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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 29 January, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

61. The value of the integral $\int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to

(1) $\pi \log_e 2$ (2) $\frac{1}{2}\log_e 2$ (3) $\frac{\pi}{2}\log_e 2$ (4) $\frac{\pi}{4}\log_e 2$

Ans. **(3)**

Sol. Let $I = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x} dx$ ____(1)

replace x by $\frac{1}{x}$

$$I = \int_{2}^{\frac{1}{2}} x\left(\tan^{-1}\frac{1}{x}\right)\left(-\frac{1}{x^2}dx\right)$$

$$I = \int_{\frac{1}{2}}^{2} \frac{1}{x}\tan^{-1}\frac{1}{x}dx \quad \text{\_\_\_\_(2)}$$

Adding (1) & (2)

$$2I = \int_{\frac{1}{2}}^{2} \frac{1}{x}\left(\tan^{-1}x + \tan^{-1}\frac{1}{x}\right)dx$$

$$2I = \frac{\pi}{2}\int_{\frac{1}{2}}^{2} \frac{1}{x}dx$$

$$2I = \frac{\pi}{2}(\ln x)_{\frac{1}{2}}^{2}$$

$$I = \frac{\pi}{2}\ln 2$$

62. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}K}{a} = \frac{2^{l} m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to

(1) (51, 99) (2) (50, 101) (3) (50, 51) (4) (51, 101)

Ans. **(2)**

Sol. Sum of odd coeff. of x = $\frac{2^{99}-0}{2}$ = k

k = 2^{98}

Middle term α = T101 = ${}^{200}C_{100}\ \frac{(2)^{100}}{2^{50}}$

= ${}^{200}C_{100}\ 2^{50}$

Now $\frac{{}^{200}C_{99} \times k}{\alpha} = \frac{200!}{100!\ 99!} \times \frac{2^{98}}{{}^{200}C_{100} \times 2^{50}}$

= $\frac{100}{101} \times 2^{48}$

= $\frac{25}{101} \times 2^{50}$

63. Let $S = \{w_1, w_2, ...\}$ be the sample space associated to a random experiment.

Let $p(w_n) = \frac{p(w_{n-1})}{2}, n \ge 2$. Let $A = \{2k + 3l : k, l \in N\}$ and $B = \{w_n : n \in A\}$. Then P (B) is equal to

(1) $\frac{1}{16}$ (2) $\frac{3}{64}$ (3) $\frac{3}{32}$ (4) $\frac{1}{32}$

Ans. **(2)**

Sol. $P(w_n) = \frac{P(w_{n)}}{2}$

A = {2k + 3 ℓ , K, ℓ ∈ N}

A = {5, 7, 8, 9, 10, …….}

$\frac{P(w_2)}{P(w_1)} = \frac{1}{2}$

$\frac{P(w_3)}{P(w_2)} = \frac{1}{2}$

.

.

$\frac{P(w_n)}{P(w_{n+1})} = \frac{1}{2}$

$P(w_n) = \frac{1}{2^{n-1}} P(w_1)$

P(B) = P (w_5) + P (w_7) + P (w_8) + P (w_9) + ……

= $\frac{1}{24} P(w_1) + P(w_1)\left[\frac{1}{26} + \frac{1}{26} + \frac{1}{26} + ...\right]$

$\frac{1}{24} P(w_1) + P(w_1) \times \frac{1/26}{1 - 1/2}$

= P (w_1) $\left[\frac{1}{24} + \frac{1}{25}\right)$

$= P(w_1)\left(\frac{1}{16}+\frac{1}{32}\right)$

$= P(w_1).\frac{3}{32}$

$\Rightarrow P(w_1)\left(1+\frac{1}{2}+\frac{1}{22}+......\right)=1$

$\Rightarrow P(w_1).2=1$

$\Rightarrow P(w_1)=\frac{1}{2}$

$\Rightarrow P(B)=\frac{1}{2}\times\frac{3}{32}=\frac{3}{64}$

64. The set of all values of λ for which the equation $\cos^2 2x-2\sin^4 x-2\cos^2 x=\lambda$ has a real solution x, is

(1) $\left[-1,-\frac{1}{2}\right]$ (2) $\left[-2,-\frac{3}{2}\right]$ (3) $[-2,-1]$ (4) $\left[-\frac{3}{2},-1\right]$

Ans. **(4)**

Sol. Let f(x) = $\cos^2 2x - 2\sin^4 x - 2\cos^2 x$

$= \cos^2 2x - 2\left(\frac{1-\cos^2 2x}{2}\right)^2-(1+\cos 2x)$

$= \cos^2 2x - \frac{1}{2}(1-2\cos 2x+\cos^2 2x)-(1+\cos 2x)$

$= \frac{1}{2}\cos^2 2x-\frac{3}{2}$

$\therefore \cos^2 2x \in [0,1]$

$\Rightarrow \frac{1}{2}\cos^2 2x\in\left[0,\frac{1}{2}\right]$

$\Rightarrow \frac{1}{2}\cos^2 2x-\frac{3}{2}\in\left[-\frac{3}{2},-1\right]$

65. The plane 2x – y + z = 4 intersects the line segment joining the points A (a, -2, 4) and B (2, b, -3) at the point C in the ratio 2 : 1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab\langle 0$ and P is the point (a - b, b, 2b - a) then CP^2 is equal to

