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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 24 January, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 06.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | **FAX No. :** +91-022-39167222

To Know more : sms **RESO** at **56677** | **Website :** www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN :** U80302RJ2007PLC024029

Toll Free : 1800 258 5555 | 7340010333 | facebook.com/ResonanceEdu | twitter.com/ResonanceEdu | www.youtube.com/resowatch | blog.resonance.ac.in

PART : MATHEMATICS

61. If the foot of the perpendicular drawn from (1 , 9, 7) to the line passing through the point (3, 2,,1) and parallel to the planes x +2y +z = 0 and 3y – z =3 is (α,β,γ), the $\alpha + \beta + \gamma$ is equal to

(1) –1 (2) 1 (3) 3 (4) 5

Ans. **(4)**

Sol. D.R's of line be

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$$

$i(-2-3) - j(-1) + \hat{k}(3)$

$= -5i + j + 3\hat{k}$

Equation of line

$$\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3} = \lambda$$

$N(-5\lambda+3, \lambda+2, 3\lambda+1)$

D.R's of AN, : $-5\lambda + 2, \lambda - 7, 3\lambda - 6$

Now, $(-5\lambda + 2)(-5) + (\lambda - 7)(1) + (3\lambda - 6)3 = 0$

$25\lambda - 10 + \lambda - 7 + 9\lambda - 18 = 0$

$35\lambda = 35 \Rightarrow \lambda = 1$

$N(-2, 3, 4)$

$\alpha + \beta + \gamma = -2 + 3 + 4 = 5$

62. The number of real solutions of equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(1) 4 (2) 3 (3) 2 (4) 0

Ans. **(4)**

Sol. $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$

Let $x + \frac{1}{x} = t$; $t \in (-\infty - 2] \cup [2, \infty)$

$3t^2 - 6 - 2t + 5 = 0$

$3t^2 - 2t - 1 = 0$

$t = 1, t = \frac{-1}{3}$ But $t \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow$ No. real solution

63. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let the $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2.(\hat{i} + \hat{j} + \hat{k})$ is.

(1) 9 (2) 11 (3) 7 (4) 6

Ans. **(3)**

Sol. $\vec{B}_1 = \lambda(4i + 3j + 5k)$, $\vec{B}_2.\alpha = 0$

$\vec{B} = \vec{B}_1 + \vec{B}_2$

$\vec{B}\vec{\alpha} = \vec{B}_1.\vec{\alpha} + \vec{B}_2.\alpha$

$-10 = \lambda(50) \Rightarrow \lambda = -1/5$

$\vec{B}_2 = \vec{B} - \vec{B}_1 = (\hat{i} + 2\hat{J} - 4\hat{K}) + \frac{1}{5}(4\hat{i} + 3\hat{J} + 5\hat{K})$

$\vec{B}_2 = \frac{(9\hat{i} + 13\hat{J} - 15\hat{K})}{5}$

Now , $5B_2.(\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$

64. If $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in R$ then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{3}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$ is equal to

(1) 1011 (2) 2010 (3) 1010 (4) 2011

Ans. **(1)**

Sol. $f(x) = \frac{4^x}{4^x + 2}$

$\because\ f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2} = \frac{4^x+2}{4^x+2} = 1$

$f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{3}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$

= 1 + 1+ .... + 1 = 1011
(1011 time)

65. The value of $\left(\frac{1+\sin\frac{2\pi}{9} + i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9} - i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $-\frac{1}{2}\left(1 - i\sqrt{3}\right)$ (2) $\frac{1}{2}\left(\sqrt{3} + i\right)$ (3) $-\frac{1}{2}\left(\sqrt{3} - i\right)$ (4) $\frac{1}{2}\left(1 - i\sqrt{3}\right)$

Ans. **(3)**

Sol. $\left(\frac{1+\sin\frac{2\pi}{9} + i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9} - i\cos\frac{2\pi}{9}}\right)^3 = \left(\frac{1+\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18} - i\sin\frac{5\pi}{18}}\right)^3$

$= \left(\frac{2\cos^2\frac{5\pi}{36} + 2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36} - 2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}\right)^3$

$= \left(\frac{\cos\frac{5\pi}{36} + i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36} - i\sin\frac{5\pi}{36}}\right)^3 = \left(\cos\frac{10\pi}{36} + i\sin\frac{10\pi}{36}\right)^3$

