JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 29 January, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

61. Let the tangents at the points A (4, –11) and B (8, –5) on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$, intersect at the point C. Then the radius of the circle, whose centre is C and the line joining A and B is its tangent, is equal to

(1) $2\sqrt{13}$ (2) $\frac{2\sqrt{13}}{3}$ (3) $\sqrt{13}$ (4) $\frac{3\sqrt{3}}{4}$

NTA Ans. **(2)**
Reso Ans. **(2)**

Sol. Equation of line AB is $y + 5 = \frac{3}{2}(x-8)$

$2y + 10 = 3x - 24$

$\Rightarrow 3x - 2y - 34 = 0$ ………………. (1)

let C be (h, k) then equation of AB

$hx + ky - \frac{3}{2}(x+h) + 5(y + k) - 15 = 0$

$x\left(h - \frac{3}{2}\right) + y(k+5) - \frac{3}{2}h + 5k - 15 = 0$ ….. (2)

comparing (1) and (2)

$\frac{h - \frac{3}{2}}{3} = \frac{k+5}{-2} = \frac{\frac{-3}{2}h + 5k - 15}{-34}$

$(h, k) \equiv \left(8, \frac{-28}{3}\right)$

$\therefore$ required radius = length of perpendicular drawn for (h, k) to line AB = $\frac{2\sqrt{13}}{3}$

62. Three rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X, respectively, then $10(\mu^2 + \sigma^2)$ is equal to

(1) 250 (2) 20 (3) 25 (4) 30

NTA Ans. **(2)**
Reso Ans. **(2)**
Sol.

x_i	0	1	2	3
p_i	$\frac{35}{210} = \frac{1}{6}$	$\frac{105}{210} = \frac{1}{2}$	$\frac{3\times 21}{210} = \frac{3}{10}$	$\frac{7}{210} = \frac{1}{30}$

$\mu = \Sigma p_i x_i = \frac{1}{2}\times 1 + \frac{3}{10}\times 2 + \frac{1}{30}\times 3 = \frac{1}{2} + \frac{3}{5} + \frac{1}{10} = \frac{5+6+1}{10} = \frac{6}{5}$

$\sigma^2 = \Sigma p_i x_i^2 - \mu^2 = \frac{1}{2} + \frac{3}{10}\times 4 + \frac{1}{30}\times 9 - \frac{36}{25} = \frac{14}{25}$

$10(\mu^2 + \sigma^2) = 10\left(\frac{36}{25} + \frac{14}{25}\right) = 20$

63. Let $A = \left\{(x, y) \in R^2 : y \geq 0, 2x \leq y \leq \sqrt{4-(x-1)^2}\right\}$ and

$$B = \left\{(x, y) \in R \times R : 0 \leq y \leq \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}$$

Then the ratio of the area of A to the area of B is

(1) $\frac{\pi}{\pi+1}$ (2) $\frac{\pi-1}{\pi+1}$ (3) $\frac{\pi+1}{\pi-1}$ (4) $\frac{\pi}{\pi-1}$

NTA Ans. (2)

Reso Ans. (2)

Sol. $2x \leq y \leq \sqrt{4-(x-1)^2} : y \geq 0$

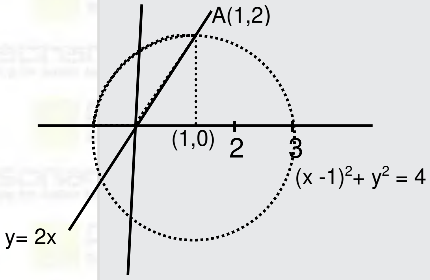


figure -1

area of A : area of shaded Region as show in above figure -1

$\therefore$ area of A = $\frac{\pi \times 4}{2} - \left(\frac{1}{2} \times 1 \times 2\right)$

$= \pi - 1$

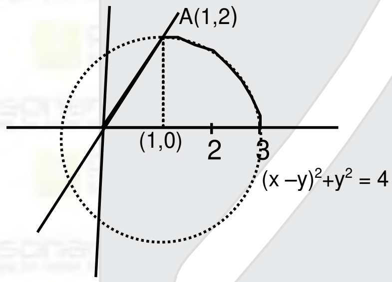


figure -2

area of B : area of shaded Region in as show in above figure -2

$\therefore$ area of B = $\frac{1}{2} \times 1 \times 2 + \frac{\pi \times 4}{4} = (1+\pi)$

$$\frac{A}{B} = \frac{\text{area of A}}{\text{area of B}} = \frac{\pi-1}{\pi+1}$$

64. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation

(1) $7x^2 + 245x - 250 = 0$ (2) $49x^2 + 245x + 250 = 0$

(3) $49x^2 - 245x + 250 = 0$ (4) $7x^2 - 245x + 250 = 0$

NTA Ans. (3)

Reso Ans. (3)

Sol. $14\alpha^2 - 31\alpha + 3\lambda = 0$ ......(i)

$35\alpha^2 - 53\alpha + 4\lambda = 0.$ ......(ii)

