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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)
Official Based Questions & Solutions

Date: 24 January, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

61. Let y = y(x) be the solution of the differential equation $x^3dy + (xy - 1)\,dx = 0$, x > 0, $y\left(\frac{1}{2}\right) = 3 - e$. Then y(1) is equal to .

(1) 2 – e (2) e (3) 1 (4) 3

Ans. **(3)**

Sol. $x^3.dy + (xy - 1)dx = 0,\ y\left(\frac{1}{2}\right) = 3 - e$

$$\frac{dy}{dx} = \frac{1-xy}{x^3}$$

$$\frac{dy}{dx} + \frac{1}{x^2}.y = \frac{1}{x^3}$$

$$\text{I.F.} = e^{\int \frac{1}{x^2}dx} = e^{-\frac{1}{x}}$$

$$y.\ e^{-\frac{1}{x}} = \int \frac{1}{x^3}.e^{-\frac{1}{x}}dx + C,\ \text{put}\quad -\frac{1}{x} = t$$

$$\frac{1}{x^2}dx = dt$$

$$= \int -t.e^t\,dt$$

$$= -\left(e^t(t-1)\right) + C$$

$$ye^{-\frac{1}{x}} = -e^{-\frac{1}{x}}\left(\frac{-1}{x} - 1\right) + C$$

$$y = 1 + \frac{1}{x} + Ce^{1/x}$$

$$3 - e = 1 + 2 + C\,.\,e^2$$

$$C = -\frac{1}{e}$$

$$y = 1 + \frac{1}{x} - \frac{1}{e}.\ e^{\frac{1}{x}}$$

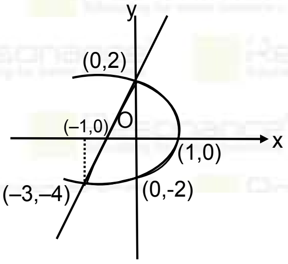
$$y(1) = 1 + 1 - \frac{1}{e}.\,e = 1$$

62. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is:

(1) $\frac{23}{3}$ (2) $\frac{22}{3}$ (3) 9 (4) $\frac{25}{3}$

Ans. **(3)**

Sol.



Required area = $\int_{-3}^{0}\sqrt{4-4x}\,dx - \frac{1}{2}(2)(4) + \frac{1}{2}(1)(2) + 2\int_{0}^{1}\sqrt{4-4x}\,dx$

$= 2\int_{-3}^{0}\sqrt{1-x}\,dx + 4\int_{0}^{1}\sqrt{1-x}\,dx - 3$

$= -2 \times \frac{2}{3}\left[(1-x)^{3/2}\right]_{-3}^{0} - 4\frac{2}{3}\left[(1-x)^{3/2}\right]_{0}^{1} - 3$

$= -\frac{4}{3}(1-8) - \frac{8}{3}(0-1) - 3$

$= \frac{28}{3} + \frac{8}{3} - 3 = 9$

63. Let Ω be the sample space and $A \subseteq \Omega$ be an event.
Given below are two statements:
(S1) : If P(A) = 0, then A = $\emptyset$
(S2) : If P(A) = 1, then A = Ω

(1) both (S1) and (S2) are true
(2) only (S1) is true
(3) only (S2) is true
(4) both (S1) and (S2) are false

Ans. (1)

Sol. both (S1) and (S2) are true

64. The value of $\sum_{r=0}^{22} {}^{22}C_r\, {}^{23}C_r$ is

(1) ${}^{45}C_{24}$
(2) ${}^{45}C_{23}$
(3) ${}^{44}C_{23}$
(4) ${}^{44}C_{22}$

Ans. (2)

Sol. $\sum_{r=0}^{22} {}^{22}C_{22-r}\, {}^{23}C_r$

${}^{22+23}C_{22} = {}^{45}C_{22} = {}^{45}C_{23}$

65. Let α be a root of the equation $(a-c)x^2 + (b-a)x + (c-b) = 0$ where a, b, c are distinct real numbers such that the matrix $\begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$ is singular. Then, the value of

$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$ is

(1) 6
(2) 3
(3) 9
(4) 12

Ans. (2)