$= \left(\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right)^3 = \left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$= -\frac{\sqrt{3}}{2} + i\frac{1}{2}$

66. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}$ dx is equal to

(1) 2π (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Ans. (1)

Sol. $\frac{48}{2}\left(\text{Sin}^{-1}\left(\frac{2x}{3}\right)\right)_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} = 24\left(\text{Sin}^{-1}\frac{\sqrt{3}}{2} - \text{Sin}^{-1}\frac{1}{\sqrt{2}}\right)$

$= 24\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = 24\left(\frac{\pi}{12}\right) = 2\pi$

67. Let y = y[x] be the solution of the differential equation $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$, y(1) = 1. Then $6y^2(e)$ is equal to

(1) e^2 (2) $\frac{3}{2}e^2$ (3) $3e^2$ (4) $2e^2$

Ans. (4)

Sol. D.E. $\rightarrow \frac{2ydy}{dx} - \frac{2y^2}{x} = \frac{-2x}{3}$

Let $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

D.E. $\rightarrow \frac{dt}{dx} - \frac{2t}{x} = -\frac{2x}{3}$

I.F. $= e^{\int -\frac{2}{x}dx} = e^{-2\ell n|x|} = e^{\ell n\frac{1}{x^2}} = \frac{1}{x^2}$

solution $\Rightarrow t.\frac{1}{x^2} = \int \frac{1}{x^2}\left(\frac{-2x}{3}\right)dx + c \Rightarrow \frac{y^2}{x^2} = \frac{-2}{3}\ell n|x| + c$

$\therefore$ x = 1, y = 1 $\Rightarrow$ 1 = 0 + c $\Rightarrow$ c = 1

$\Rightarrow \frac{y^2}{x^2} = -\frac{2}{3}\ell n|x| + 1$

$x = e \Rightarrow \frac{y^2(e)}{e^2} = \frac{-2}{3} + 1 \Rightarrow y^2(e) = \frac{e^2}{3} \Rightarrow 6y^2(e) = 2e^2$

68. Let f(x) be a function such that f(x +y) = f(x). f(y) for all x,y $\in$ N. If f(1) = 3 and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(1) 8 (2) 6 (3) 7 (4) 9

Ans. (3)

Sol. f(x+y) = f(x) f(y)

$f(k) = (f(1))^k = 3^k$

Now, $\sum_{k=1}^{n} f(k) = 3 + 3^2 + 3^3 + \ldots\ldots + 3^n$

$\Rightarrow \frac{3(3^n - 1)}{3-1} = 3279$

$\Rightarrow 3^n = 2187 = 3^7$

$\Rightarrow n = 7$

69. The set of all values of a for which $\lim\limits_{x\to a}([x-5]-[2x+2])=0$ where $[\alpha]$ denotes the greatest integer lass than or equal to α is equal to

(1) [–7.5, –6.5) (2) (–7.5, –6.5) (3) (–7.5, –6.5] (4) [–7.5, – 6.5]

Ans. **(2)**

Sol. $\lim\limits_{x\to a}([x-5]-[2x+2])=0$

$\lim\limits_{x\to a}([x]-5-[2x]-2)=0$

$\lim\limits_{x\to a}([x]-[2x])=7$

Let $a\in[n,n+\frac{1}{2})$ then n – 2n = 7

n = –7

$\Rightarrow a\in[-7,-6.5)$

Let $a\in[n+\frac{1}{2},n+1)$ then

n – (2n +1) = 7

– n = 8

n = –8

$a\in[-7\frac{1}{2},-7)$

but limit does not exists at a = – 7.5

Hence $a\in(-7.5,-6.5)$

70. The equations of the sides AB and AC of a triangle ABC are (λ + 1) x + λy = 4 and λx +(1 - λ) y + λ = 0 respectively. its vertex A is on the y –axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is:

(1) $2\sqrt{2}$ (2) 2 (3) $\sqrt{6}$ (4) 4

Ans. **(1)**

Sol. equation of AB : (λ+1) x + λy – 4= 0

equation of AC : λx + (1–λ)y + λ = 0

$$x=\frac{\lambda^2+4(1-\lambda)}{(\lambda+1)(1-\lambda)-\lambda^2}=0$$

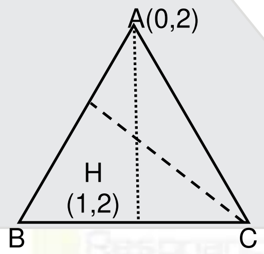
$\Rightarrow \lambda^2 - 4\lambda + 4 = 0 \Rightarrow \lambda = 2$

equation of line AB : 3x + 2y = 4

equation of line AC : 2x – y = –2

A(0, 2)

c(α, 2α+ 2)