(1) $\frac{73}{3}$ (2) $\frac{97}{3}$ (3) $\frac{16}{3}$ (4) $\frac{17}{3}$

Ans. **(4)**

Sol. $C\equiv\left(\frac{4+a}{3},\frac{2b-2}{3},\frac{-2}{3}\right)$

also lies on plane

$\Rightarrow 2\left(\frac{4+a}{3}\right)-\left(\frac{2b-2}{3}\right)+\left(\frac{-2}{3}\right)=4$

$\Rightarrow a + 4 - (b - 1) - 1 = 6$

$\Rightarrow a - b = 2$ ...(1)

$\because$ OC = $\sqrt{5} \Rightarrow \left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \frac{4}{9} = 5$

$\Rightarrow a^2 + 8a + 16 + 4b^2 - 8b + 4 + 4 = 45$

$\Rightarrow a^2 + 4b^2 + 8a - 8b = 21$

$\Rightarrow (b + 2)^2 + 4b^2 + 8(2) = 21$

$\Rightarrow 5b^2 + 4b - 1 = 0 \Rightarrow b = -1$ or $\frac{1}{5}$

$\Rightarrow a = 1$ or $\frac{11}{5}$

$\because ab < 0 \Rightarrow a = 1, b = -1$

$P \equiv (2, -1, -3)$ & $C \equiv \left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3}\right)$

$CP^2 = \left(2 - \frac{5}{3}\right)^2 + \left(\frac{4}{3} - 1\right)^2 + \left(3 - \frac{2}{3}\right)^2$

$= \frac{1}{9} + \frac{1}{9} + \frac{49}{9} = \frac{51}{9} = \frac{17}{3}$

66. Let R be a relation defined on N as a R b if 2a + 3b is a multiple of 5, a, b $\in$ N. Then R is

(1) transitive but not symmetric
(2) an equivalence relation
(3) symmetric but not transitive
(4) not reflexive

Ans. (2)

Sol. For (a, a) $\Rightarrow$ 2a + 3b = 2a + 3b = 5a $\Rightarrow$ divisible by 5

$\Rightarrow (a, a) \in R \ \forall\ a \in N \Rightarrow$ reflexive

Let $(a, b) \in R \Rightarrow 2a + 3b = 5k_1$

and $5a + 5b = 5k_2$ then

$5a + 5b - 2a - 3b = 5(k_2 - k_1) \Rightarrow 2b + 3a = 5k$

$\Rightarrow (b, a) \in R \Rightarrow$ symmetric

Let (a, b) & (b, c) both belong to R

$\Rightarrow 2a + 3b = 5k_1$ & $2b + 3c = 5k_2$

then $2a + 3b + 2b + 3c = 5(k_1 + k_2) \Rightarrow 2a + 3c = 5k - 5b = 5\lambda$

$\Rightarrow (a, c) \in R \Rightarrow$ transitive

67. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

(1) $A \Rightarrow (A \Leftrightarrow B)$
(2) $B \Rightarrow (A \Rightarrow B)$
(3) $B \Rightarrow ((\sim A) \Rightarrow B)$
(4) $A \Rightarrow ((\sim A) \Rightarrow B)$

NTA Ans. (1)

Reso Ans. (2) or (3) or (4)

Sol. $B \to (\sim A \vee B) = B \to (A \to B) = \sim B \vee (\sim A \vee B) = t$

Similarly $B \Rightarrow ((\sim A) \Rightarrow B) = t$ and $A \Rightarrow ((\sim A) \Rightarrow B) = t$

68. If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane z = a is :

(1) 22 (2) 10 (3) 28 (4) 16

Ans. **(3)**

Sol. $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}=\lambda$

a pt. on line ≡ (λ + 1, 2λ + 2, λ – 3)

$\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}=\mu$

a pt. on line ≡ (2μ + a, 3μ – 2, μ + 3)

Fro p^t of intersection

2λ + 2 = 3μ – 2 & λ – 3 = μ + 3

⇒ 2λ –3μ = –4 .... (1) λ –μ = 6 ... (2)

Solving (1) & (2) ⇒ μ = 16, λ = 22

and 2μ + a = λ + 1

⇒ 32 + a = 22 +1 ⇒ a = –9

P ≡ (23, 46, 19)

Distance of P from plane Z = – 9 is equal to 19 – (–9) = 28

69. The shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is

(1) $2\sqrt{3}$ (2) $5\sqrt{3}$ (3) $4\sqrt{3}$ (4) $3\sqrt{3}$

Ans. **(3)**

Sol. $L_1:\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$

$A\rightarrow\vec{a}=\hat{i}-8\hat{j}+4\hat{k},\ \vec{b}=2\hat{i}-7\hat{j}+5\hat{k}$