Sol. 'α' is a root of given quadratic equation

$(a-c)\alpha^2 + (b-a)\alpha + (c-b) = 0$ ……(1)

and matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular so,

$\Rightarrow \alpha^2(c-b) + \alpha(a-c) + (b-a) = 0$ …….(2)

both equation (1) and (2) identical

$\frac{a-c}{c-b} = \frac{b-a}{a-c} = \frac{c-b}{b-a}$

$\frac{(a-c)^2}{(c-b)(b-a)} = 1$ ....(3)

$\frac{(b-a)^2}{(a-c)(c-b)} = 1$ ....(4)

$\frac{(c-b)^2}{(a-c)(b-a)} = 1$ ....(5)

by adding equation (3) , (4) and (5) we get $\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)} = 3$

66. $\lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(1) n^2 (2) $n^2 + n$ (3) n (4) $\frac{n(n+1)}{2}$

Ans. **(3)**

Sol. $\lim_{t\to 0}\left(n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}\left[1+\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t} + \left(\frac{2}{n}\right)^{\operatorname{cosec}^2 t} + \ldots\ldots\left(\frac{n-1}{n}\right)^{\operatorname{cosec}^2 t}\right]^{\sin^2 t}$

as $0 < \frac{1}{n} < 1$, So $\left(\frac{1}{n}\right)^{\infty} = 0$

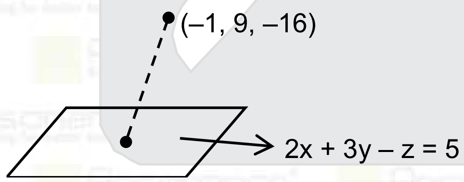
So, $n\,[1 + 0]^0 = n$

67. The distance of the point (–1, 9,–16) from the plane 2x + 3y – z = 5 measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

(1) $20\sqrt{2}$ (2) 31 (3) $13\sqrt{2}$ (4) 26

Ans. **(4)**

Sol. We find the equation of line which is parallel to line



$\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ and passes through the point (–1, 9, –16)

So line of equation a

$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12} = r$

$x = 3r - 1$
$y = 9 - 4r$
$z = 12r - 16$

The above point is lies or plane

So, $2(3r - 1) + 3(9 - 4r) - (12r - 16) = 5$

$\Rightarrow -18r + 41 = 5$

$\Rightarrow 18r = 36$
$r = 2$
So point is (5, 1, 8)
distance = $\sqrt{6^2 + (8)^2 + 24^2}$
= $\sqrt{36 + 64 + 576}$ = 26

68. Let N denote the number that turns up then a fair die is rolled. If the probability that the system of equations.
$x + y + z = 1$
$2x + Ny + 2z = 2$
$3x + 3y + Nz = 3$
has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is
(1) 18 (2) 19 (3) 20 (4) 21

Ans. **(3)**

Sol. N = {1, 2, 3, 4, 5, 6}
$\Delta \neq 0$ for unique solution

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix} \neq 0$$

$1(N^2 - 6) - 1(2N - 6) + 1(6 - 3N) \neq 0$
$N^2 - 5N + 6 \neq 0$
$N \neq 2,3$
So, we take only
N = {1,4,5,6}
Probability = $\frac{4}{6}$
K = 4
required ans is (1+ 4 + 5 + 6) + 4 = 20

69. Let f(x) = $\begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , \; x \neq 0 \\ 0 & , \; x = 0 \end{cases}$

then at x = 0
(1) f is continuous but not differentiable (2) f is continuous but f' is not continuous
(3) f' is continuous but not differentiable (4) f and f' both are continuous

Ans. **(2)**

Sol. at x = 0

$$\text{LHD} = \lim_{h\to 0^+} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0^+} \frac{-h^2\sin(1/h)}{-h} = \lim_{h\to 0} \frac{\sin(1/h)}{(1\backslash h)} = 0$$

$$\text{RHD} = \lim_{h\to 0^+} \frac{f(o+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{h^2\sin(1/h)}{h} = \lim_{h\to 0^+} \frac{\sin(1/h)}{1/h} = 0$$

$\Rightarrow$ f(x) is continuous and differential at x = 0

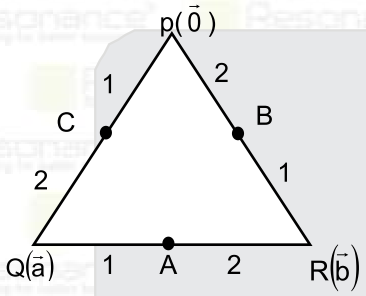
Now $f'(x) = \begin{cases} 2x\sin(1/x) - \cos(1/x) & , \; x \neq 0 \\ 0 & , \; x = 0 \end{cases}$

clearly f' (x) is discontinuous at x = 0

70. Let PQR be a triangle. The point A,B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR}=\frac{RB}{BP}=\frac{PC}{CQ}=\frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