$\because$ α will satisfy both give equations

$\therefore\ 14\alpha^2 - 31\alpha + 3\lambda = 0 \Rightarrow \alpha^2 - \frac{31\alpha}{14} + \frac{3\lambda}{14} = 0$ .....(iii)

$35\alpha^2 - 53\alpha + 4\lambda = 0 \Rightarrow \alpha^2 - \frac{53\alpha}{35} + \frac{4\lambda}{35} = 0$ .....(iv)

from (iii) – (iv) we get

$\lambda = 7\alpha$ put in ... (iii)

$\alpha^2 - \frac{31\alpha}{14} + \frac{3}{14}(7\alpha) = 0$

$\alpha^2 - \frac{31\alpha}{14} + \frac{3\alpha}{2} = 0$

$\Rightarrow \alpha = 0$ or $\alpha = \frac{5}{7}$

($\alpha = 0$ not acceptable as $\lambda \neq 0$)

$\therefore\ \alpha = \frac{5}{7}$

$\because\ \alpha + \beta = \frac{31}{14} \Rightarrow \beta = \frac{3}{2}$

$\because\ \alpha + \gamma = \frac{53}{35} \Rightarrow \gamma = \frac{4}{5}$

$\therefore\ \frac{3\alpha}{\beta} = \frac{10}{7}$ and $\frac{4\alpha}{\gamma} = \frac{25}{7}$

$\therefore$ Reqd. Q. E is $\left(x - \frac{10}{7}\right)\left(x - \frac{25}{7}\right) = 0$

$(7x - 10)(7x - 25) = 0$

$49x^2 - 245x + 250 = 0$

65. Let $y = f(x)$ be the solution of the differential equation $y(x+1)dx - x^2dy = 0,\ y(1) = e.$ Then $\lim\limits_{x\to 0^+} f(x)$ is equal to

(1) $\frac{1}{e^2}$ (2) $\frac{1}{e}$ (3) 0 (4) e^2

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $\frac{x+1}{x^2}dx = \frac{dy}{y}$

$\ln x - \frac{1}{x} = \ln y + c \quad \because y(1) = e$

$\therefore\ 0 - 1 = 1 + c \Rightarrow c = -2$

$\ln x - \frac{1}{x} = \ln y - 2$

$\ln y = \ln x - \frac{1}{x} + 2$

$y = x.e^{2 - \frac{1}{x}} \qquad \therefore\ \lim\limits_{x\to 0^+} y = 0$

66. Let $[x]$ denote the greatest integer $\leq x$. Consider the function $f(x)=\max\{x^2, 1+[x]\}$. Then the value of the integral $\int_0^2 f(x)dx$ is

(1) $\frac{5+4\sqrt{2}}{3}$ (2) $\frac{8+4\sqrt{2}}{3}$ (3) $\frac{1+5\sqrt{2}}{3}$ (4) $\frac{4+5\sqrt{2}}{3}$

NTA Ans. (1)

Reso Ans. (1)

Sol. $\int_0^2 f(x)dx = \int_0^1 1dx + \int_1^{\sqrt{2}} 2dx + \int_{\sqrt{2}}^2 x^2dx = 1+2(\sqrt{2}-1)+\frac{8-2\sqrt{2}}{3} = \frac{5}{3}+\frac{4\sqrt{2}}{3}$

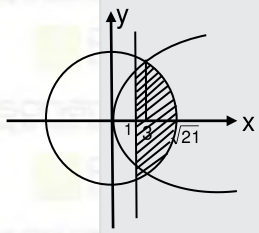
67. Let Δ be the area of the region $\{(x, y)\in R^2 : x^2+y^2 \leq 21,\ y^2 \leq 4x,\ x \geq 1\}$. Then $\frac{1}{2}\left(\Delta - 21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to

(1) $\sqrt{3}-\frac{2}{3}$ (2) $2\sqrt{3}-\frac{2}{3}$ (3) $2\sqrt{3}-\frac{1}{3}$ (4) $\sqrt{3}-\frac{4}{3}$

NTA Ans. (4)

Reso Ans. (4)

Sol.



Required area = $2\left(\int_1^3 2\sqrt{x}dx + \int_3^{\sqrt{21}} \sqrt{21-x^2}dx\right) = 2\left(2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^3 + \left[\frac{x}{2}\sqrt{21-x^2}+\frac{21}{2}\sin^{-1}\left(\frac{x}{\sqrt{21}}\right)\right]_3^{\sqrt{21}}\right)$

$=2\left(\frac{4}{3}(3\sqrt{3}-1)+\frac{21\pi}{4}-\frac{3}{2}\times 2\sqrt{3}-\frac{21}{2}\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)\right) = 8\sqrt{3}-\frac{8}{3}+\frac{21\pi}{2}-6\sqrt{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

$\therefore \Delta = 2\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

$\therefore \frac{1}{2}\left(\Delta - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right)$

$=\frac{1}{2}\left[2\sqrt{3}-\frac{8}{3}+\frac{21\pi}{2}-21\left(\sin^{-1}\left(\frac{\sqrt{3}}{7}\right)+\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right)\right]$ $\quad \because \sin^{-1}\left(\frac{\sqrt{3}}{\sqrt{7}}\right)+\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)=\frac{\pi}{2}$