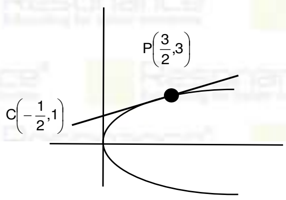


$M_{CH} = 2/3$

$$\frac{2\alpha}{\alpha-1}=\frac{2}{3}\Rightarrow\alpha=-\frac{1}{2}$$

$c\left(-\frac{1}{2},1\right)$

equation fo tangent : $y = mx + \frac{3}{2m}$



if passes through $\left(-\frac{1}{2},1\right)$

$\Rightarrow m^2 + 2m - 3 = 0$

$\Rightarrow m = 1, -3$

point of contact in first quadrant = $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = (3/2, 3)$

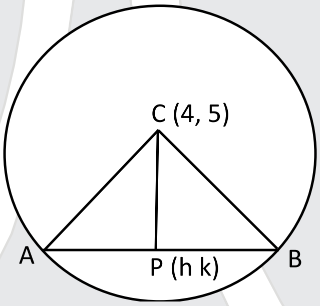
Length of tangent pc = $\sqrt{(3-1)^2 + \left(\frac{3}{2}+\frac{1}{2}\right)^2} = \sqrt{8} = 2\sqrt{2}$

71. The locus of the mid point of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend and angle θ_1 a t the centre of the circle C_1, is circle of radius r_1. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$ then θ_2 is equal to

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{3\pi}{4}$

Ans. **(1)**

Sol. In ΔPCB



$\cos\frac{\theta}{2} = \frac{PC}{2}$

$PC = 2\cos\frac{\theta}{2}$

$(h-4)^2 + (k-s)^2 = \left(2\cos\frac{\theta}{2}\right)^2$

$r_1 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$r_3 = 2\cos\frac{\pi}{3} = 1$

$r_2 = 2\cos\frac{\theta_2}{2}$

Now, $r_1^2 = r_2^2 + r_3^2$

$3 = 4\cos^2\frac{\theta_2}{2} + 1$

$\cos^2\frac{\theta_2}{2} = \frac{1}{2}$

$\theta_2 = \frac{\pi}{2}$

72. Let p and q be two statements. Then $\sim(P \wedge (P \Rightarrow \sim q))$ is equivalent to-

(1) $p \vee (p \wedge q)$ (2) $((\sim p) \vee q$ (3) $p \vee (p \wedge (\sim q))$ (4) $p \vee ((\sim p) \wedge q)$

Ans. **(2)**

Sol. $\sim P \vee \sim (p \rightarrow \sim q)$

$\Rightarrow \sim P \vee (p \wedge q)$

$\Rightarrow (\sim p \vee p) \wedge (\sim p \vee q)$

$\Rightarrow t \wedge (\sim p \vee q)$

$\Rightarrow \sim p \vee q$

73. If $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3)$, $x \in R$ then

(1) $2 f(0) - f(1) + f(3) = f(2)$

(2) $f(3) - f(2) = f(1)$

(3) $3f(1) + f(2) = f(3)$

(4) $f(1) + f(2) + f(3) = f(0)$

Ans. **(1)**

Sol. $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$ ....(1)

$f'(x) = 3x^2 - 2x f'(1) + f''(2)$ ....(2)

$f''(x) = 6x - 2 f'(1)$ ....(3)

$f'''(x) = 6 \Rightarrow f'''(3) = 6$

from (3)$\rightarrow f''(2) = 12 - 2 f'(1)$ ....(4)

from (2) $\rightarrow f'(1) = 3(1)^2 - 2 f'(1) + f''(2)$

$\Rightarrow f''(2) = 3 f'(1) - 3$ ....(5)

$\Rightarrow 12 - 2 f'(1) = 3 f'(1) - 3 \Rightarrow f'(1) = 3$

$f''(2) = 12 - 6 = 6$

$f(x) = x^3 - 3x^2 + 6x - 6$, $f(0) = -6$

$f(1) = -2$, $f(2) = 2$, $f(3) = 12$

74. Let the plane containing the line of intersection of the planes P1 : $x + (\lambda + 4)y + z = 1$ and P2 : $2x + y + z = 2$ pass through the points (0, 1,0) and (1,0,1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$