(1) 4 (2) 3 (3) 2 (4) $\frac{5}{2}$

Ans. **(2)**



Let position vector of P, Q R be $\bar{o}, \bar{a}$ & $\bar{b}$ respectively

$\Rightarrow$ P.V of C = $\frac{\bar{a}}{3}$, P.V of A = $\frac{2\bar{a}+\bar{b}}{3}$ and P.V of B = $\frac{2\bar{b}}{3}$

$\therefore \overline{CA} = \frac{\vec{a}+\vec{b}}{3}$ & $\overline{AB} = \frac{\bar{b}-2\bar{a}}{3}$

$\Delta PQR = \frac{1}{2}\left|\overline{PQ}\times\overline{PR}\right| = \frac{1}{2}\left|\bar{a}\times\bar{b}\right|$

$\Delta ABC = \frac{1}{2}\left|\overline{CA}\times\overline{AB}\right| = \frac{1}{2}\left|\left(\frac{(\bar{a}+\bar{b})}{3}\times\frac{(\bar{b}-2\bar{a})}{3}\right)\right| = \frac{1}{2}\left|\frac{\bar{a}\times\bar{b}}{3}\right|$

$= \frac{\Delta PQR}{\Delta ABC} = 3$

71. For the positive integers $p,q,r, x^{pq^2} = y^{qr} = z^{p^2r}$ and r = pq + 1 such that 3, $3\log_y x$, $3\log_z y$, $7\log_x z$ are in A.P with common difference $\frac{1}{2}$. The r –p–q is equal to

(1) 6 (2) 2 (3) 12 (4) –6

Ans. **(2)**

Sol. Here

$\frac{3\log x}{\log y} - 3 = \frac{1}{2} \Rightarrow \frac{\log x}{\log y} = \frac{7}{6}$ ....(1)

$\Rightarrow \frac{3\log y}{\log z} - \frac{3\log y}{\log z} = \frac{1}{2} \Rightarrow \frac{\log y}{\log z} = \frac{4}{3}$ ....(2)

$\Rightarrow \frac{7\log z}{\log x} - \frac{3\log y}{\log 2} = \frac{1}{2} \Rightarrow \frac{\log z}{\log x} = \frac{9}{14}$ ....(3)

Now by $x^{pq^2} = y^{qr} = z^{p^2r}$

$pq^2\log x = qr \log y = p^2r \log z$ ......(4)

by equation (1), (2) , (3) and (4)

$\frac{\log x}{\log y} = \frac{r}{pq} = \frac{7}{6}$ ......(5)

$\frac{\log y}{\log z} = \frac{p^2}{q} = \frac{4}{3}$ ......(6)

$\frac{\log z}{\log x} = \frac{q^2}{pr} = \frac{9}{14}$ ......(7)

r = pq + 1 (given) ....(8)

By solving equation (5), (6), (7) & (8) we get

r = 7, p = 2 & q = 3

So, r–p–q = 7– 2 – 3 = 2

72. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to:

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Ans. **(4)**

Sol. $E = \tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)}\right) + \sec^{-1}\left(\frac{16+8\sqrt{3}}{12+6\sqrt{3}}\right)^{1/2}$

$\because \frac{16+8\sqrt{3}}{12+6\sqrt{3}} = \frac{4(\sqrt{3}+1)^2}{(3+\sqrt{3})^2}$

$\therefore \left(\frac{16+8\sqrt{3}}{12+6\sqrt{3}}\right)^{1/2} = \frac{2(\sqrt{3}+1)}{3+\sqrt{3}} = \frac{2}{\sqrt{3}}$

$E = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$E = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

73. The equation $x^2 - 4x + [x] + 3 = x[x]$, where [x] denotes the greatest integer function, has:

(1) no solution (2) exactly two solutions $(-\infty, \infty)$

(3) a unique solution in $(-\infty, 1)$ (4) a unique solution in $(-\infty, \infty)$