$\frac{1}{2}\left(\Delta - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right) = \frac{1}{2}\left[2\sqrt{3}-\frac{8}{3}+\frac{21\pi}{2}-21\left(\frac{\pi}{2}\right)\right] = \sqrt{3}-\frac{4}{3}$

68. Let $x=2$ be a root of the equation $x^2+px+q=0$ and

$$f(x)=\begin{cases}\dfrac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4}, & x\neq 2p\\ 0, & x=2p\end{cases}$$

Then $\lim\limits_{x\to 2p^+}[f(x)]$, where $[.]$ denotes greatest integer function, is

(1) 2 (2) 1 (3) -1 (4) 0

NTA Ans. (4)

Reso Ans. (4)

Sol. Put x = 2 $\Rightarrow$ 4+ 2p + q = 0 $\Rightarrow$ q + 4 = – 2p

$\because x^2 - 4px + q^2 + 8q + 16 = x^2 - 4px + (q+4)^2 \quad \because q + 4 = -2p$

$= x^2 - 4px + 4p^2$

$= (x-2p)^2$

$$\underset{x\to 2p^+}{\text{Lt}}[f(x)] = \underset{x\to 2p^+}{\text{Lt}}\left[\frac{1-\cos\left((x-2p)^2\right)}{(x-2p)^4}\right] \qquad \text{Let } x-2p=\theta$$

$$\underset{\theta\to 0^+}{\text{Lt}}[f(x)] = \underset{\theta\to 0^+}{\text{Lt}}\left[\frac{1-\cos(\theta^2)}{(\theta)^4}\right] \qquad \because \underset{\theta\to 0}{\text{Lt}}\left(\frac{1-\cos(\theta^2)}{\theta^4}\right)=\frac{1}{2}$$

$$\therefore \underset{\theta\to 0^+}{\text{Lt}}[f(x)]=0$$

69. A light ray emits from the origin making an angle 30^0 with the positive $x-$axis After getting reflected by the line $x+y=1$, if this ray intersects $x-$axis at Q, then the abscissa of Q is

(1) $\frac{2}{(\sqrt{3}-1)}$ (2) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$ (3) $\frac{2}{3+\sqrt{3}}$ (4) $\frac{2}{3-\sqrt{3}}$

NTA Ans. (3)

Reso Ans. (3)

Sol. Image of O (0 ,0) in line x +y – 1= 0 lies on reflected ray.

$$\frac{x-0}{1}=\frac{y-0}{1}=\frac{-2(0+0-1)}{2} \Rightarrow B\ (1\ ,1).$$

Also, upon solving $y=\frac{x}{\sqrt{3}}$ and x + y – 1 = 0 we get $P\equiv\left(\frac{3-\sqrt{3}}{2},\frac{\sqrt{3}-1}{2}\right)$

equation of reflected ray is same as line passing through BP.

$$\text{slope} = \frac{\frac{\sqrt{3}}{2}-\frac{1}{2}-1}{\frac{3}{2}-\frac{\sqrt{3}}{2}-1}=\frac{\sqrt{3}-3}{1-\sqrt{3}}=\sqrt{3}$$

Equation of line BP is

$y-1=\sqrt{3}(x-1)$

Put y = 0 $\Rightarrow -\frac{1}{\sqrt{3}}=x-1$

Required point $\left(1-\frac{1}{\sqrt{3}},0\right)$

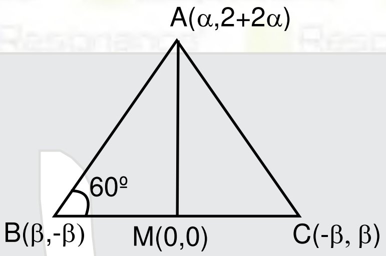
70. Let B and C be the two points on the line y + x = 0 such that B and C are symmetric with respect to the origin. Suppose A is a point on y – 2x = 2 such that ΔABC is an equilateral triangle. Then, the area of the ΔABC is

(1) $2\sqrt{3}$ (2) $\frac{8}{\sqrt{3}}$ (3) $3\sqrt{3}$ (4) $\frac{10}{\sqrt{3}}$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. Slope of AM = $\frac{2+2\alpha}{\alpha} = 1$



$\Rightarrow \alpha = -2$ $\therefore$ A (–2, 2)

$AM = 2\sqrt{2}$ $\because \sin 60^\circ = \frac{AM}{AB}$

$AB = 2\sqrt{2} \times \frac{2}{\sqrt{3}} = \frac{4\sqrt{2}}{\sqrt{3}}$

area = $\frac{\sqrt{3}}{4}(AB)^2 = \frac{\sqrt{3}}{4} \times \frac{32}{3} = \frac{8}{\sqrt{3}}$

71. If p, q and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false ?

(1) p = T, q = T, r = F (2) p = T, q = F, r = T

(3) p = F, q = T, r = F (4) p = T, q = F, r = F

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $\because ((p \vee q) \cap ((\sim p) \vee r)) \rightarrow ((\sim q) \vee r)$ is false.