(1) $5\sqrt{6}$ (2) $2\sqrt{6}$ (3) $3\sqrt{6}$ (4) $4\sqrt{6}$

Ans. **(3)**

Sol. Eq. of plane containing the line of intersection of planes is given by

$p_1 + k p_2 = 0$

$X + (\lambda + 4)\, y + z - 1 + k\,(2x + y + z - 2) = 0$

It passes through (0, 1, 0) & (1, 0, 1)

$\lambda + 3 + k\,(1 - 2) = 0 \Rightarrow \lambda - k = -3$ ....(1)

And $(1 + 1 - 1) + k\,(2 + 1 - 2) = 0 \Rightarrow 1 + k = 0$

$\Rightarrow k = -1, \ \lambda = -4$

Now given point (-8, -4, 4)

Its distance from plane $p_2 = \left|\dfrac{-16-4+4-2}{\sqrt{6}}\right| = 3\sqrt{6}$

75. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in AP. and $a_1 + a_3 = 10$, If the mean of these six numbers $\frac{19}{2}$ and their variance is σ^2 , then $8\sigma^2$ is equal to .

(1)105 (2) 210 (3) 200 (4) 220

Ans. **(2)**

Sol. $\dfrac{\frac{6}{2}[2a+5d]}{6} = \dfrac{19}{2}$

$2a + 5d = 19$.......(i)

$a_1 + a_3 = 10$

$a + a + 2d = 10$

$2a + 2d = 10$......(ii)

from (i) & (ii)

$d = 3$

$a = 2$

Numbers are 2, 5, 8, 11, 14, 17

$\sigma^2 = \dfrac{2^2+5^2+8^2+11^3+14^2+17^2}{6} - \left(\dfrac{19}{2}\right)^2$

$= \dfrac{699}{6} - \left(\dfrac{19}{2}\right)^2 = \dfrac{233}{2} - \dfrac{361}{4}$

$\sigma^2 = \dfrac{466-361}{4} = \dfrac{105}{4}$

$\Rightarrow 8\sigma^2 = 210$

76. If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots\ldots\ldots + 30.\ ({}^{30}C_{30})^2 = \dfrac{\alpha 60!}{30!\,30!}$ then α is equal to

(1) 60 (2) 10 (3) 15 (4) 30

Ans. **(3)**

Sol. LHS $= \sum_{r=1}^{30} r\cdot\left({}^{30}C_r\right)^2$

$= \sum_{r=1}^{30} r\,.{}^{30}C_r\,.{}^{30}C_r = \sum_{r=1}^{30} r\cdot\dfrac{30}{r}.{}^{29}C_{r-1}.{}^{30}C_r$

$= 30\sum_{r=1}^{30} {}^{29}C_{r-1}.{}^{30}C_r$

$= 30 \times$ coefficient of x^{29} in $(1+x)^{59}$

$= 30 \times {}^{59}C_{29} \times \dfrac{60}{30} \times \dfrac{30}{60}$

$= \dfrac{900}{60} \times {}^{60}C_{30} = 15 \cdot {}^{60}C_{30}$

$\Rightarrow \alpha = 15$

77. The number of square matrices of order 5 with entries form the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 225 (2) 120 (3) 125 (4) 150

Ans. **(2)**

Sol. Total number of matrices = 5.4.3.2.1 = 5! = 120

78. The number of integers, greater than 7000 that can be formed, using the digits 3,5,6,7,8 without repetition, is

(1) 168 (2) 120 (3) 220 (4) 48

Ans. **(1)**

Sol. Number can be 5 digits of 4 digits

5 digits numbers = 5!