Ans. **(4)**

Sol. $x^2 - x[x] - x - 3x + [x] + 3 = 0$

$x^2 - x[x] - (x - [x]) - 3(x - 1) = 0$

$x(x - [x]) - 1(x - [x]) - 3(x - 1) = 0$

$(x - 1)(x - [x]) - 3(x - 1) = 0$

$(x - 1)(x - [x] - 3) = 0$

$\Rightarrow x = 1$ as $\{x\} \neq 3$

74. Let $p, q \in R$ and $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$, $i = \sqrt{-1}$ then $p + q + q^2$ and $p - q + q^2$ are roots of the equation

(1) $x^2 - 4x + 1 = 0$ (2) $x^2 + 4x + 1 = 0$ (3) $x^2 - 4x - 1 = 0$ (4) $x^2 + 4x - 1 = 0$

Ans. **(1)**

Sol. $2^{200}\left(\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\right)^{200} = 2^{200}\left(\cos\frac{200\pi}{3} - i\left(\sin\frac{200\pi}{3}\right)\right)$

$= 2^{200}\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$

$= 2^{200}\left(\frac{-1}{2} - \frac{\sqrt{3}i}{2}\right)$

$= 2^{199}\left(-1 - \sqrt{3}i\right) \Rightarrow P = -1, q = -\sqrt{3}$

roots are $(p - q + q^2, p + q + q^2) = \left(2 + \sqrt{3}, 2 - \sqrt{3}\right)$

equation is $x^2 - 4x + 1 = 0$

75. If A and B are two non-zero n × n matrices such that $A^2 + B = A^2 B$, then

(1) $A^2 = I$ or $B = I$ (2) $A^2 B = BA^2$ (3) $AB = I$ (4) $A^2 B = I$

Ans. **(2)**

Sol. $A^2 + B = A^2B$

$A^2 - A^2B + B = 0$

$A^2(I - B) + B - I = -I$

$A^2(I - B) - (I - B) = -I$

$(A^2 - I)(B - I) = I = (B - I)(A^2 - B)$

$A^2B - B - A^2 + I = BA^2 - B - A^2 + I$

$A^2B = BA^2$

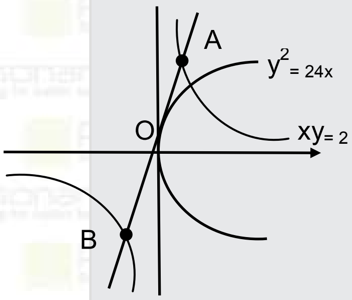
76. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at points A and B. Then the mid points of such line segments AB lie on a parabola with the

(1) length of latus rectum $\frac{3}{2}$ (2) length of latus rectum 2

(3) directrix $4x = 3$ (4) directrix $4x = -3$

Ans. **(3)**

Sol.



$y^2 = 24x$ .... (1)

$xy = 2$ .... (2)

let mid-point of chord AB of $xy = 2$

be m (x_1, y_1)

$\therefore$ equation of AB be $T = S_1$

$$\frac{x(y_1) + y(x_1)}{2} - 2 = x_1y_1 - 2$$

$$\Rightarrow x(y_1) + y(x_1) = 2x_1y_1 \;\; \Rightarrow y(x_1) = -x(y_1) + 2x_1y_1$$

$$\Rightarrow y = x\left(\frac{-y_1}{x_1}\right) + 2y_1 \;\; ...(3)$$

$\because$ (3) is tanget to (1)

$$\therefore \; c = \frac{a}{m} \;\; \Rightarrow 2y_1 = -\frac{6x_1}{y_1}$$

$$\therefore \; y_1^2 = -3x$$

$\therefore$ locus of mid-point m (x_1, y_1) is

$y^2 = -3x$

length of latus rectum = 3

Directrix $\Rightarrow x = a$

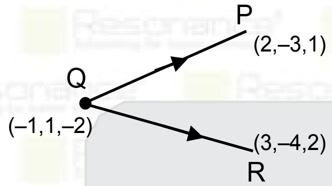
$x = \frac{3}{4} \Rightarrow 4x = 3$

77. The distance of the point (7,–3,–4) form the plane passing through the points (2,–3,1), (–1,1,–2) and (3,–4, 2) is

(1) $5\sqrt{2}$ (2) $4\sqrt{2}$ (3) 4 (4) 5

Ans. (1)

Sol.