$L_2:\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$

$C\rightarrow\vec{c}=\hat{i}+2\hat{j}+6\hat{k},\vec{d}=2\hat{i}+\hat{j}-3\hat{k}$

$$\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}=16\hat{i}+16\hat{j}+16\hat{k}\Rightarrow\hat{i}+\hat{j}+\hat{k}$$

$$\text{S.D.}=\left|\frac{\overrightarrow{AC}.(\vec{b}\times\vec{d})}{|\vec{b}\times\vec{d}|}\right|=\left|\frac{(10\hat{j}+2\hat{k}).(\hat{i}+\hat{j}+\hat{k})}{|\vec{b}\times\vec{d}|}\right|$$

$$\text{S.D.}=\frac{12}{\sqrt{3}}=4\sqrt{3}$$

70. The set of all values of $t\in R$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t-2\cos t) & e^{-t}(-2\sin t-\cos t) \\ e^t & e^{-t}(2\sin t+\cos t) & e^{-t}(\sin t-2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ is invertible, is

(1) R (2) $\left\{(2k+1)\frac{\pi}{2}, k\in Z\right\}$ (3) $\{k\pi, k\in Z\}$ (4) $\left\{k\pi+\frac{\pi}{4}, k\in Z\right\}$

Ans. (1)

Sol. $|A| = \begin{bmatrix} e^t & e^{-t}(\sin t-2\cos t) & e^{-t}(-2\sin t-\cos t) \\ e^t & e^{-t}(2\sin t+\cos t) & e^{-t}(\sin t-2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$

$= e^t.\ e^{-t}.\ e^{-t} \begin{vmatrix} 1 & \sin t-2\cos t & -2\sin t-\cos t \\ 1 & 2\sin t+\cos t & \sin t-2\cos t \\ 1 & \cos t & \sin t \end{vmatrix}$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$|A| = e^{-t} \begin{vmatrix} 0 & \sin t-3\cos t & -3\sin t-\cos t \\ 0 & 2\sin t & -2\cos t \\ 1 & \cos t & \sin t \end{vmatrix}$

$= e^{-t}\left(-2\cos t(\sin t-3\cos t)+2\sin t(3\sin t+\cos t)\right)$

$= e^{-t}(6\cos^2 t + 6\sin^2 t) = 6e^{-t}$

$|A| \neq 0\ \forall\ t \in R$

71. Consider a function $f: IN \to IR$, satisfying

$f(1)+2f(2)+3f(3)+\ldots+xf(x) = x(x+1)f(x) : x \geq 2$ with $f(1)=1$. Then $\frac{1}{f(2022)}+\frac{1}{f(2028)}$ is equal to

(1) 8400 (2) 8100 (3) 8200 (4) 8000

Ans. (2)

Sol. $f(1) + 2f(2) + 3f(3)$ …………… $n\,f(n) = n\,(n+1)\,f(n)$ …………..(1)

replace in by $n + 1$

$f(1) + 2f(2) + 3f(3)$ ……….. $nf(n) + (n+1)\,f(n+1) = (n+1)\,(n+2)\,f(n+1)$ ………….(2)

Subtracting (1) from (2)

$(n+1)\,f(n+1) = (n+1)\,(n+2)\,f(n+1) - n(n+1)\,f(n)$ or $(n+1)f(n+1) - nf(n) = 0$ ………..(3)

put n = 2, 3, 4 ………… n + 1 in eq. (3)

$3f(3) - 2\,f(2) = 0$

$4f(4) - 3f(3) = 0$

,,

$(n+1)\,f(n+1) - n\,f(n) = 0$

Add $\Rightarrow (n+1)\,f(n+1) = 2f(2)$

$\Rightarrow f(n+1) = \frac{2f(2)}{n+1}$ …………(4)

again put n = 2 in eq. (1)

$f(1) + 2f(2) = 6f(2)$

$\Rightarrow 1 = 4f(2) \Rightarrow f(2) = \frac{1}{4}$

Now by (4) $f(n) = \frac{1}{2n}$

$\Rightarrow \frac{1}{f(2022)} + \frac{1}{f(2028)} = 4044 + 4056 = 8100$

72. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

(1) 79 (2) 89 (3) 86 (4) 84

Ans. **(2)**

Sol. G,H,O,T,U

No. of words starting from G = 4! = 24

No. of words starting from H = 4! = 24

No. of words starting from O = 4! = 24

No. of words starting from TG = 3! = 6

No. of words starting from TH = 3! = 6

No. of words starting from TOG = 2! = 2

No. of words starting from TOH = 2! = 2

Next "TOUGH" = 1

Rank of "TOUGH" = 24 + 24+ 24 + 6 + 6 + 2 + 2 +1 = 89 Ans.

73 Let f and g be twice differentiable functions on R such that

$f''(x) = g''(x) + 6x$

$f'(1) = 4g'(1) - 3 = 9$

$f(2) = 3g(2) = 12.$

Then which of the following is NOT true ?