$\because$ s $\rightarrow$ m becomes false when T $\rightarrow$ F

$\therefore$ s must be true and m must be false.

for m (i.e $((\sim q) \vee r)$ to be false $q \equiv T$ and $r \equiv F$ and now for $((p \vee q) \wedge ((\sim p) \vee r))$ to be true p must be false.

72. Consider the following system of equations

$\alpha x + 2y + z = 1$

$2\alpha x + 3y + z = 1$

$3x + \alpha y + 2z = \beta$

for some $\alpha, \beta \in R$. Then which of the following is NOT correct.

(1) It has no solution if $\alpha = -1$ and $\beta \neq 2$

(2) It has no solution for $\alpha = -1$ and for all $\beta \in R$

(3) It has a solution for all $\alpha \neq -1$ and $\beta = 2$

(4) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $\because D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = \alpha(6-\alpha) - 2(4\alpha - 3) + 1(2\alpha^2 - 9)$

$= \alpha^2 - 2\alpha - 3$

$\therefore \quad D = (\alpha - 3)(\alpha + 1)$

$\because D = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ \beta & \alpha & 2 \end{vmatrix} = 1(6-\alpha) - 2(2-\beta) + 1(\alpha - 3\beta)$

$= 6 - \alpha - 4 + 2\beta + \alpha - 3\beta$

$= 2 - \beta$

Similarly $D_2 = \alpha(\beta - 2)$

$D_3 = \alpha^2 - \alpha\beta - 3$

Let $\alpha = -1$ and $\beta = 2$

$\therefore$ system of equations will become.

$P_1 : x - 2y - z = -1$

$P_2 : 2x - 3y - z = -1$

$P_3 : 3x - y + 2z = 2$

$\because x = t,\ y = t,\ z = 1 - t,\quad t \in R$ is satisfying all the three planes P_1, P_2 and P_3.

$\therefore$ if $\alpha = -1$ and $\beta = 2$ the system of equations will have infinite solutions

$\therefore$ (2) is not correct.

73. Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ and

$S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$. If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to

(1) $\frac{11}{8}$ (2) $\frac{3}{2}$ (3) $\frac{9}{8}$ (4) $\frac{5}{4}$

NTA Ans. **(4)**

Reso Ans. **(4)**

Sol. $f(\theta) = 3(\cos^4\theta + \sin^4\theta) - 2(1 - \sin^2 2\theta)$

$f(\theta) = 3[(\sin^2\theta + \cos^2\theta) - 2\sin^2\theta\cos^2\theta)] - 2 + 2\sin^2 2\theta$

$= 3 - \frac{3}{2}\sin^2 2\theta - 2 + 2\sin^2 2\theta$

$1 + \frac{1}{2}\sin^2 2\theta$ .... (1)

$\because f'(\theta) = \sin 4\theta$

$f'(\theta) = -\frac{\sqrt{3}}{2} \Rightarrow \sin 4\theta = -\frac{\sqrt{3}}{2}$ $\quad \because \theta \in [0, \pi] \therefore 4\theta \in [0, 4\pi]$

$\therefore 4\theta = \pi - \frac{\pi}{3};\ 4\theta = 2\pi - \frac{\pi}{3};\ 4\theta = 3\pi + \frac{\pi}{3};\ 4\theta = 4\pi - \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{4} + \frac{\pi}{12}$; $\theta = \frac{\pi}{2} - \frac{\pi}{12}$; $\theta = \frac{3\pi}{4} + \frac{\pi}{12}$; $\theta = \pi - \frac{\pi}{12}$

$\therefore 4\beta = \frac{5\pi}{2} \Rightarrow \beta = \frac{5\pi}{8}$

$\therefore f(\beta) = 1 + \frac{1}{2}\sin^2\frac{5\pi}{4}$ from (1)

$= 1 + \frac{1}{2} \times \frac{1}{2} = 1 + \frac{1}{4}$

$\therefore f(\beta) = \frac{5}{4}$

74. For two non-zero complex numbers z_1 and z_2, if $\text{Re}(z_1 z_2) = 0$ and $\text{Re}(z_1 + z_2) = 0$, then which of the following are possible ?

A. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$

B. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$

C. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) < 0$

D. $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) < 0$

Choose the correct answer from the options given below :

(1) A and B (2) B and C (3) B and D (4) A and C

NTA Ans. (2)

Reso Ans. (2)

Sol. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$\because$ $\text{Re}(z_1 z_2) = 0$ and $\text{Re}(z_1 + z_2) = 0$,

$x_1x_2 - y_1y_2 = 0$ ....(i)

$x_1 + x_2 = 0$ ....(ii)

$x_1^2 + y_1y_2 = 0$

$y_1y_2 = - x_1^2$

$\Rightarrow$ $\text{Im}(z_1)$ and $\text{Im}(z_2)$ are of opposite sign.

