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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 30 January, 2023 (SHIFT-1) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

61. If the coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real number, then for each such ordered pair (a, b) :

(1) a = 3b (2) a = b (3) ab = 1 (4) ab = 3

Ans. **(3)**

Sol. $(r + 1)^{th}$ term in the expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \left(ax^3\right)^{15-r}\left(\frac{1}{bx^{\frac{1}{3}}}\right)^r$$

$$= {}^{15}C_r\, a^{15-r}\, x^{45-3r-\frac{r}{3}}\, b^{-r}$$

for coefficient of x^{15}

$$45 - 3r - \frac{r}{3} = 15$$

$$30 = \frac{10r}{3}$$

$$r = 9$$

and $(r+1)^{th}$ term in $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \left(ax^{\frac{1}{3}}\right)^{15-r}\left(\frac{1}{bx^3}\right)^r$$

$${}^{15}C_r\, a^{15-r}\, b^{-r}\, x^{\frac{15-r}{3}-3r}$$

for coefficient of x^{-15}

$$\frac{15-r}{3} - 3r = -15$$

$$15 - r - 9r = -45$$

$$60 = 10r \Rightarrow r = 6$$

$$\text{Now } {}^{15}C_9\, a^6\, b^{-9} = {}^{15}C_6\, a^9\, (-b)^{-6}$$

$$1 = a^3 b^3$$

$$ab = 1$$

62. Let the solution curve y = y(x) of the differential equation

$$\frac{dy}{dx} - \frac{3x^5\tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}}y = 2x\exp\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{(1+x^6)}}\right\}$$ pass through the origin. Then y(1) is equal to :

(1) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$ (2) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$ (3) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$ (4) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

Ans. (2)

Sol. $$\frac{dy}{dx} - \frac{3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}y = 2x\exp\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right\}$$

I.f. = $e^{-\int\frac{3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}dx}$

= $e^{-\int\frac{3x^2.x^3\tan^{-1}x^3}{(1+x^6)^{3/2}}dx}$

Let $\tan^{-1}x^3 = t \Rightarrow \frac{3x^2}{1+x^6}dx = dt$

= $e^{-\int\frac{x^3}{\sqrt{1+x^6}}.\frac{3x^2\tan^{-1}x^3}{(1+x^6)}dx}$

= $e^{-\int\frac{\tan t}{\sec t}.t.dt}$

= $e^{-\int t\sin t.dt}$ = $e^{(t\cos t - \sin t)}$

= $e^{\left(\frac{\tan^{-1}x^3}{\sqrt{1+x^6}}-\frac{x^3}{\sqrt{1+x^6}}\right)}$

Hence solutions of the differential equation is

$$y.\ e^{\frac{\tan^{-1}x^3}{\sqrt{1+x^6}}-\frac{x^3}{\sqrt{1+x^6}}} = \int 2\times e^{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}}.e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}.dx + C$$

= x^2 + C

Passes through (0,0) $\Rightarrow$ C = 0

Hence y(1) = $e^{\frac{1}{\sqrt{2}}\left(1-\frac{\pi}{4}\right)}$

63. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to x + 90y + 2 = 0 is :

(1) 2 (2) 4 (3) 0 (4) 3

Ans. (2)

Sol. $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$

P(x,y)

$g(x) = f'(x) = 270x^4 - 540^3 - 210x^2 + 360x + 210$

f'(x) = 90 (slope of Tangent)

No. of roots of $g(x) = 270x^4 - 540x^3 - 210x^2 + 360x + 210 = 90$

$g(x) = 9x^4 - 18x^3 - 7x^2 + 12x + 4 = 0$

$= (x-1)(9x^3 - 9x^2 - 16x - 4) = 0$

$= (x-1)(x-2)(9x^2 + 9x + 2) = 0$

$= (x-1)(x-2)(3x+2)(3x+1) = 0$

Total 4 points option (2)

64. If $\tan 15° + \frac{1}{\tan 75°} + \frac{1}{\tan 105°} + \tan 195° = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is :

(1) 4 (2) 2 (3) $4 - 2\sqrt{3}$ (4) $5 - \frac{3}{2}\sqrt{3}$

Ans. (1)