4 digits numbers = $2 \times 4 \times 3 \times 2 = 48$

Total numbers = 120 + 48 = 168

79. Let A be a 3×3 matrix such that $|adj\,(adj\,(adj\,A))| = 12^4$ Then $|A^{-1}\,adj\,A|$ is equal to

(1) 1 (2) $\sqrt{6}$ (3) 12 (4) $2\sqrt{3}$

Ans. **(4)**

Sol. $|\,Adj\,(\,Adj\,(\,Adj\,A\,)\,)\,|$

$= |Adj\,(Adj\,A)|^2$

$= |Adj\,A|^4$

$\Rightarrow |A|^8 = 12^4$

$\Rightarrow |A| = 12^{1/2}$

Now, $|A^{-1}\,Adj\,A| = |A^{-1}|\,|Adj\,A|$

$= \dfrac{1}{|A|^2}|A|^2 = |A| = 12^{1/2} = 2\sqrt{3}$

80. If the system of equation

$x + 2y + 3z = 3$

$4x + 3y - 4z = 4$

$8x + 4y - \lambda z = 9 + \mu$

has infinitely many solution, then the ordered pair (λ, μ) is equal to:

(1) $\left(\frac{75}{5}, -\frac{21}{5}\right)$ (2) $\left(\frac{72}{5}, \frac{21}{5}\right)$ (3) $\left(-\frac{72}{5}, -\frac{21}{5}\right)$ (4) $\left(-\frac{72}{5}, \frac{21}{5}\right)$

Ans. **(1)**

Sol. $D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & -4 \\ 8 & 4 & -\lambda \end{vmatrix} = 0$

$\Rightarrow 5\lambda = 72 \quad \Rightarrow \quad \lambda = 72/5$.

and $D_3 = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & 4 \\ 8 & 4 & 9 + -4 \end{vmatrix} = 0$

$\Rightarrow 5\mu + 21 = 0$

$\mu = -21/5$.

81. Let f be a differentiable function defined on $\left[0,\frac{\pi}{2}\right]$ such that f(x) > 0 and $f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt = e, \forall x \in \left[0,\frac{\pi}{2}\right]$. Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to.

Ans. (27)

Sol. $f(x) = \int_0^x f(t)\sqrt{1-\ell n(f(x)))^2}\,dt = e : \forall x \in \left[0, \frac{\pi}{2}\right]$

Differentiate on both side

$f'(x) + f(x)\sqrt{1-\ell n f(x)))^2} = 0$

$\int \frac{f'(x)dx}{f(x)\sqrt{1-\ell n f(x)))^2}} = -\int dx$

Let $\ell n f(x) = u$

$\frac{1}{f(x)} . f'(x)\,dx = du$

$\int \frac{du}{\sqrt{1-u^2}} = -\int dx$

$\sin^{-1}(\ell n(f(x)) = -x + c$

put x = 0

$\sin^{-1}(\ell n(f(0)) = 0 + c$

$\sin^{-1}(1) = c$

$c = \frac{\pi}{2}$

$\sin^{-1}(\ell n(f(x)) = -x + \frac{\pi}{2}$

$\ell n(f(x)) = \sin\left(\frac{\pi}{2} - x\right) = \cos x$

$f(x) = e^{\cos x}$

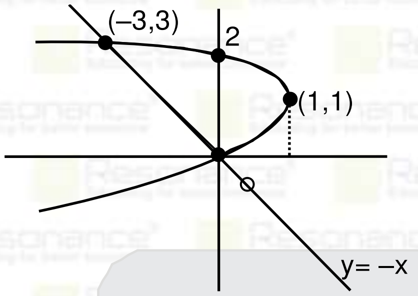
$f\left(\frac{\pi}{6}\right) = e^{\frac{\sqrt{3}}{2}}$

Now $\left(6\ell n e^{\frac{\sqrt{3}}{2}}\right)^2 = \left(6 \times \frac{\sqrt{3}}{2}\right)^2 = 27$

82. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8 A is equal to

Ans. **(36)**

Sol.



$$A = \int_0^3 \left[\left(2y - y^2\right) - (-y)\right]dy = \int_0^3 \left(3y - y^2\right)dy = \left[\frac{3y^2}{2} - \frac{y^3}{3}\right]_0^3$$

$= \frac{3\times 9}{2} - \frac{27}{3} = \frac{9}{2}$ So, 8A = 36

83. Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}.\vec{c} = 7, 2\vec{b}.\vec{c} + 43 = 0, \vec{a}\times\vec{c} = \vec{b}\times\vec{c}$. Then $|\vec{a}.\vec{b}|$ is equal to

Ans. **(8)**

Sol. $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}.\vec{c} = 7$

$2\vec{b}.\vec{c} + 43 = 0$, $\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$