75. Let α and β be real numbers. Consider a 3x3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

(1) $\beta = -8$ (2) $\beta = 8$ (3) $\alpha = 4$ (4) $\alpha = 1$

NTA Ans. (1)

Reso Ans. (1)

Sol. $\because A^2 = 3A + \alpha I$ and $A^4 = 21A + \beta I$

$\because A^4 = (A^2)^2 = (3A + \alpha I)^2$

$= (3A + \alpha I)(3A + \alpha I)$ $= 9A^2 + 6\alpha A + \alpha^2 I$

$= 9(3A + \alpha I) + 6\alpha A + \alpha^2 I$

$= (27 + 6\alpha) A + (9\alpha + \alpha^2) I$

$\therefore 27 + 6\alpha = 21$ and $9\alpha + \alpha^2 = \beta$.

$\therefore \alpha = -1$ and $\beta = -8$

76. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is

(1) $\frac{5}{36}$ (2) $\frac{2}{15}$ (3) $\frac{5}{24}$ (4) $\frac{1}{6}$

NTA Ans. (4)

Reso Ans. (Bonus)

Sol. 15 players and 15. T-shirts

The answer of this question given by NTA is $\frac{1}{6}$

Which might be calculated by them like $\frac{{}^{15}C_3 \times 12!}{15!} = \frac{1}{6}$.

But calculating favourable case by $({}^{15}C_3)12$! will be wrong because it will included repetitions also

The correct answer would be $\frac{15!-({}^{15}C_2D_{13} + {}^{15}C_1D_{14} + D_{15}!}{15!}$

Where D_n = De-arrangement of 'n' things

$\therefore$ P (Reqd) = $\frac{15!-(105D_{13} + 15D_{14} + D_{15})}{15!}$

77. Let $f:R \to R$ be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then

(1) $f(x)$ is many – one in $(-\infty, -1)$

(2) $f(x)$ is one – one in $(-\infty, -\infty)$

(3) $f(x)$ is many – one in $(1, \infty)$

(4) $f(x)$ is one – one in $[1, \infty)$ but not in $(-\infty, \infty)$

NTA Ans. (4)

Reso Ans. (4)

Sol. $\because f(x) = \frac{(x+1)^2}{(x+1)}$

$\because f'(x) = \frac{(x^2+1)2(x+1)-(x+1)^2(2x)}{(x^2+1)^2} = -\frac{2(x+1)(x-1)}{(x^2+1)^2}$

clearly f(x) is one – one is $(-\infty, -1)$ and also in $(1, \infty)$ but f(x) is not one – one is $(-\infty,\infty)$

78. If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$, $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to

(1) 18 (2) 6 (3) 24 (4) 0

NTA Ans. (3)

Reso Ans. (3)

Sol. $\because [\bar{a}\bar{b}\bar{c}] = 0 \Rightarrow \begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow \lambda (4 + 6) - \mu (-2 + 4) + 4 (-6 - 8) = 0$

$5\lambda - \mu - 28 = 0$ ....(1)

$\bar{a}.\hat{b} = \sqrt{54}$

$$\frac{-2\lambda+4\mu-8}{\sqrt{4+16+4}}=\sqrt{54}$$

$\lambda - 2\mu + 22 = 0$ .... (2)

Solving (1) and (2), we get

$\lambda = \frac{78}{9}$ and $\mu = \frac{39}{9} + 11$

$\therefore \lambda + \mu = 24$

79. Let $f(x) = x + \frac{a}{\pi^2-4}\sin x + \frac{b}{\pi^2-4}\cos x, x \in R$ be a function which satisfies

$f(x) = x + \int_0^{\pi/2} \sin(x+y) f(y) dy$. Then (a + b) is equal to

(1) $-\pi(\pi+2)$ (2) $-\pi(\pi-2)$ (3) $-2\pi(\pi+2)$ (4) $-2\pi(\pi-2)$

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $\because f(x) = x + \left(\frac{a}{\pi^2-4}\right)\sin x + \left(\frac{b}{\pi^2-4}\right)\cos x$, $x \in R$

$\because f(x) = x + \int_0^{\frac{\pi}{2}} \sin(x+y) f(y) dy$

$\Rightarrow f(x) = x + \int_0^{\frac{\pi}{2}} (\sin x \cos y + \cos x \sin y) f(y) dy$

$\Rightarrow f(x) = x + (\sin x) \int_0^{\frac{\pi}{2}} \cos y . f(y) dy + (\cos x) \int_0^{\pi} \sin y . f(y) dy$ .... (1)

let $A = \int_0^{\frac{\pi}{2}} \cos y . f(y) dy$ and $B = \int_0^{\frac{\pi}{2}} \sin y . f(y) dy$

$\therefore (1)$ becomes

f(x) = x+ A sin x +B cos x

$\therefore$ f(y) = y+ A sin y +B cosy

$\therefore A = \int_0^{\frac{\pi}{2}} \cos y f(y) dy = \int_0^{\frac{\pi}{2}} (y + A \sin y + B \cos y) \sin y \, dy$

$\therefore A = \int_0^{\frac{\pi}{2}} y \cos y \, dy + A \int_0^{\frac{\pi}{2}} \sin y \cos y \, dy + B \int_0^{\frac{\pi}{2}} \cos^2 dy$