(1) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

(2) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

(3) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$

(4) $g(-2) - f(-2) = 20$

Ans. **(2)**

Sol. $f''(x) = g''(x) + 6x$

$\Rightarrow \int f''(x)dx = \int g''(x)dx + \int 6x\,dx + C$

$\Rightarrow f'(x) = g'(x) + 3x^2 + C$ ______(1)

$x = 1 \Rightarrow f'(1) = g'(1) + 3 + C$

$\Rightarrow 9 = 3 + 3 + c \Rightarrow C = 3$

from (1) $\int f'(x)dx = \int g'(x)dx + \int 3x^2\,dx + \int 3dx + k$

$\Rightarrow f(x) = g(x) + x^3 + 3x + K$

$x = 2 \Rightarrow f(2) = g(2) + 8 + 6 + K$

$\Rightarrow 12 = 4 + 8 + 6 + K$

$\Rightarrow K = -6$

$\Rightarrow f(x) = g(x) + x^3 + 3x - 6$

Let $h(x) = f(x) - g(x) = x^3 + 3x - 6$

$\Rightarrow h'(x) = f'(x) - g'(x) = 3x^2 + 3 > 0 \ \forall\ x$

$\Rightarrow h'(x)$ is S.I. $\Rightarrow h(x) = 0$ has no solution

$\Rightarrow h(x) \in (h(-1), h(2)) \Rightarrow h(x) \in (-8, 8)$

$| f'(x) - g'(x)| < 6 \Rightarrow |3x^2 + 3| < 6 \Rightarrow 3x^2 + 3 < 6$

$\Rightarrow x^2 - 1 < 0 \Rightarrow x \in (-1, 1)$

$h(-2) = f(-2) - g(-2) = -20$

74. The value of the integral $\int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$ is

(1) $\tan^{-1}\frac{1}{2} + \frac{1}{3}\tan^{-1}8 - \frac{\pi}{3}$

(2) $\tan^{-1}\frac{1}{2} - \frac{1}{3}\tan^{-1}8 + \frac{\pi}{3}$

(3) $\tan^{-1}2 - \frac{1}{3}\tan^{-1}8 + \frac{\pi}{3}$

(4) $\tan^{-1}2 + \frac{1}{3}\tan^{-1}8 - \frac{\pi}{3}$

Ans. **(4)**

Sol. $= \int_1^2 \frac{(t^4+1)}{(t^6+1)} \frac{(t^2+1)}{(t^2+1)} dt$

$= \int_1^2 \frac{t^6+t^4}{(t^6+1)} \frac{t^2+1}{(t^2+1)} dt$

$= \int_1^2 \frac{t^6+1}{(t^6+1)(t^2+1)} dt + \int_1^2 \frac{t^4+t^2}{(t^6+1)(t^2+1)} dt$

$= \int_1^2 \frac{1}{t^2+1} dt + \int_1^2 \frac{t^2}{t^6+1} dt,$

$I = \left[\tan^{-1} t\right]_1^2 + I_1$

$\tan^{-1} - \frac{\pi}{4} + I_1$

$I_1 = \int_1^2 \frac{t^2}{t^6+1} dt$

put $t^3 = \theta$. $3t^2\, dt = d\theta$

$t = 1$ then $\theta = 1$

$t = 2$ then $\theta = 8$

$= \frac{1}{3} \int_1^8 \frac{d\theta}{\theta^2+1}$

$= \frac{1}{3}\left[\tan^{-1}(\theta)\right]_1^8$

$= \frac{1}{3}\left[\tan^{-1}8 - \frac{\pi}{4}\right]$

75. Let y = y (x) be the solution of the differential equation $x\log_e x\frac{dy}{dx}+Y=X^2\log_e x(x\rangle 1)$.

If y (2) = 2, then y (e) is equal to

(1) $\frac{4+e^2}{4}$ (2) $\frac{1+e^2}{2}$ (3) $\frac{2+e^2}{2}$ (4) $\frac{1+e^2}{4}$

Ans. **(1)**

Sol. $x\,\ell n\,x\frac{dy}{dx}+y=x^2\,\ell nx$

$$\frac{dy}{dx}+\frac{y}{x\ell nx}=x$$

$$\text{I.F.} = e^{\int\frac{dx}{x\ell nx}} = e^{\ell n(\ell nx)} = \ell nx$$

now solution is

$$y\ell nx=\int x\ell nx\,dx$$

$$y\ell nx = (\ell nx)\frac{x^2}{2}-\frac{x^2}{4}+c$$

$$y=\frac{x^2}{2}-\frac{x^2}{4\ell nx}+\frac{c}{\ell nx}$$

$$y(2) = \frac{4}{2}-\frac{4}{4\ell n2}+\frac{c}{\ell n2}$$

$$2 = 2-\frac{1}{\ell n2}+\frac{c}{\ell n2}$$

c = 1

$$y=\frac{x^2}{2}-\frac{x^2}{4\ell nx}+\frac{1}{\ell nx}$$

$$y(e)=\frac{e^2}{2}-\frac{e^2}{4}+1$$

$$y(e)=\frac{e^2}{4}+1$$

76. If $\vec{a}=\hat{i}+2\hat{k},\vec{b}=\hat{i}+\hat{j}+\hat{k},\vec{c}=7\hat{i}-3\hat{j}+4\hat{k},\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$ and $\vec{r}.\vec{a}=0$. Then $\vec{r}.\vec{c}$ is equal to

(1) 30 (2) 36 (3) 32 (4) 34

Ans. **(4)**

Sol. $\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=0$

$\Rightarrow \vec{r}\times\vec{b}-\vec{c}\times\vec{b}=0$

$\Rightarrow (\vec{r}-\vec{c})\times\vec{b}=0$

$\Rightarrow \vec{r}-\vec{c} \parallel \vec{b} \Rightarrow \vec{r}-\vec{c}=\lambda\vec{b}$