Sol. $\Rightarrow \tan 15° + \tan 15° - \tan 15 + \tan 15 = 2a$

$\Rightarrow a = \tan 15°$

Now $\left(a + \frac{1}{a}\right) = \tan 15° + \frac{1}{\tan 15°}$

$= \frac{1 + \tan^2 15°}{\tan 15°} = \frac{2}{\sin 30°} = 4$

65. If [t] denotes the greatest integer $\le t$, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2\, e^{[x]+[x^3]}dx$ is :

(1) $e^8 - 1$ (2) $e^7 - 1$ (3) $e^9 - e$ (4) $e^8 - e$

Ans. (4)

Sol. $\frac{3(e-1)}{e}\int_1^2 x^2\, e^{[x]+[x^3]}dx$

let $k = \int_1^2 x^2\, e^{1+[x^3]}dx$

$x^3 = t$

$3x^2\, dx = dt$

$k = \frac{1}{3}\int_1^8 e.e^{[t]}dt$

$\frac{e}{3}\left[e + e^2 + e^3 + ... + e^7\right] = \frac{e^2}{3}\left(\frac{e^7 - 1}{e - 1}\right)$

$\frac{3(e-1)}{e}k = \frac{3(e-1)}{e}\frac{e^2}{3}\left(\frac{e^7-1}{e-1}\right) = e(e^7 - 1)$

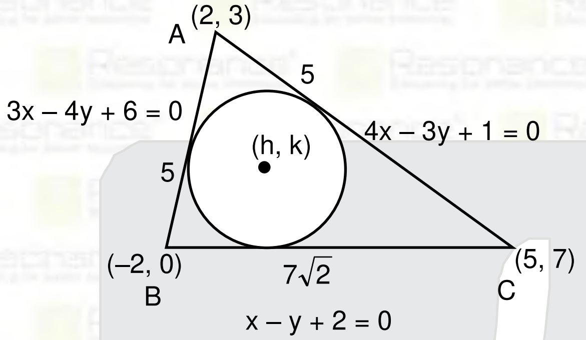
$= e^8 - e$

66. Let y = x + 2, 4y = 3x + 6 and 3y = 4x + 1 be three tangent lines to the circle $(x - h)^2 + (y - k)^2 = r^2$. Then h + k is equal to :

(1) 5 (2) $5(1+\sqrt{2})$ (3) 6 (4) $5\sqrt{2}$

Ans. **(1)**

Sol.



Δ ABC is isosceles (AB = AC) so angle bisector & altitude are same then
Equation of angle bisector of angle A is equal to altitude from vertex A
$x + y = \lambda$ (Passes through A)
$2 + 3 = \lambda \Rightarrow \lambda = 5$
x + y = 5 (Passes through Centre)
h + k = 5

67. If the solution of the equation $\log_{\cos x}\cot x + 4\log_{\sin x}\tan x = 1$, $x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha + \beta$ is equal to :

(1) 6 (2) 4 (3) 5 (4) 3

Ans. **(2)**

Sol. $\log_{\cos x}\cot x + 4\log_{\sin x}\tan x = 1$

$(\ln\cos x - \ln\sin x)\ln\sin x + 4(\ln\sin x - \ln\cos x)\ln\cos x = \ln\sin x \cdot \ln\cos x$

$(\ln\sin x - 2\ln\cos x)^2 = 0$

$\ln\left(\frac{\sin x}{\cos^2 x}\right) = 0$

$\frac{\sin x}{\cos^2 x} = 1$

$\sin^{2x} + \sin x - 1 = 0$

$\sin x = \frac{-1+\sqrt{5}}{2}$

$x = \sin^{-1}\left(\frac{-1+\sqrt{5}}{2}\right) = \sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$

$\alpha = -1, \beta = 5, \alpha + \beta = 4.$

68. If an unbiased die, marked with –2, –1, 0, 1, 2, 3 on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is :

(1) $\frac{440}{2592}$ (2) $\frac{881}{2592}$ (3) $\frac{27}{288}$ (4) $\frac{521}{2592}$