$A = (y \sin y)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin y \, dy + \frac{A}{2} \int_0^{\frac{\pi}{2}} \sin 2y dy + \frac{B}{2} \int_0^{\frac{\pi}{2}} (2\cos^2 y) dy$

$A = \frac{\pi}{2} + (\cos y)_0^{\frac{\pi}{2}} + \frac{A}{2}\left(\frac{-\cos 2y}{2}\right)_0^{\frac{\pi}{2}} + \frac{B}{2} \int_0^{\frac{\pi}{2}} (1+\cos 2y) dy$

$\Rightarrow 2A - B\pi - (2\pi - 4) = 0$ .... (2)

$\because B = \int_0^{\frac{\pi}{2}} \sin y f(y) dy = \int_0^{\frac{\pi}{2}} \sin y (y + A\sin y + B\cos y) dy$

$\Rightarrow B = \int_0^{\frac{\pi}{2}} y \sin y dy + \frac{A}{2}\int_0^{\frac{\pi}{2}} (2\sin^2 y) dy + \frac{B}{2}\int_0^{\frac{\pi}{2}} \sin 2y \, dy$

$\Rightarrow B = 0 + \int_0^{\frac{\pi}{2}} \cos y dy + \frac{A}{2}\int_0^{\frac{\pi}{2}} (1 - \cos 2y) dy + \frac{B}{2}\left(-\frac{\cos 2y}{2}\right)_0^{\frac{\pi}{2}}$

$B = 1 + \frac{A\pi}{4} + \frac{B}{2}$

$A\pi - 2B + 4 = 0$ .... (3)

and $2A - B\pi - (2\pi - 4) = 0$ .... (2)

solving (2) and (3), we get

$A = \frac{-8\pi + 8}{\pi^2 - 4}$ and $B = \frac{-2(\pi^2 - 2\pi + 4)}{\pi^2 - 4}$

$\because$ f(x) = x +A sinx +B cos x

$\Rightarrow f(x) = x + \frac{8(1-\pi)}{\pi^2 - 4}\sin x + \frac{(4\pi - 8 - 2\pi^2)}{\pi^2 - 4}\cos x$

$\therefore$ $a + b = 8 - 8\pi + 4\pi - 8 - 2\pi^2$

$= -2\pi(\pi + 2)$

80. The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log x} - (2x+3)}, x \in R$ is

(1) $(2,\infty) - \{3\}$ (2) $R - \{3\}$ (3) $R - \{-1, 3\}$ (4) $(-1,\infty) - \{3\}$

NTA Ans. **(1)**

Reso Ans. **(1)**

Sol. $\therefore f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log x} - (2x+3)}$

(i) $x - 2 > 0$

$x > 2$

$x \in (2, \infty)$

(ii) $x + 1 > 0$ & $x + 1 \neq 1$

$\Rightarrow x > -1$ $\Rightarrow x \neq 0$

$x \in (-1, 0) \cup (0, \infty)$

(iii) $x > 0$

$x \in (0, \infty)$

(iv) $e^{2\ell nx} - (2x + 3) \Rightarrow x^2 - 2x - 3 \neq 0$

$\Rightarrow (x - 3)(x + 1) \neq 0$

$\Rightarrow x \neq 3, x \neq -1$

from (i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv)

$x \in (2, \infty) - \{3\}$

81. Let $a_1, a_2, a_3, \ldots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to ....

NTA Ans. (60)

Reso Ans. (60)

Sol. $\because a_1, a_2, a_3$ .... increasing G.P of positive nos.

$\because a_4a_6 = 9$ and $a_5 + a_7 = 24$

$(ar^3)(ar^5) = 9$

$a^2r^8 = 9$ and $ar^4 + ar^6 = 24$

$\Rightarrow ar^4 = 3$ .... (1) $ar^4(1+r^2) = 24$ .... (2)

$\therefore$ from (1) and (2) we get $1 + r^2 = 8$

$r^2 = 7 \Rightarrow r = \sqrt{7}$

$\therefore a = \dfrac{3}{49}$

$\therefore a_1.a_9 + a_2\,a_4a_9 + a_5 + a_7 = a(ar^8) + (ar)(ar^3)(ar^8) + ar^4 + ar^6$

$= a^2r^8 + a^3r^{12} + ar^4 + ar^6$

$= 9 + 3^3 + 3 + \dfrac{3}{49} \times 7^3$

$= 9 + 27 + 3 + 21$

$= 60$

82. If the co-efficient of x^9 in $\left(\alpha x^3 + \dfrac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \dfrac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to ....