$$\hat{n} = \overline{QP} \times \overline{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 3 \\ 4 & -5 & 4 \end{vmatrix}$$

$= \hat{i}(-16+15) - \hat{j}(12-12) + \hat{k}(-15+16)$

$= -\hat{i} + 0\hat{j} + \hat{k}$

So equation of plane

$-1(x + 1) - 0(y - 4) + 1(z + 2) = 0$

$-x + z + 1 = 0$

$\because$ distance $= \dfrac{|-7-4+1|}{\sqrt{2}} = \dfrac{10}{\sqrt{2}} = 5\sqrt{2}$

78. The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

(1) $((\sim(P) \vee Q) \wedge ((\sim Q) \vee P)$ (2) $(\sim Q) \vee P)$

(3) $((\sim(P) \vee Q) \wedge (\sim Q)$ (4) $(\sim P) \vee Q)$

Ans (1)

Sol. $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q)) \Rightarrow ((\sim P) \wedge (\sim Q))$

$(\sim P \vee \sim Q) \vee (\sim P \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$

$[(\sim P \vee \sim Q) \vee (\sim P)] \wedge [(\sim P \vee \sim Q) \vee Q] \Rightarrow ((\sim P) \wedge (\sim Q))$

$(\sim P \vee \sim P) \vee (\sim Q \vee \sim P) \Rightarrow ((\sim P) \wedge (\sim Q))$

$\sim P \vee \sim Q \vee \sim P \Rightarrow ((\sim P) \wedge (\sim Q))$ $[\because a \Rightarrow b = \sim a \vee b]$

$\sim(\sim P \vee \sim Q) \vee ((\sim P) \wedge (\sim Q))$

$(P \wedge Q) \vee (\sim P \wedge \sim Q)$

$[(P \wedge Q) \vee \sim P] \wedge [(P \wedge Q) \vee \sim Q]$

$[(P \vee \sim P) \wedge (Q \vee \sim P)] \wedge [(P \vee \sim Q) \wedge (Q \vee \sim Q)]$

$(Q \vee \sim P) \wedge (P \vee \sim Q)$

$((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$ Ans.

79. The relation R = {(a,b) : gcd (a,b) = 1 ; 2a $\neq$ b, a, b $\in$ z} is:
(1) reflexive but not symmetric
(2) transitive but not reflexive
(3) symmetric but not transitive
(4) neither symmetric not transitive

Ans. (3)

Sol. (1) Reflexive
Let a $\in$ z

aRa $\Rightarrow$ gcd(a,a) = a

$\Rightarrow$ R is not reflexive

(2) Let a, b $\in$ z and aRb

So, aRb $\Rightarrow$ gcd (a,b) = 1

$\Rightarrow$ gcd (b,a) = 1, $\forall$ a, b $\in$ z $\Rightarrow$ b Ra, $\forall$ a, b $\in$ z $\Rightarrow$ R is symmetric relation on z

80. **Let** $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}, \vec{v} = 2\hat{i} + \hat{j} - \hat{k}, \vec{v}.\vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u}.\vec{w}$ is equal to

(1) 1 (2) 2 (3) $\frac{3}{2}$ (4) $-\frac{2}{3}$

Ans. (3)

Sol. Given $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$

$(\vec{v} \times \vec{w}).\vec{v} = \vec{u}.\vec{v} + \lambda\vec{v}.\vec{v}$

$0 = \vec{u}.\vec{v} + \lambda|v|^2$

$$\lambda = -\left(\frac{\vec{u}.\vec{v}}{|v|^2}\right) = -\left(\frac{2-1+2}{6}\right) = -\frac{1}{2}$$

Now, $(\vec{v} \times \vec{w}).\vec{w} = \vec{u}.\vec{w} + \lambda\,\vec{v}.\vec{w}$

$\Rightarrow 0 = \vec{u}.\vec{w} + 2\lambda \Rightarrow \vec{u}.\vec{w} = -2\lambda$

$\Rightarrow \vec{u}.\vec{w} = -2\left(-\frac{1}{2}\right) = 1$

81. Let $\lambda \in$ R and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set S= {x + λ ; x is an integer solution of E} is

Ans. (5)

Sol. Let l $\in$ R and Let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set S = {x + λ : x is an integer of solution E} is.