Ans. **(4)**

Sol. All positive or 3 positive and 2 negative or 1 positive and 4 negative

$$\Rightarrow P = \left(\frac{3}{6}\right)^5 + {}^5C_2.\left(\frac{2}{6}\right)^2.\left(\frac{3}{6}\right)^3 + {}^5C_1.\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4$$

$$= \frac{3^5}{6^5} + \frac{10.2^2.3^3}{6^5} + \frac{5.3.2^4}{6^5}$$

$$= \frac{243+1080+240}{6^5} = \frac{1563}{6^5} = \frac{521}{2592}$$

69 The coefficient of x^{301} in $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + ... + x^{500}$ is :

(1) $500_{C_{301}}$ (2) $501_{C_{200}}$ (3) $500_{C_{300}}$ (4) $501_{C_{302}}$

Ans. **(2)**

Sol. $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498}$ - - - - - - - + $(1 + x)^0\ x^{500}$

$$= \frac{(1+x)^{500}\left[1-\left(\frac{x}{1+x}\right)^{501}\right]}{1-\frac{x}{1+x}}$$

$$= \frac{(1+x)^{501}\left[1-\left(\frac{x}{1+x}\right)^{501}\right]}{1+x-x}$$

$= (1 + x)^{501} - x^{501}$

Now coefficient of x^{301} in $(1 + x)^{501} - x^{501}$

$= {}^{501}C_{301} = {}^{501}C_{200}$

70. If $a_n = \dfrac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + ... + a_{25}$ is equal to :

(1) $\dfrac{52}{147}$ (2) $\dfrac{50}{141}$ (3) $\dfrac{51}{144}$ (4) $\dfrac{49}{138}$

Ans. **(2)**

Sol. $a_1 + a_2$ + - - - - - - + $a_{25} = \sum_{n=1}^{25} a_n$

$$= \Sigma\frac{-2}{4n^2-16n+15} = \Sigma\frac{-2}{(2n-5)(2n-3)}$$

$$= \sum_{n=1}^{25}\left(\frac{1}{2n-3} - \frac{1}{2n-5}\right)$$

$$= \left[\left(\frac{1}{-1} - \frac{1}{-3}\right) + \left(\frac{1}{1} - \frac{1}{-1}\right) + \left(\frac{1}{3} - \frac{1}{1}\right) + ...\right.$$

$$= \frac{1}{2(25)-3} + \frac{1}{3} = \frac{50}{141}$$

71. The minimum number of elements that must be added to the relation R = {(a, b), (b, c)} on the set {a, b, c} so that it becomes symmetric and transitive is :

(1) 3 (2) 7 (3) 4 (4) 5

Ans. **(4)**

Sol. Required elements in sets for symmetric and transitive (a,a) (b,b) (c,c) (b,a) (c,b) (a,c) (c,a) Minimum no. of elements required = 7

72. If P(h, k) be a point on the parabola $x = 4y^2$, which is nearest to the point Q(0, 33), then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to :

(1) 4 (2) 2 (3) 8 (4) 6

Ans. **(4)**

Sol. $4y^2 = x$

$8y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{8y}$

$m_n = -8K = \frac{K-33}{h}$

$-8K = \frac{K-33}{4k^2}$

$-32k^2 = k - 33$

$32k^2 + k - 33 = 0$

$(32k + 33)(k - 1) = 0$

$k = -\frac{33}{32},$

$k = 1 \quad h = 4$

P (4, 1)

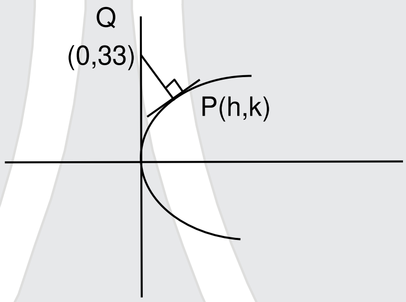
curve $y^2 + 4y = 4x$

$(y + 2)^2 = 4(x + 1)$

dire. $x + 1 = -1$

$x + 2 = 0$

req. dist. = |4 + 2| = 6



73. Let the system of linear equations

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

have infinitely many solutions. Then the system

$(k + 1)x + (2k - 1)y = 7$

$(2k + 1)x + (k + 5)y = 10$

has :

(1) infinitely many solution

(2) unique solution satisfying $x - y = 1$

(3) no solution

(4) unique solution satisfying $x + y = 1$

Ans. **(4)**

Sol. D = 0

$$\begin{vmatrix} 1 & 1 & K \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

6 + 8K – 3 – 9K + 4 – 4 = 0

k = 3

4x + 5y = 7 ..... (1)

7x +8y = 10 ......(2)

(2) –(1) ⇒ 3x +3y = 3

x + y = 1

These are two intersecting line has unique solution

Option (4) is correct.