$\Rightarrow \vec{r}=\vec{c}+\lambda\vec{b}$

$\because \vec{r}.\vec{a}=0 \Rightarrow \vec{a}.(\vec{c}+\lambda\vec{b})=0$

$\Rightarrow \vec{a}.\vec{c}+\lambda(\vec{a}.\vec{b})=0$

$$\lambda=-\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}=\frac{-15}{3}=-5$$

$\vec{r}.\vec{c}=(\vec{c}+\lambda\vec{b}).\vec{c}=\left|\vec{c}\right|^2+\lambda(\vec{b}.\vec{c})$

$=74-5(8)=34$

77. If the tangent at a point P on the parabola $y^2=3x$ is parallel to the line x + 2y = 1 and the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line x – y = 2, then the area of the triangle PQR is :

(1) $3\sqrt{5}$ (2) $\frac{9}{\sqrt{5}}$ (3) $\frac{3}{2}\sqrt{5}$ (4) $5\sqrt{3}$

Ans. (1)

Sol.

Slope of tangent at $P\left(\frac{\alpha^2}{3},\alpha\right)$

$$\Rightarrow \frac{dy}{dx}=\frac{3}{2y}=\frac{3}{2\alpha}=-\frac{1}{2}$$

$\Rightarrow \alpha=-3 \Rightarrow P(3,-3)$

slope of tangent to ellipse $=-1$

$$\text{point of contacts}=\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}},\mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$$

$$\Rightarrow\left(\pm\frac{(-4)}{\sqrt{5}},\mp\frac{1}{\sqrt{5}}\right)$$

$$\Rightarrow Q\left(-\frac{4}{5},-\frac{1}{5}\right) R\left(\frac{4}{5},\frac{1}{5}\right)$$

$$\Delta PQR=\frac{1}{2}\begin{vmatrix}3 & -3 & 1\\ -\frac{4}{5} & -\frac{1}{5} & 1\\ \frac{4}{5} & \frac{1}{5} & 1\end{vmatrix}=\frac{1}{2}\left[-\frac{6}{5}-\frac{8}{5}(3)+0\right]$$

$=3\sqrt{5}$

78. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(1) 400 (2) 472 (3) 432 (4) 507

Ans. **(3)**

Sol. no. divisible by 3 = 102, 105, ……., 999

= 300

no. divisible by 4 = 100, 104, …. , 996

= 225

now no. divisible by 12

= 75

So numbers divisible by 3 or 4

= 300 + 225 – 75 = 450

now numbers divisible by 48 are = 18

So total numbers = 450 – 18

= 432

79. Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}.(\vec{a} \times \vec{b}) + 25 = 0, \vec{c}.(\hat{i} + \hat{j} + \hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals

(1) $\frac{5}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{5}$ (4) $\frac{3}{\sqrt{2}}$

Ans. **(1)**

Sol. Let $\bar{c} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

$\therefore$ $(x\,\hat{i} + y\,\hat{j} + z\,\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$

$\Rightarrow$ x + y + z = 4 ... (1)

Projection of $\bar{c}$ on $\bar{a}$ = $\bar{c} \cdot \hat{a} = 1$

$\Rightarrow$ 4x + 3y = 5 ... (2)

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ 3 & -4 & 5 \end{vmatrix} = 15\hat{i} - 20\hat{j} - 25\hat{k}$$

$\bar{c}(\bar{a} \times \bar{b})$ = 15 x – 20y – 25z

$\therefore$ $\bar{c} \cdot (\bar{a} \times \bar{b}) + 25 = 0$

$\Rightarrow$ 3x – 4y – 5z = – 5 ... (3)

5 × (1) + 3 $\Rightarrow$ 8x + y = 15 ... (4)

(4) – 2 × 2 $\Rightarrow$ – 5y = 5 $\Rightarrow$ y = 1 $\Rightarrow$ x = 2 $\Rightarrow$ z = 3

$\bar{c} = 2\,\hat{i} - \hat{j} + 3\,\hat{k}$

Projection of $\bar{c}$ on $\bar{b}$ = $\bar{c} \cdot \hat{b}$ = $\frac{\bar{c} \cdot \bar{b}}{|\bar{b}|}$

$= \frac{6+4+15}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$

80. The area of the region $A=\left\{(x,y):|\cos x-\sin x|\le y\le \sin x, 0\le x\le\frac{\pi}{2}\right\}$ is

(1) $\sqrt{5}-2\sqrt{2}+1$ (2) $1-\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{5}}$ (3) $\frac{3}{\sqrt{5}}-\frac{3}{\sqrt{2}}+1$ (4) $\sqrt{5}+2\sqrt{2}-4.5$

Ans. **(1)**

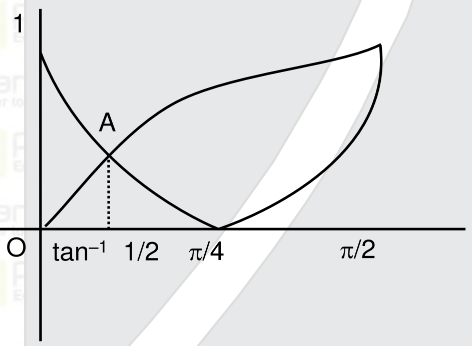
Sol. $y=\begin{cases}\cos x-\sin x & : 0\le x\le\frac{\pi}{4}\\ \sin x-\cos x & : \frac{\pi}{4}<x\le\frac{\pi}{2}\end{cases}$