$\vec{a}\times\vec{c} - \vec{b}\times\vec{c} = 0$

$(\vec{a} - \vec{b})\,x\vec{c} = 0$

$\Rightarrow \vec{a} - \vec{b} = \mu\vec{c}$

$\Rightarrow \vec{a} = \vec{b} + \mu\vec{c}$

Now $\vec{a}.\vec{a} = \vec{a}.\vec{b} + \mu\vec{a}.\vec{c}$

$\Rightarrow 2\lambda^2 + 12 = 7\mu$ .....(1)

and $\vec{a}.\vec{b} = \vec{b}.\vec{b} + \mu\vec{b}.\vec{c}$

$\frac{43\mu}{2} = 41 + 2\lambda^2$ ....(2)

From (1) and (2)

$\mu = 2$, $\lambda^2 = 1$

Now $\vec{a}.\vec{b} = -7 - \lambda^2$

$= -7 - 1 = -8$

$|\vec{a}.\vec{b}| = 8$

84. The minimum number of elements that must be added to the relation R = {(a,b), (b,c), (b,d)} on the set {a, b, c, d) so that it is an equivalence relation, is .

Ans. **(13)**

Sol. **Sol.**

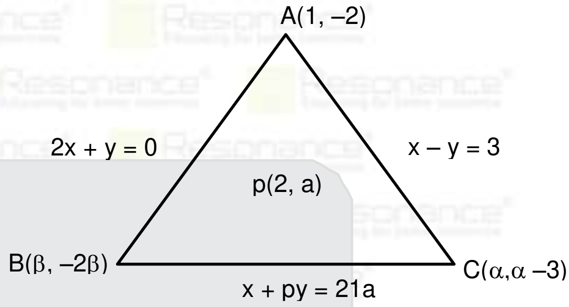
Reflexive	(a,a),	(b,b),	(c,c),	(d,d)
Symmetric	(b,a),	(c,b),	(d,b)	
Transitive	(a,c)	(a,d)	(d,c)	
	↓	↓	↓	
	(c,a)	(d,a)	(c,d)	

So 13 ordered pair

85. The equation of the sides AB, BC and CA of triangle ABC are 2x+ y = 0, x + py =21α, (a ≠ 0) and x – y = 3 respectively. Let P(2,a) be the centroid of ΔABC. Then $(BC)^2$ is equal to

Ans. **(122)**

Sol. Now $\frac{1+\alpha+\beta}{3} = 2$



$\alpha + \beta = 5$ ....(1)

and $\frac{-2-2\beta+\alpha-3}{3} = a \Rightarrow \alpha - 2\beta = 3a + 5$ ....(2)

from (1) and (2) $\alpha = a + 5,\ \beta = -a$

B(–a, 2a), C(a + 5, a + 2)

B and C lies on x + py = 21a

$-a + p(2a) = 21a \Rightarrow p = 11$

and $a + 5 + 11(a + 2) = 21a$

$\Rightarrow a = 3$

B(–3, 6), C(8, 5), $BC^2 = 122$

86. Let the sum of coefficient of first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$. $n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is-

Ans. **(405)**

Sol. ${}^nC_0 - {}^nC_1.3 + {}^nC_2.3^2 = 376$

$\Rightarrow n=10$

Now $T_{r+1} = {}^{10}C_r.\left(\frac{-3}{x^2}\right)^r.(x)^{10-r}$

$= {}^{10}C_r.(-3)^r.x^{10-3r}$

For $x^4 \Rightarrow 10 - 3r = 4 \Rightarrow r = 2$

Coff. of $x^4 = {}^{10}C_2.(-3)^2 = 405$

87. There urns A, B and C contain 4 Red , 6 black balls, 5 Red , 5 black and λ red 4 black balls. respectively. One of the urns is selected at random and a ball is drawn. It the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola.

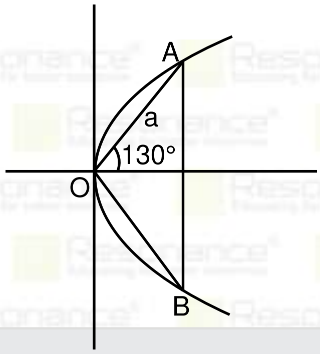
Ans. **(432)**

Sol. Probability = $\dfrac{\frac{1}{3}.\frac{\lambda}{10}}{\frac{1}{3}.\frac{4}{10}+\frac{1}{3}.\frac{5}{10}+\frac{1}{3}.\frac{\lambda}{10}} = 0.4$

$\Rightarrow \frac{\lambda}{9+\lambda} = \frac{2}{5} \Rightarrow 5\lambda = 18 + 2\lambda$