$$|x| = \frac{2 \pm \sqrt{4 - 4|\lambda - 3|}}{2 \times 1}$$

$|x| = 1 \pm \sqrt{1 - |\lambda - 3|}$

$1 - |\lambda - 3| \geq 0$

$|\lambda - 3| \leq 1$

$-1 \leq \lambda - 3 \leq 1$

$+2 \leq l \leq 4$

here 'x' is an integer so , possible value of λ = 2, 3, 4

for $\lambda = 3 \Rightarrow |x| = 2 , 0 \Rightarrow x = \pm 2, 0$

$\lambda = 2 \Rightarrow |x| = 1, \Rightarrow x = \pm 1$

$\lambda = 4 \Rightarrow |x| = 1, \Rightarrow x = \pm 1$

So, S = {5,1,3}

hence largest elements of S is 5

82. A boy needs to select five courses from 12 available courses, out of which 5 courses are language, course. If he can choose at most 2 language courses, then the number of ways he can choose five courses is

Ans. **(546)**

Sol. Required number of ways to select = $^7C_5 . ^5C_0 + ^7C_4 . ^5C_1 + ^7C_3 . ^5C_2$

= 21 + 175 +350

= 546

83. The value of $\frac{8}{\pi}\int_0^{\pi/2} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ is

Ans. **(2)**

Sol. $I = \frac{8}{\pi}\int_0^{\pi/2} \frac{(\cos x)^{2023}}{(\cos x)^{2023} + (\sin x)^{2023}} dx$ ......(i)

Applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$I = \frac{8}{\pi}\int_0^{\pi/2} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ ....(ii)

add (i) and (ii), we get

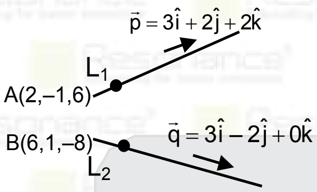
$2I = \frac{8}{\pi}\int_0^{\pi/2} (1)dx = \frac{8}{\pi}\left(\frac{\pi}{2}\right)$

$\therefore I = 2$

84. The shortest distance between the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to

Ans. (14)

Sol.



Required Shortest Distance = $\left|\frac{\overline{AB}.(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$\because p\times q=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 2\\ 3 & -2 & 0\end{vmatrix}$

$= \hat{i}(4)-\hat{j}(-6)+\hat{k}(-12)$

$= 4\hat{i}+6\hat{j}-12\hat{k}$

$\because \overline{AB}=4\hat{i}+2\hat{j}-14\hat{k}$

$\because$ Distance = $\left|\frac{16+12+168}{\sqrt{16+36+144}}\right|$ = $\frac{196}{14}$ = 14 Ans.

85. The 4th term of GP is 500 and its common ratio is $\frac{1}{m}, m\in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is

Ans. (12)

Sol. Given 4th term of GP = 500

C.R. = $\frac{1}{m}$

$\frac{a}{m^3}$ = 500 ...(1)

$S_n = \frac{a\left[\left(\frac{1}{m}\right)^n-1\right]}{\frac{1}{m}-1}$

$S_n = \frac{a\left(1-m^n\right)}{m^n\frac{(1-m)}{m}}$

$S_n = \frac{a\left(1-m^n\right)}{m^{n-1}(1-m)}$

$S_6 > S_5 + 1$

$\frac{a(1-m^6)}{m^5(1-m)} > \frac{a(1-m^5)}{m^4(1-m)} + 1$

$a - am^6 > m^5 - m^6 + am - am^6$

$a(1 - m) > m^5(1 - m)$

$a > m^5$ ... (2)

$S_7 < S_6 + \frac{1}{2}$

$\frac{a(1-m^7)}{m^6(1-m)} < \frac{a(1-m^6)}{m^5(1-m)} + \frac{1}{2}$

$a - am^7 < am - am^7 + \frac{1}{2}m^6 - \frac{m^7}{2}$

$a(1 - m) < \frac{1}{2}m^6(1-m)$

$a < \frac{m^6}{2}$ ...(3)

So $m^5 < a < \frac{m^6}{2}$ ... (4)

$m^2 < \frac{a}{m^3} < \frac{m^3}{2}$

$m^2 < 500 < \frac{m^3}{2}$

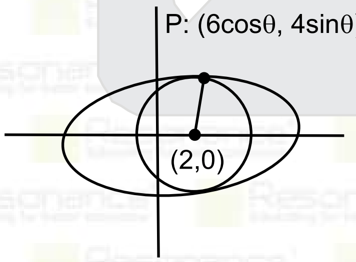
$m \in [11, 22]$

so, no. of possible value of m is 12

86. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. If $(1,\alpha)$ lies on C, then $10\,\alpha^2$ is equal to ……….