$y^1=\begin{cases}-\sin x-\cos x & : 0\le x\le\frac{\pi}{4}\\ \cos x+\sin x & : \frac{\pi}{4}<x\le\frac{\pi}{2}\end{cases}$

For pt. A

cosx – sinx = sinx

$\tan x=\frac{1}{2}$



Let $x=\tan^{-1}\frac{1}{2}$

Area = $\int_{\tan^{-1}1/2}^{\pi/4}(\sin x-(\cos x-\sin x))\,dx$

$+\int_{\pi/4}^{\pi/2}(\sin x-\sin x+\cos x)\,dx$

$=(-2\cos x-\sin x)_x^{\pi/4}+(\sin x)_{\pi/4}^{\pi/2}$

$=\left(\frac{-2}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)-(-2\cos x-\sin x)+\left(1-\frac{1}{\sqrt{2}}\right)$

$=-\sqrt{2}-\frac{1}{\sqrt{2}}+2\cos x+\sin x+1-\frac{1}{\sqrt{2}}=-2\sqrt{2}+1+\frac{2\times2}{\sqrt{5}}+\frac{1}{\sqrt{5}}$

$=\sqrt{5}+1-2\sqrt{2}$

81. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}, \forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to ………

Ans. **(461)**

Sol. $S = \frac{b_1}{2} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \ldots\ldots\ldots + \frac{b_{10}}{2^{10}}$ (1)

$\frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + \frac{b_3}{2^4} + \ldots\ldots\ldots + \frac{b_{10}}{2^{11}}$ (2)

(1) – (2) $\Rightarrow \frac{S}{2} = \frac{b_1}{2} + \frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\ldots\ldots + \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$

$\Rightarrow S = b_1 + \frac{a_1}{2} + \frac{a_2}{2^2} + \ldots\ldots\ldots + \frac{a_9}{2^9} - \frac{b_{10}}{2^{10}}$ (3)

$\Rightarrow \frac{S}{2} = \frac{b_1}{2} + \frac{a_1}{2^2} + \ldots\ldots\ldots + \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$ (4)

(2) – (4) $\Rightarrow \frac{S}{2} = \frac{b_1}{2} + \frac{a_1}{2} + .\frac{1}{2^2} + \frac{2}{2^3} \ldots\ldots\ldots + \frac{8}{2^9} - \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$

$\Rightarrow \frac{S}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2^2}\left(\frac{1}{1} + \frac{2}{2} + \ldots.. + \frac{8}{2^7}\right) - \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$

$\Rightarrow \frac{S}{2} = 1 + \frac{1}{4}(T) - \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$

$\Rightarrow 2S = 4 + T - \frac{a_9}{2^8} - \frac{b_{10}}{2^9}$

$\Rightarrow 2S - T = 4 - \frac{a_9}{2^8} - \frac{b_{10}}{2^9}$

$\Rightarrow 2^7 (2S - T) = 2^9 - \frac{37}{2} - \frac{130}{2^2}$

$= 461$

82. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is …………… .

Ans **(3000)**

Sol. Four digit numbers which are divisible by 2 $\Rightarrow$ 1000 to 9998 Þ 4500

Four digit numbers divisible by 6

$\Rightarrow$ 1002 to 9996 $\Rightarrow$ 1500

Four digit integer divisible by 2 but not divisible by 3 $\Rightarrow$ 4500 – 1500 = 3000

83. Let A be a symmetric matrix such that |A| = 2 and $\begin{vmatrix}2 & 1\\ 3 & \frac{1}{2}\end{vmatrix}A - \begin{vmatrix}1 & 2\\ \alpha & \beta\end{vmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to

Ans (5)

Sol. $A^T = A$ Þ So $A = \begin{bmatrix}a & b\\ b & d\end{bmatrix}$

now $ad - b^2 = 2$ ....(1)

also $BA = \begin{bmatrix}2 & 1\\ 3 & 3/2\end{bmatrix}\begin{bmatrix}a & b\\ b & d\end{bmatrix}$

$\Rightarrow$ 2a + b = 1 ....(2)

$3a + \frac{3b}{2} = \alpha$ .... (3)

$3b + \frac{3d}{2} = \beta$ .... (4)

2b + d = 2 .... (5)

Solving equations

$a = \frac{3}{4}, \alpha = \frac{3}{2}, b = -\frac{1}{2}$

$d = 3, \beta = 3$

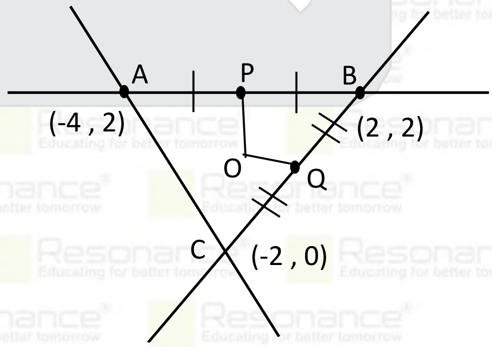
So $\frac{\beta \times S}{\alpha} = 5$

84. A triangle is foprmed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line x + y + 2 = 0. If r is the radius of its circumcircle, the r^2 is equal to............ .