NTA Ans. (1)

Reso Ans. (1)

Sol. $\left(\alpha x^3 + \dfrac{1}{\beta x}\right)^{11}$

$\because T_{r+1} = {}^{11}C_r\left(\alpha x^3\right)^{11-r}\left(\dfrac{1}{\beta x}\right)^r = {}^{11}C_r\alpha^{11-r}\beta^{-r}.x^{33-4r}$

for coefficient of x^9 ; $33 - 4r = 9 \Rightarrow 4r = 24$

$\Rightarrow r = 6$

$\therefore$ coefficient of x^9 in $\left(\alpha x^3 + \dfrac{1}{\beta x}\right)^{11}$ is $= {}^{11}C_6 \,.\, \alpha^5.\beta^{-6}$

for $\left(\alpha x - \dfrac{1}{\beta x^3}\right)^{11}$, $T_{r+1} = {}^{11}C_r(\alpha x)^{11-r}\left(\dfrac{-1}{\beta x^3}\right)^r$

$= {}^{11}C_r\,\alpha^{11-r}(-1)^r\beta^{-r}x^{11-4r}$

for coefficient of x^{-9} :- $11 - 4r = -9 \Rightarrow r = 5$

$\therefore$ coefficient x^{-9} in $\left(\alpha x - \dfrac{-1}{\beta x^3}\right)^{11}$ is $= -\,{}^{11}C_5\,\alpha^6(-1)^5\beta^{-5}$

$= {}^{11}C_5\,\alpha^6\beta^{-5}$

Q coefficient x^9 = coefficient of x^{-9}

$\Rightarrow {}^{11}C_6\,\alpha^5\beta^{-6} = -\,{}^{11}C_5\,\alpha^6\beta^{-5} \Rightarrow \alpha\beta = -1$

$\Rightarrow (\alpha\beta)^2 = 1$

83. Suppose f is a function satisfying $f(x+y)=f(x)+f(y)$ for all $x, y \in N$ and $f(1)=\frac{1}{5}$. If $\sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$, then m is equal to ....

NTA Ans. **(10)**

Reso Ans. **(10)**

Sol. $f(x+y) = f(x) + f(y) \Rightarrow f(x) = ax$

$f(1) = \frac{1}{5} \Rightarrow a.1 = \frac{1}{5} \Rightarrow a = \frac{1}{5}$

$f(n) = \frac{n}{5}$

$\therefore \sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12} \Rightarrow \sum_{n=1}^{m}\frac{(n)}{5n(n+1)(n+2)} = \frac{1}{12}$

$\left(\sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\right) = \frac{5}{12}$

$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+.......+\left(\frac{1}{m+1}-\frac{1}{m+2}\right) = \frac{5}{12}$

$\frac{1}{2}-\frac{1}{m+2} = \frac{5}{12} \Rightarrow \frac{1}{2}-\frac{5}{12} = \frac{1}{m+2}$

$\Rightarrow \frac{2}{2\times 12} = \frac{1}{m+2} \Rightarrow m + 2 = 12 \Rightarrow m = 10$

84. Let the coefficients of three consecutive terms in the binomial expansion of $(1+2x)^n$ be in the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is ....

NTA Ans. **(1120)**

Reso Ans. **(1120)**

Sol. $(1+2x)^n$

$T_{r+1} = {}^nC_r\ (2x)^r$

Let T_r, T_{r+1} T_{r+2} are three consecutive terms

$\therefore {}^nC_{r-1}.2^{r-1} : {}^nC_r.2^r : {}^nC_{r+1}2^{r+1} :: 2 : 5 : 8$

$\Rightarrow \frac{{}^nC_r.2^r}{{}^nC_{r-1}.2^{r-1}} = \frac{5}{2} \Rightarrow 2\left(\frac{n-r+1}{r}\right) = \frac{5}{2}$

$4n - 9r + 4 = 0$ ....(1)

$\because \frac{{}^nC_{r+1}.2^{r+1}}{{}^nC_r.2^r} = \frac{8}{5}$

$\Rightarrow \frac{n-r}{r+1} = \frac{4}{5} \Rightarrow 5n - 9r - 4 = 0$ .... (2)

On solving (1) and (2) we get n = 8, r = 4

consecutive terms T_r, T_{r+1}, T_{r+2} will be T_4, T_5, T_6, and T_5, is middle term of these three consecutive terms.

$\therefore$ coefficient of $T_5 = {}^8C_4 2^4$

$= 1120$

85. Let $f:R \to R$ be a differentiable function that satisfies the relation $f(x+y) = f(x) + f(y) - 1, \forall x, y \in R$.

If f' (0)=2, then $|f(-2)|$ is equal to

NTA Ans. **(3)**

Reso Ans. **(3)**

Sol. $f(x+y) = f(x) + f(y) - 1$ ....(1)

Partial differential w.r.t. x

$f'(x + y) = f'(x)$

put x = 0

$f'(y) = f'(0) = 2$

$\Rightarrow f(y) = 2y + c$

$f(x) = 2x + c$

Now put x = y = 0 in (1) we get f(0)=1

$\Rightarrow f(0) = 0 + c$

$\Rightarrow c = 1$

$\Rightarrow f(x) = 2x + 1$

$\Rightarrow |f(-2)| = 3$

86. Let the equation of the plane P containing the line $x + 10 = \frac{8-y}{2} = z$ be ax + by + 3z = 2 (a+b) and the distance of the plane P from the point (1, 27, 7) be c. Then $a^2 + b^2 + c^2$ is equal to........