74. The line ℓ_1 passes through the point (2, 6, 2) and is perpendicular to the plane 2x + y – 2z = 10.

Then the shortest distance between the line ℓ_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :

(1) $\frac{19}{3}$ (2) 7 (3) 9 (4) $\frac{13}{3}$

Ans. **(3)**

Sol. Line l_1 : $\frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

2^{nd} line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$

shortest distance $\left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = \frac{\begin{vmatrix} 3 & 10 & 2 \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix}}{\begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix}}$

$= \left|\frac{6-12-40-4-18-40}{\sqrt{16+64+64}}\right|$

$= \frac{108}{12} = 9$

Option (3) is correct.

75. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha\vec{b} - \hat{n}, (\alpha \neq 0)$ and $\vec{b}.\vec{c} = 12$, then $\left|\vec{c} \times (\vec{a} \times \vec{b})\right|$ is equal to :

(1) 9 (2) 15 (3) 6 (4) 12

Ans. **(4)**

Sol. $\vec{n} \cdot \vec{c} = 0,\ |\vec{n}| = 1$

$\vec{a} = \alpha\vec{b} - \hat{n}\ \&\ \vec{b} \cdot \vec{c} = 12$

$\vec{a} \cdot \vec{c} = \alpha\vec{b} \cdot \vec{c} - 0$

$\vec{a}\cdot\vec{c} = 12\alpha$

$\left|\vec{c}\times\left(\vec{a}\times\vec{b}\right)\right|$

$\left|\left(\vec{c}\cdot\vec{b}\right)\cdot\vec{a}-\left(\vec{c}\cdot\vec{a}\right)\cdot\vec{b}\right|$

$|12\,\vec{a} - 12\alpha\vec{b}\,|$

$12\,\left|\vec{a}-\alpha\vec{b}\right|$

$12\left|-\hat{n}\right| = 12.$

76. Among the statements :

(S1) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r)$

(S2) $((p\vee q)\Rightarrow r)\Leftrightarrow((p\Rightarrow r)\vee(q\Rightarrow r))$

(1) only (S2) is a tautology
(2) both (S1) and (S2) are tautologies
(3) only (S1) is a tautology
(4) neither (S1) nor (S2) is a tautology

Ans. **(4)**

Sol. If p = F, q = T, r = F

S1: $((F \vee T) \to F) \leftrightarrow (F \to F) = (T \to F) \leftrightarrow T = F \leftrightarrow T = F$

S2: $((F \vee T) \to F) \leftrightarrow ((F \to F) \vee (T\to F) = (T\to F) \leftrightarrow (T \vee F)$

$= F \leftrightarrow T = F$

77. Let a unit vector $\overrightarrow{OP}$ make angles α, β, γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta \in \left(0,\frac{\pi}{2}\right)$. If $\overrightarrow{OP}$ is perpendicular to the plane through points (1, 2, 3), (2, 3, 4) and (1, 5, 7), then which one of the following is true ?

(1) $\alpha \in \left(0,\frac{\pi}{2}\right)$ and $\gamma \in \left(0,\frac{\pi}{2}\right)$
(2) $\alpha \in \left(\frac{\pi}{2},\pi\right)$ and $\gamma \in \left(\frac{\pi}{2},\pi\right)$
(3) $\alpha \in \left(\frac{\pi}{2},\pi\right)$ and $\gamma \in \left(0,\frac{\pi}{2}\right)$
(4) $\alpha \in \left(0,\frac{\pi}{2}\right)$ and $\gamma \in \left(\frac{\pi}{2},\pi\right)$

Ans. **(2)**

Sol. $\overrightarrow{OP} = \vec{n} = \pm\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)$

A = (1, 2, 3)
B = (2, 3, 4)
C(1, 5, 7)

$$\vec{n} = \pm\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 0 & 3 & 4\end{vmatrix}$$