Ans (10)

Sol. Tangent to parabola = T = 0

$\Rightarrow \quad 2y = \frac{2(x+2)}{2} \Rightarrow x - 2y + 2 = 0$ ... (1)



Tangent to circle $\Rightarrow$ T = 0

$\Rightarrow \quad 2x + 2y = 4\left(\frac{x+2}{2}\right) \Rightarrow y = 2$ ... (2)

Line x + y + 2 = 0 ... (3)

P = (– 1, 2)

Perpendicular bisector of AB $\Rightarrow$ x = – 1

Q = (0,1) , Shape of BC = $\frac{1}{2}$

Perpendicular bisector of BC $\Rightarrow$ y – 1 = – 2 (x – 0)

$\Rightarrow$ 2x + y = 1

$\Rightarrow$ Circumcenter $\Rightarrow$ O(–1,3)

Circum radius $\Rightarrow$ r = OC = $\sqrt{10}$ $r^2 = 10$

85. Let X = {11, 12, 13, … , 40, 41} and Y = {61, 62, 63, … 90 , 91} be the two sets of observaztion. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations is X ∪ Y, then $\left|\bar{x}+\bar{y}-\sigma^2\right|$ is equal to ……….. .

Ans. (603)

Sol. Mean of observations 1,2 , 3 , ... 31 = $\frac{1+2+....+31}{31} = \frac{\left(\frac{31(32)}{2}\right)}{31} = 16$

$\bar{x}$ = 16 + 10 = 26, $\bar{y}$ = 16 + 60 = 76

Variance of observation 1,2 31

$$= \frac{1^2+2^2+....31^2}{31} - (16)^2 = \frac{\left(\frac{31(32)(63)}{6}\right)}{31} - 256 = (16)(21) - 256 = 80$$

Variance of the two given sets = 80

$$\sigma^2 = \frac{31(80)+31(80)}{31+31} + \frac{31.31}{(31+31)^2}(76-16)^2$$

$$= 80 + \frac{2500}{4} = 705$$

$$\left|\bar{x}+5-\Sigma^2\right| = |76+26-705| = 603$$

86. If the equation of the normal to the curve y = $\frac{x-a}{(x+b)(x-2)}$ at the point (1, -3) is x – 4y = 13, then the value of a + b is equal to ……….. .

Ans. (4)

Sol. (1, – 3) lies on curve

⇒ $-3 = \frac{1-a}{(1+b)(1-2)} \Rightarrow 3(1+b) = 1-a$

⇒ a + 3b = –2 .... (1)

∴ $y = \frac{x-a}{x^2+(b-2)x-2b}$

$$\frac{dy}{dx} = \frac{(x^2+(b-2)x-2b)-(x-a)(2x+b-2)}{(x^2+(b-2)x-2b)^2}$$

∴ Slope of tangent at (1,–3) = –4

⇒ $\frac{1+b-2-2b-(1-a)(2+b-2)}{(1+b-2-2b)^2} = -4$

⇒ – b – 1 – b (1 + 3b + 2) = – 4 (–1 – b)^2

⇒ –3b^2 – 4b – 1 = –4 (1 + 2b + b^2)

⇒ b^2 + 4b + 3 = 0 ⇒ b = –1 or –3

⇒ a = 1 or 7

∴ b ≠ – 1 for (1, –3)

So a + b = 7 + (–3) = 4

87. Let $\alpha = 8 - 14i$, $A = \left\{ z \in c : \dfrac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i} = 1 \right\}$ and B = { z ∈ C: |z +3i| = 4}.

The $\sum_{z \in A \cap B} (\text{Re}\, z - \text{Im}\, z)$ is equal to ………… .

Ans. **(14)**

Sol. Let z = x + i y

$A : (8-14i)(x+iy) - (8+14i)(x-iy)$

$= (x+iy)^2 - (x-iy)^2 - 112i$

$\Rightarrow (-28x + 16y)i = 4ixy - 112i$

$\Rightarrow -28x + 16y = 4xy - 112$

$\Rightarrow -7x + 4y = xy - 28$

$\Rightarrow 7x - 4y + xy - 28 = 0$

$\Rightarrow (x-4)(y+7) = 0$

$\Rightarrow x = 4 \text{ or } y = -7$

B : $|z + 3i| = 4$

$\Rightarrow x^2 + (y+3)^2 = 4^2$

For $x = 4 \Rightarrow y = -3 \Rightarrow A(4, -3)$

For $y = -7 \Rightarrow x = 0 \Rightarrow B(0, -7)$

$\sum_{z \in A \cap B} (R_e z - I_{mz}) = (4 - (-3)) + (0 - (-7))$

$= 14$

88. Let {a_k} and {b_k}, k ∈ N, be two G.P.s with common ratios r_1 and r_2 respectvely such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$,. k ∈ N. If $c_2 = 5$ and $C_3 = \dfrac{13}{4}$