NTA Ans. **(355)**

Reso Ans. **(355)**

Sol. $\frac{x+10}{1} = \frac{y-8}{-2} = \frac{z}{1}$ .... (1)

and ax + by +3z = 2 (a + b) .... (2)

$\because$ (2) contains the line (1)

$\therefore$ (–10, 8,0) will lie on (2)

$\Rightarrow$ -10a + 8b = 2a +2b

b = 2a ..... (3)

and a – 2b +3 = 0 .... (4)

form (3) and (4), we get

a = 1 & b= 2

$\therefore$ eq^n of plane (2) is x + 2y +3z – 6 = 0

$\therefore c = \left| \frac{1+54+21-6}{\sqrt{1+4+5}} \right| = 5\sqrt{14}$

$\therefore a^2 + b^2 + c^2 = 1 + 4 + 25 \times 14 = 5 + 350 = 355$

87. Five digit numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is

NTA Ans. **(1436)**

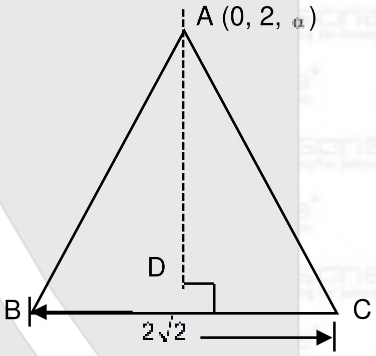
Reso Ans. **(1436)**

Sol. Number of numbers starting with 7 $\rightarrow$ 625
Number of numbers starting with 5 $\rightarrow$ 625
Number of numbers starting with 37 $\rightarrow$ 125
Number of numbers starting with 357 $\rightarrow$ 25
Number of numbers starting with 355 $\rightarrow$ 25
Number of numbers starting with 3537 $\rightarrow$ 5
Number of numbers starting with 3535 $\rightarrow$ 5
Number of numbers starting with 35337 $\rightarrow$ 1
1436
The position of the number 35337 is 1436

88. Let the co-ordinates of one vertex of ΔABC be A (0, 2, α) and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in Z$, if the area of ΔABC is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to ....

NTA Ans. **(9)**
Reso Ans. **(9)**

Sol. $\because$ Area = 21



$\Rightarrow \frac{1}{2}(BC)(AD) = 21$

$\Rightarrow \frac{1}{2}(2\sqrt{21})(AD) = 21$

AD = $\sqrt{21}$

$\because$ eq^n of BC is $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$

$\because$ D is a point on line BC
$\therefore$ let D ($5\lambda - \alpha$, $2\lambda + 1$, $3\lambda - 4$)
$\because$ dr's of AD : $5\lambda - \alpha$, $2\lambda - 1$, $3\lambda - 4 - \alpha$
$\because$ AD $\perp$ BC
$\Rightarrow 5(5\lambda - \alpha) + 2(2\lambda - 1) + 3(3\lambda - 4 - \alpha) = 0$
$19\lambda - 4\alpha - 7 = 0$

$\Rightarrow \lambda = \frac{4\alpha + 7}{19}$

$\because (AD)^2 = 21$

$\Rightarrow (5\lambda - \alpha)^2 + (2\lambda - 1)^2 + (3\lambda - 4 - \alpha)^2 = 21$ ....(1)

$\because \lambda = \frac{4\alpha + 7}{19}$ put in (1), we get

$\therefore \frac{(\alpha + 35)^2}{19^2} + \frac{(8\alpha - 5)^2}{19^2} + \frac{(7\alpha + 55)^2}{19^2} = 21$

$(\alpha + 35)^2 + (8\alpha - 5)^2 + (7\alpha + 55)^2 = 361 \times 21$

$\because \alpha = 3$ satisfies it

$\Rightarrow \alpha^2 = 9$

89. If all the six digit numbers $x_1, x_2, x_3, x_4, x_5, x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72^{th} number is ....

NTA Ans. **(32)**

Reso Ans. **(32)**

Sol. Number of numbers starting with 1 = ${}^8C_5 = 56$

Number of numbers starting with 23 = ${}^6C_4 = 15$

Next number at 72^{nd} position is 245678.

$\therefore$ sum of digits = 2+ 4 + 5 + 6 + 7 + 8 = 32

90. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a}-\vec{b}+\vec{c}, \lambda\vec{a}-3\vec{b}+4\vec{c}, -\vec{a}+2\vec{b}-3\vec{c}$ and $2\vec{a}-4\vec{b}+6\vec{c}$ respectively. If $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, then λ is equal to ....

NTA Ans. **(2)**

Reso Ans. **(2)**

Sol. $\because \overrightarrow{AB} = (\lambda-1)\vec{a} - 2\vec{b} + 3\vec{c}$

and $\overrightarrow{AC} = -2\vec{a} + 3\vec{b} - 4\vec{c}$

and $\overrightarrow{AD} = \vec{a} - 3\vec{b} + 5\vec{c}$

$\because [\overrightarrow{AB} \quad \overrightarrow{AC} \quad \overrightarrow{AD}] = 0$

$$\Rightarrow \begin{vmatrix} \lambda-1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} [\vec{a}\,\vec{b}\,\vec{c}] = 0 \because [\vec{a}\vec{b}\vec{c}] \neq 0$$

$3\lambda - 3 - 12 + 9 = 0$

$\lambda = 2$