$\vec{n} = \pm\left(\hat{i}-4\hat{j}+3\hat{k}\right)$

$\beta \in (0, \pi/2)$

So, $\overrightarrow{OP} = \vec{n} = -\hat{i}+4\hat{j}-3\hat{k}$

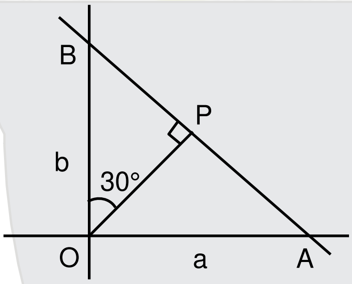
$\widehat{OP} = \alpha \in (\pi/2, \pi)$ & $\gamma = (\pi/2, \pi)$

78. A straight line cuts off the intercepts OA = a and OB = b on the positive directions of x-axis and y-axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of ΔOAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to :

(1) $\frac{392}{3}$ (2) $\frac{196}{3}$ (3) 196 (4) 98

Ans. **(1)**

Sol. Area of $\Delta OAB = \frac{98}{3}\sqrt{3}$



$\Rightarrow \frac{1}{2}ab = \frac{98}{3}\sqrt{3} \Rightarrow \sqrt{3}ab = 196$ ....(1)

Now OP = OB cos 30° = OA cos 60º

$\Rightarrow b.\frac{\sqrt{3}}{2} = a.\frac{1}{2}$

$\Rightarrow \sqrt{3}b = a$ ....(2)

by (1) and (2)

$a^2 = 196$

and $b^2 = \frac{a^2}{3}$

so $a^2 - b^2 = a^2 - \frac{a^2}{3} = \frac{2a^2}{3} = \frac{2}{3} \times 196 = \frac{392}{3}$

79. Let A = $\begin{pmatrix} m & n \\ p & q \end{pmatrix}$, d = $|A| \neq 0$ and $|A - d(\text{Adj } A)|$ = 0. Then

(1) $1 + d^2 = m^2 + q^2$ (2) $1 + d^2 = (m + q)^2$

(3) $(1 + d)^2 = m^2 + q^2$ (4) $(1 + d)^2 = (m + q)^2$

Ans. **(4)**

Sol. A = $\begin{bmatrix} m & n \\ p & q \end{bmatrix}$

$d = mq - np$

adj A = $\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

$|A - d \text{ Adj } A| = 0$

$\begin{bmatrix} m - dq & n + dn \\ p + dp & q - dm \end{bmatrix}$

$(m - dq)(q - dm) - (d + 1)^2 np = 0$

$mq - dm^2 - dq^2 + d^2mq = (d + 1)^2 np$

$mq(1 + d^2) - d(m^2 + q^2) = (d + 1)^2 (mq - d)$

$mq(1 + d^2) - d(m^2 + q^2) = (d^2 + 2d + 1)mq - (d + 1)^2 d$

$(m + q)^2 = (d + 1)^2$

80. Suppose $f : R \to (0, \infty)$ be a differentiable function such that $5f(x + y) = f(x) . f(y), \forall x, y \in R$.

If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

(1) 6575 (2) 6825 (3) 6875 (4) 6525

Ans. **(2)**

Sol. $5f(x + y) = f(x). f(y)$

$f(x) = 5a^x$

$x = 3 \quad 320 = 5.a^3$

$64 = a^3$

$a = 4$

$f(x) = 5. 4^x$

$\sum_{n=0}^{5} f(x) = 5\sum_{n=0}^{5} 4^n =$

$\frac{5(4^6 - 1)}{4 - 1}$

$\frac{5 \times 4095}{3} = 5 \times 1365 = 6825$ Ans. Option (2)

81. $\lim_{x \to 0} \frac{48}{x^4}\int_0^x \frac{t^3}{t^6 + 1} dt$ is equal to ________ .

