^2 - 5x - 10 + 6 = 0$

$x^2 - 5x - 4 = 0$

$x = \frac{5 \pm \sqrt{41}}{2}$

$x = \frac{5+\sqrt{41}}{2}$, $\qquad x \ne \frac{5-\sqrt{41}}{2}$

Number of solution = 3.

20. Let the determinant of a square matrix A of order m be m – n , where m and n satisfy 4m + n = 22 and 17m + 4n = 93 . If det (n adj(adj(mA))) = $3^a5^b6^c$, then a + b + c is equal to

(1) 84 (2) 96 (3) 109 (4) 101

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $17m + 4n = 93$ ....(i)

$4m + n = 22$ ....(ii)

from (i) & (ii) m = 5, n = 2

$|A| = m - n = 3$

Now |n adj(adj (MA))|

$= n^m |MA|^{(m-1)^2}$

$= n^m M^{m(m-1)^2} |A|^{(m-1)^2}$

$= 2^5.5^{80}.3^{16}$

$= 3^{11}5^{80}6^5$

a = 11, b = 80, c = 5

$\Rightarrow$ a + b + c = 96

21. Consider the triangle with vertices A(2,1), B (0,0) and C (t,4), $t \in [0,4]$. If the maximum and the minimum perimeters of such triangles are obtained at $t = \alpha$ and $t = \beta$ respectively, then $6\alpha + 21\beta$ is equal to

NTA Ans. **(48)**

Reso Ans. **(48)**

Sol. A(2, 1), B(0, 0), C(t, 4), $t \in [0, 4]$

Image of B in y = 4

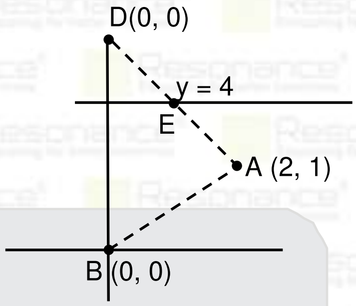
D(0, 8)

Equation of AD:

$$y - 1 = \frac{7}{-2}(x - 2)$$

$$7x + 2y = 16$$

$$E\left(\frac{8}{7}, 4\right) \Rightarrow \beta = \frac{8}{7}$$

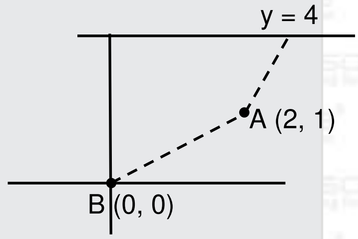


For maximum perimeter

Maximum perimeter at C(4, 4)

$\therefore t \in [0, 4]$

Now, $6\alpha + 21\beta = 6 \times 4 + 21\left(\frac{8}{7}\right) = 48$.



22. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$ and outside the circle $(x - 3)^2 + y^2 = 2$ is A, then $4(\pi + 4A)$ is equal to

NTA Ans. **(42)**

Reso Ans. **(42)**

Sol. By solving

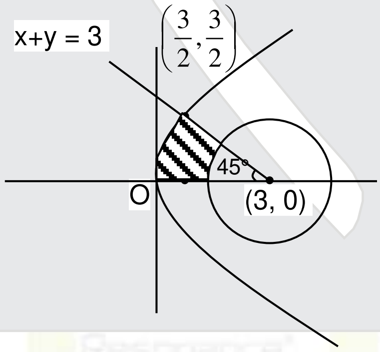
$x + y = 3$ with $2y^2 = 3x$

We have $x = \frac{3}{2}$ & $y = \frac{3}{2}$

So required area

$$= \int_0^{3/2} \sqrt{\frac{3}{2}x}\, dx + \int_{\frac{3}{2}}^{3} (3 - x) dx$$

–area of sector of circle



$$= \sqrt{\frac{3}{2}} \left. \frac{x^{3/2}}{3/2} \right|_0^{3/2} + \frac{1}{2}\left(\frac{3}{2}\right)\left(-\frac{3}{2}\right) - \frac{\pi}{4}\left(\frac{(\sqrt{2})^2}{2}\right)$$

$$A = \frac{3}{2} + \frac{9}{8} - \frac{\pi}{4} = \frac{21 - 2\pi}{8}$$

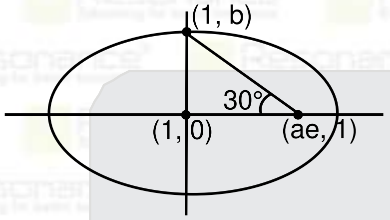
$\Rightarrow 4(\pi + 4A) = 42$

23. Let an ellipse with centre (1, 0) and latus rectum of length $\frac{1}{2}$ have its major axis along x-axis. If its minor axis subtends an angle 60º at the foci, then the square of the sum of the length of its minor and major axes is equal to

NTA Ans. (9)

Reso Ans. (9)

Sol.