then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

Ans. **(9)**

Sol. a = b = 4

$C_2 = a_2 + b_2 = 5$

$\Rightarrow ar_1 + br_2 = 5$

$\Rightarrow 4r_1 + 4r_2 = 5 \Rightarrow r_1 + r_2 = \dfrac{5}{4}$

$C_3 = a_3 + b_3 = \dfrac{13}{4} \Rightarrow ar_1^2 + br_2^2 = \dfrac{13}{4}$

$\Rightarrow 4r_1^2 + 4r_2^2 = \dfrac{13}{4}$

$\Rightarrow 4\left(\dfrac{5}{4} - r_2\right)^2 + 4r_2^2 = \dfrac{13}{4}$

$\Rightarrow 16\left(\dfrac{25}{16} - \dfrac{5}{2}r_2 + r_2^2\right) + 16r_2^2 = 13$

$\Rightarrow 25 - 40r_2 + 16r_2^2 + 16r_2^2 - 13 = 0$

$\Rightarrow 32r_2^2 - 40r_2 + 12 = 0$

$\Rightarrow 8r_2^2 - 10r_2 + 3 = 0$

$\Rightarrow (4r_2 - 3)(2r_2 - 1) = 0$

$\Rightarrow r_2 = \frac{3}{4}$ or $\frac{1}{2}$

$r_1 = \frac{1}{2}$ or $\frac{3}{4}$

$r_1 < r_2 \Rightarrow r_1 = \frac{1}{2},\ r_2 = \frac{3}{4}$

$$\sum_{k=1}^{\infty} C_k = \sum a_k + \sum b_k = \frac{a}{1-r_1} + \frac{b}{1-r_2} = \frac{4}{1-\frac{1}{2}} + \frac{4}{1-\frac{3}{4}}$$

$= 8 + 16 = 24$

$$\sum_{k=1}^{\infty} C_k - 12a_6 - 8b_4 = 24 - 12\left(4\left(\frac{1}{2}\right)^5\right) - 8.4.\left(\frac{3}{4}\right)^3$$

$= 24 - \frac{3}{2} - \frac{27}{2} = 9$

89. A circle with centre (2,3) and radius 4 inersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q inersect at the point S (α,β), then $4\alpha - 7\beta$ is equal to

Ans. (11)

Sol. Circle $(x-2)^2 + (y-3)^2 = 4^2$

$\Rightarrow\ x^2 + y^2 - 4x - 6y - 3 = 0$

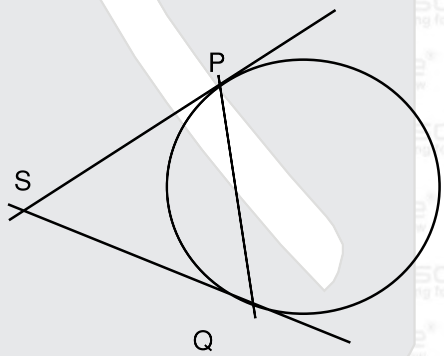
Equation of PQ

$\Rightarrow x + y - 3 = 0 \quad ....(1)$

$\because$ AB is also the chord of contact

w.r.t

Equation of chord of contact $\Rightarrow T = 0$



$\Rightarrow\ x\alpha + y\beta - 4\left(\frac{x+\alpha}{2}\right) - 6\left(\frac{y+\beta}{2}\right) - 3 = 0$

$\Rightarrow (\alpha - 2)x + (\beta - 3)y - 2\alpha - 3\beta - 3 = 0\ .....(2)$

Comparing (1) and (2)

$\Rightarrow \frac{\alpha-2}{1} = \frac{\beta-3}{1} = \frac{2\alpha-3\beta-3}{-3}$

$\Rightarrow \alpha - 2 = \beta - 3 \Rightarrow \beta = \alpha + 1$

and $\alpha - 2 = \frac{2\alpha+3\beta+3}{3}$

$\Rightarrow 3\alpha - 6 = 2\alpha + 3(\alpha + 1) + 3$

$\Rightarrow \alpha = -6 \Rightarrow \beta = -5$

$4\alpha - 7\beta = -24 + 35 = 11$

90. Let $\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \ldots\ldots \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to..............

Ans. (9)

Sol. $x^7 + 3x^5 - 13x^3 - 15x = 0$

$\Rightarrow \quad x = 0 \,,\, x^6 + 3x^4 - 13x^2 - 15 = 0$

Let $\quad x^2 = t$

Equation $\quad t^3 + 3t^2 - 13t - 15 = 0$

$\Rightarrow \quad (t-3)(t^2 + 6t + 5) = 0$

$\Rightarrow \quad t = 3, \quad t^2 + 6t + 5 = 0$

$\Rightarrow \quad t = -1, -5$

$x^2 = 3 \Rightarrow x = \sqrt{3}, -\sqrt{3}$

$x^2 = -1 \Rightarrow x = i, -i$

$x^2 = -5 \Rightarrow x = -\sqrt{5}\,i, \sqrt{5}\,i$

$a_1a_2 - a_3a_4 + a_5a_6$

$= (\sqrt{5}\,i)(-\sqrt{5}\,i) - (\sqrt{3})(-\sqrt{3}) + (i)(-i)$

$= 5 + 3 + 1 = 9$

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