Ans. **(118)**

Sol.



equation of ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

$\frac{x}{18}+\frac{yy'}{8}=0$

$y' = \frac{-x}{18}\cdot\frac{8}{y}$

$Y_N = \frac{9y}{4x} = 9.\frac{4\sin\theta}{4.6\cos\theta} = \frac{3}{2}\frac{\sin\theta}{\cos\theta}$

$Y_N(\text{Slope}) = \frac{4\sin\theta}{6\cos\theta-2} = \frac{2\sin\theta}{3\cos\theta-1}$

$\frac{3}{2}\frac{\sin\theta}{\cos\theta} = \frac{2\sin\theta}{3\cos\theta-1}$

$\cos\theta = \frac{3}{5}$

$P : \left(\frac{18}{5}, \frac{16}{5}\right)$

$S = \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{16}{5}\right)^2}$

Now $(x-2)^2 + y^2 = r^2$

$(1, \alpha) \rightarrow 1 + \alpha^2 = \frac{8^2}{5^2} + \frac{16^2}{5^2} = 12.8$

$\alpha^2 = 11.8$

$10\alpha^2 = 118$

87. Let a tangent to the curve $9x^2 + 16y^2 = 144$, intersects the coordinate axes at A and B. Then the minimum length of the line segment AB is

Ans. **(7)**

Sol. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Equation of tangent $\frac{x}{4}\cos\theta + \frac{y}{3}\sin\theta = 1$

A $(4\sec\theta, 0)$ B$(0, 3(\text{cosec}\theta)$

$AB = \sqrt{16\sec^2\theta + 9\text{cosec}^2\theta} = \sqrt{25 + (4\tan\theta - 3\cot\theta)^2 + 24} \geq \sqrt{49} \geq 7$

$AB_{min} = 7$

88. Suppose $\sum_{r=0}^{2023} r^2 . {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is

Ans. **(1012)**

Sol. $\sum r.r^{2023}C_r = \sum r.(2023)^{2022}C_{r-1}$

$= 2023\sum r.^{2022}C_{r-1}$

$= 2023\left(\sum_{1}^{2023}(r-1)^{2022}C_{r-1} + {}^{2022}C_{r-1}\right)$

$= 2023\left(\left(\sum_{r=2}^{2023}(2022)^{2021}C_{r-2}\right) + \sum_{r=1}^{2023}{}^{2022}C_{r-1}\right) = 2023(2022.2^{2021} + 2^{2022})$

$= 2^{2022}.2023(1011+1) = 2^{2022}(1012).(2023)$

$\because \alpha = 1012$

89. The value of $12\int_0^3 \left|x^2 - 3x + 2\right| dx$ is

Ans. **(22)**

Sol. $I = \int_0^3 |(x-1)(x-2)|dx$

$I = \int_0^1 (x^2 - 3x + 2)dx + \int_1^2 (-x^2 + 3x - 2)dx + \int_2^3 (x^2 - 3x + 2)dx$

$= \left[\frac{x^2}{3} - \frac{3x^2}{2} + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2 + \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_2^3$

$= 2\left(\frac{1}{3} - \frac{3}{2} + 2\right) - 2\left(\frac{8}{3} - \frac{12}{2} + 4\right) + \left(\frac{27}{3} - \frac{27}{2} + 6\right)$

$= \frac{2}{3} - \frac{6}{2} + 4 - \frac{16}{3} + \frac{24}{2} - 8 + \frac{27}{3} - \frac{27}{2} + 6$

$= \frac{13}{3} - \frac{9}{2} + 2$

$= \frac{26-27}{6} + 2 = \frac{11}{6}$

$\Rightarrow 12I = 12 \times \frac{11}{6} = 22$

90. The number of 9 digit numbers, that can be formed using all the digit of the number 123412341 so that the even digits occupy only even places, is

Ans. **(60)**

Sol. Four even digits at four places in $\frac{4!}{2!\ 2!}$ = 6 ways

Five odd digits at odd places = $\frac{5!}{3!\ 2!}$ = 10 ways

Required number of numbers = 60 ways.

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