Length of L. R. = $\frac{1}{2}$

$\frac{2b^2}{a} = \frac{1}{2} \Rightarrow 4b^2 = a$ …….(i)

$\tan 30° = \frac{b}{ae}$

$\frac{1}{\sqrt{3}} = \frac{b}{ae} \Rightarrow 3b^2 = a^2e^2 = a^2 - b^2 \Rightarrow 4b^2 = a^2$ ……… (ii)

from (i) & (ii)

$a^2 = a \Rightarrow a = 1$

$b^2 = \frac{1}{4}$

Now, $(2a + 2b)^2 = (2 + 1)^2 = 9$

24. A person forgets his 4 digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digit. Then the maximum number of trials necessary to obtain the correct code is

NTA Ans. (72)

Reso Ans. (72)

Sol. Let the correct ATM PIN is abcd. (a,b,c,d $\in$ digits) such that a + b = c + d

Let maximum {a, b} = 7 ⇒ a + b can take 7 ,8, 9, 10, 11

(0, 7) a + b = 7 ⇒ c + d = 7 case (1, 6), (2 ,5), (3, 4), (4, 3), (5, 2), (6, 1)

(1, 7) a + b = 8 ⇒ c + d = 8 case (2, 6), (3 ,5), (5, 3), (6, 2)

(2, 7) a + b = 9 ⇒ c + d = 9 case (3, 6), (4 ,5), (5, 4), (6, 3)

(3, 7) a + b = 10 ⇒ c + d = 10 case (4, 6), (6, 4)

(4, 7) a + b = 12 ⇒ c + d = 11 case (5, 6), (6, 5)

Hence total Number of allow = (18)

Now a, b can be in charged by 2 method and in place of a, b we can be take c, a also.

So total number of attempt = 18 ×2 ×2 = 72

25. Let $f(x) = \int \frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$, $|x| < \frac{2}{\sqrt{3}}$. If f(0) = 0 and f(1) = $\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$, α, β > 0, then $\alpha^2 + \beta^2$ is equal to

NTA Ans. (28)

Reso Ans. (28)

Sol. $f(x) = \int \frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$

Let $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}dt$

$= \int \frac{-\frac{1}{t^2}dt}{\left(3+\frac{4}{t^2}\right)\sqrt{4-\frac{3}{t^2}}}$

$= \int \frac{-t\,dt}{(3t^2+4)\sqrt{4t^2-3}}$

Now, let $4t^2 - 3 = p^2$

4t dt = p dp

$= \int \frac{-\frac{p}{4}dp}{\left(\frac{3p^2+25}{5}\right)p} = -\int \frac{dp}{3p^2+25}$

$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}p}{5}\right) + C$

$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\cdot\sqrt{4t^2-3}}{5}\right) + C$

$= -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\cdot\sqrt{\frac{4}{x^2}-3}}{5}\right) + C$

$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\cdot\sqrt{4-3x^2}}{5|x|}\right) + C$

x = 0, y = 0

$0 = -\frac{1}{5\sqrt{3}}\tan^{-1}(\infty) + C$

$C = \frac{1}{5\sqrt{3}} \times \frac{\pi}{2}$

$$f(x) = -\frac{1}{5\sqrt{3}}\tan^{-1}\frac{\sqrt{3}\sqrt{4-3x^2}}{5x} + \frac{\pi}{2(5\sqrt{3})}$$

$$f(1) = -\frac{1}{5\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{5} + \frac{\pi}{2(5\sqrt{3})}$$

$$= \frac{1}{5\sqrt{3}}\left(\frac{\pi}{2} - \tan^{-1}\frac{\sqrt{3}}{5}\right)$$

$$= \frac{1}{5\sqrt{3}}\cot^{-1}\frac{\sqrt{3}}{5}$$

$$= \frac{1}{5\sqrt{3}}\tan^{-1}\frac{5}{\sqrt{3}}$$

Now, $\alpha = 5$, $\beta = \sqrt{3}$

$\alpha^2 + \beta^2 = 25 + 3 = 28$

26. If the line $x = y = z$ intersects the line $x\sin A + y\sin B + z\sin C - 18 = 0 = x\sin 2A + y\sin 2B + z\sin 2C - 9$, where A, B, C are the angles of a triangle ABC, then $80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ is equal to

NTA Ans. (5)

Reso Ans. (5)

Sol. $\therefore x = y = z$

$$x = \frac{18}{\sin A + \sin B + \sin C} \text{ and } x = \frac{9}{\sin 2A + \sin 2B + \sin 2C}$$

$$\frac{18}{4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = \frac{9}{4[\sin A\sin B\sin C]}$$

$$\frac{2}{\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = \frac{1}{8\cdot\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\cdot\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$$\Rightarrow \sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2} = \frac{1}{16}$$

$$80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right) = 5$$

27. If the sum of the series $\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^2}-\frac{1}{2^2.3}+\frac{1}{2.3^2}-\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{2^3.3}+\frac{1}{2^2.3^2}-\frac{1}{2.3^3}+\frac{1}{3^4}\right)+.....$ is $\frac{\alpha}{\beta}$, where α and β are co-prime , then $\alpha + 3\beta$ is equal to ___