$\Rightarrow \lambda = 6$

Parabola $y^2 = 6x$



Let side length of triangle be a

$\Rightarrow$ A (acos 30°, a Sin 30°) lies on $y^2 = 6x$

$\Rightarrow \left(\frac{a}{2}\right)^2 = 6\left(\frac{\sqrt{3}a}{2}\right) \Rightarrow \frac{a}{4} = 3\sqrt{3}$

$\Rightarrow a = 12\sqrt{3}$

$\Rightarrow a^2 = 432$

88. It the shortest between the line $\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$ is 6, then he square of all possible values of λ is

Ans. **(384)**

Sol. $A\left(-\sqrt{6}, \sqrt{6}, \sqrt{6}\right)$

$B\left(\lambda, 2\sqrt{6}, -2\sqrt{6}\right)$

$\overrightarrow{AB} = (\lambda + \sqrt{6})\hat{i} + \sqrt{6}\,\hat{j} - 3\sqrt{6}\,\hat{k}$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$

$\vec{p} \times \vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

S.D. = $\left|\frac{\overrightarrow{AB}.(\vec{p}\times\vec{q})}{\vec{p}\times\vec{q}}\right| = \frac{\left|-(\lambda+\sqrt{6}) + 2\sqrt{6} + 3\sqrt{6}\right|}{\sqrt{6}} = 6$

$\left|-\lambda + 4\sqrt{6}\right| = 6\sqrt{6}$

$\lambda - 4\sqrt{6} = \pm\, 6\sqrt{6}$

$\lambda = 10\sqrt{6}\,,\, -\,2\sqrt{6}$

Sum of all values of $\lambda = 8\sqrt{6}$

So, sum of square of these values of λ = 384

89. If $\dfrac{1^3+2^3+3^3+\ldots\text{up to n terms}}{1.3+2.5+3.7+\ldots\ldots\text{uptonterms}}=\dfrac{9}{5}$, then the value of n is

Ans. (5)

Sol. $\dfrac{\left(\dfrac{n(n+1)}{2}\right)^2}{\sum\limits_{r=1}^{n} r(2r+1)}=\dfrac{9}{5},\ \dfrac{\left(\dfrac{n(n+1)}{2}\right)^2}{2\sum\limits_{r=1}^{n} r^2+\sum\limits_{r=1}^{n} r}=$

$\Rightarrow \dfrac{\left(\dfrac{n(n+1)}{2}\right)^2}{2\left(\dfrac{n(n+1)(2n+1)}{6}\right)+\left(\dfrac{n(n+1)}{2}\right)}=\dfrac{9}{5}$

$\Rightarrow \dfrac{\left[\dfrac{n(n+1)}{2}\right]^2}{\dfrac{n(n+1)}{2}\left[\dfrac{2(2n+1)}{3}+1\right]}=\dfrac{9}{5}$

$\Rightarrow \dfrac{\left[\dfrac{n(n+1)}{2}\right]}{\dfrac{4n+5}{3}}=\dfrac{9}{5}$

$\Rightarrow 5n^2 - 19n - 30 = 0$

$\Rightarrow (5n + 6)\ (n-5) = 0$

$\Rightarrow n = \dfrac{-6}{5},\quad n = 5$

90. Let S $=\{\theta\in[0, 2\pi): \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$

Then $\sum\limits_{\theta\in S}\sin^2\left(\theta+\dfrac{\pi}{4}\right)$ is equal to

Ans. (2)

Sol. $\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0$

$\tan(\pi\cos\theta) = -\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta) = \tan(-\pi\sin\theta)$

$\pi\cos\theta = n\pi - \pi\sin\theta,\ n\in I$

$\sin\theta + \cos\theta = n$

$\therefore \sin\theta + \cos\theta \in \left[-\sqrt{2}, \sqrt{2}\right]$

Then possible value of n = –1, 0, 1

Now $\sin\theta+\cos\theta = 1$	$\sin\theta+\cos\theta = 0$	$\sin\theta+\cos\theta = -1$
$\theta = 0, \dfrac{\pi}{2},$	$\tan\theta = -1$	
	$\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4},$	$\theta = \dfrac{3\pi}{2}, \pi$

So, $\sum\limits_{\theta\in S}\sin^2\left(\theta+\dfrac{\pi}{4}\right) = 4\left(\dfrac{1}{2}\right)+2(0) = 2$

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