NTA Ans. (7)

Reso Ans. (7)

Sol. Let $x = \frac{1}{2}$, $y = \frac{4}{3}$

Now expression:

$(x - y) + (x^2 - xy + y^2) + (x^3 - x^2y + xy^2 - y^3) + \ldots\ldots\ldots$

$= \frac{1}{x+y}[(x^2 - y^2) + (x^3 + y^3) + (x^4 - y^4) + (x^5 + y^5) + \ldots\ldots..]$

$= \frac{1}{x+y}[(x^2 + x^3 + x^4 + x^5 + \ldots..) - (y^2 - y^3 + y^4 - y^5 + \ldots..)]$

$= \frac{1}{x+y}\left[\frac{x^2}{1-x} - \frac{y^2}{1+y}\right]$

Now put $x = \frac{1}{2}$, $y = \frac{1}{3}$

$\frac{\alpha}{\beta} = \frac{1}{2}$

$\Rightarrow \alpha = 1, \beta = 2$

$\alpha + 3\beta = 7$

28. Let A = {1, 2, 3, 4} and R be a relation on the set A × A defined by

R = {((a, b), (c, d)) : 2a + 3b = 4c + 5d}. Then the number of element in R is _____

NTA Ans. **(6)**

Reso Ans. **(6)**

Sol. A = {1, 2, 3, 4}

R = {((a, b)(c, d)) : 2a + 3b = 4c + 5d}

$2a \in \{2, 4, 6, 8\}$, $3b \in \{3, 6, 9, 12\}$, $4c \in \{4, 8, 12, 16\}$ $5d \in \{5, 10, 15, 20\}$

$$2a + 3b = \begin{cases} 5,8,11,14 \\ 7,10,13,20 \\ 9,12,15,18 \\ 11,14,17,20 \end{cases}$$

$$4c + 5d = \begin{cases} 9,14,19,24 \\ 13,18,23,28 \\ 17,22,27,32 \\ 21,26,31,36 \end{cases}$$

Now, 2a + 3b = 4c + 5d

for 9, 13, 14, 14, 17, 18

total pairs = 6

29. The number of element in the set $\{n \in N : 10 \le n \le 100 \text{ and } 3^n - 3 \text{ is a multiple of } 7\}$ is ______

NTA Ans. (15)

Reso Ans. (15)

Sol. $3^n - 3$ div by 7. $\Rightarrow 3^n = 7\lambda + 3$

$\Rightarrow 3^n = 3 \text{ (mod7) form}$

Now $\Rightarrow 3^1 \equiv 3 \text{ (mod7)}$

$\Rightarrow 3^2 \equiv 2 \text{ (mod7)}$

$\Rightarrow 3^3 \equiv 6 \text{ (mod7)}$

$\Rightarrow 3^4 \equiv 4 \text{ (mod7)}$

$\Rightarrow 3^5 \equiv 5 \text{ (mod7)}$

$\Rightarrow 3^6 \equiv L \text{ (mod7)}$

$\Rightarrow 3^7 \equiv 3 \text{ (mod7)}$

So we can say that $3^n - 3$ is divisible by 7 iff n = 1, 7, 13, 19, 25,……….

as $10 \le n \le 100$

$\Rightarrow n = 13, 19, 25, 31, \ldots\ldots, 97$

Hence number of element in set A is 15

30. Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$. Then the distance of the point A (8, –1, – 19) from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to _______

NTA Ans. (26)

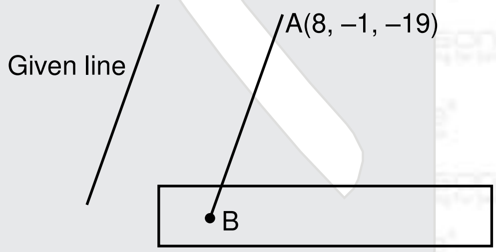
Reso Ans. (26)

Sol. Let equation of plane:

$(2x + y - z - 3) + \lambda(5x - 3y + 4z + 9) = 0$

or $x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) - 3 + 9\lambda = 0$

It is parallel to $\frac{x+2}{2} = \frac{y-3}{4} = \frac{z-7}{5}$



$\Rightarrow (2 + 5\lambda)2 + (1 - 3\lambda)4 + (-1 + 4\lambda)5 = 0$

$\Rightarrow 18\lambda + 3 = 0 \Rightarrow \lambda = -\frac{1}{6}$

Plane is $7x + 9y - 10z = 27$ …….(i)

Equation of line AB

$\frac{x-8}{-3} = \frac{y+1}{4} = \frac{z+19}{12} = \mu$

$B(-3\mu + 8, 4\mu - 1, 12\mu - 19)$

B lies on plane (i)

$7(-3\mu + 8) + 9(4\mu - 1) - 10(12\mu - 19) = 27$

$-105\mu + 210 = 0$

$\mu = 2$

B(2, 7, 5)

Now, AB = $\sqrt{36+64+576} = \sqrt{676} = 26$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