Ans. **(12)**

Sol. $\lim_{x \to 0}\left(\frac{48\int_0^x \frac{t^3}{1+t^6}dt}{x^4}\right)$ $\left(\frac{0}{0} \text{ form}\right)$

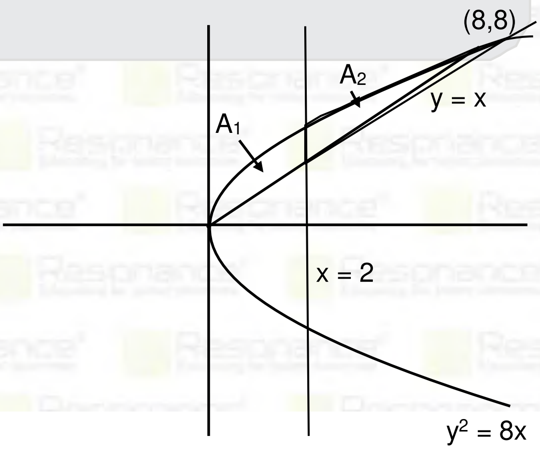
using Lebnitz theorem we have

$\lim_{x \to 0}\frac{48\frac{x^3}{1+x^6}}{4x^3} = \frac{48}{4} = 12$

82. Let α be the area of the larger region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant. Then the value of 3α is equal to _______ .

Ans. **(22)**

Sol. $y^2 = 8x$



$A_1 = \int_0^2\left(2\sqrt{2}x^{\frac{1}{2}} - x\right)dx$

$= \left[2\sqrt{2} \times \frac{2}{3}x^{\frac{3}{2}}\right]_0^2 - \left[\frac{x^2}{2}\right]_0^2 = \frac{10}{3}$

$A_2 = \int_2^8\left(2\sqrt{2}x^{\frac{1}{2}} - x\right)dx$

$$= \left[2\sqrt{2}\times\frac{2}{3}x^{\frac{3}{2}}\right]_2^8 - \left[\frac{x^2}{2}\right]_2^8 = \frac{22}{3}$$

$\because A_2 > A_1$

$\therefore$ Required area, $A_2 = \frac{22}{3} = \alpha$

So, $3\alpha = 3\times\frac{22}{3} = 22$

83. Let $\sum_{n=0}^{\infty}\frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)} = ae+\frac{b}{e}+c$, where a, b, c $\in$ Z and e = $\sum_{n=0}^{\infty}\frac{1}{n!}$ Then $a^2 - b + c$ is equal to ________ .

Ans. **(26)**

Sol. $\sum_{n=0}^{\infty}\frac{n^3(2n)! + (2n-1)n!}{2n!\ n!} = \sum_{n=0}^{\infty}\left(\frac{n^3}{n!}+\frac{2n-1}{(2n)!}\right)$

$$= \sum_{n=0}^{\infty}\left[\frac{n^2}{(n-1)!}+\frac{1}{(2n-1)!}-\frac{1}{(2n)!}\right]$$

$$= \sum_{n=0}^{\infty}\left(\frac{n^2-1}{(n-1)!}+\frac{1}{(n-1)!}\right)+\left(-e^{-1}\right)$$

$$= \sum_{n=2}^{\infty}\frac{n+1}{(n-2)}+e-\frac{1}{e}$$

$$= \sum_{n=2}^{\infty}\frac{n-2+3}{(n-2)!}+e-\frac{1}{e}$$

$$= \sum_{n=3}^{\infty}\frac{1}{(n-3)!}+\sum_{n=2}^{\infty}\frac{3}{(n-2)!}+e-\frac{1}{e}$$

$= e + 3e + e - \frac{1}{e} = 5e - \frac{1}{e}$

a = 5, b = – 1, c = 0

a2 – b + c = (5)2 – (–1) + 0 = 26

84. Number of 4-digit numbers (the repetition of digits is allowed) which are made using the digits 1, 2, 3 and 5, and are divisible by 15, is equal to ________

Ans. **(21)**

Sol. Unit place of numbers is always 5 and sum of first three digits is 4, 7, 10, 13

5, 1, 12 $\longrightarrow$ $\frac{3!}{2!}\times 1 = 3$

5, 1 33 $\longrightarrow$ $\frac{3!}{2!}\times 1 = 3$

5 5 32 $\longrightarrow$ $3! \times 1 = 6$

5 5 53 $\longrightarrow$ $\frac{3!}{2!}\times 1 = 3$

5, 2,2,3 $\longrightarrow$ $\frac{3!}{2!}\times 1 = 3$

55 11 $\longrightarrow$ $\frac{3!}{2!}\times 1 = 3$

21 Ans.

85. Let S = {1, 2, 3, 4, 5, 6}. Then the number of one-one functions f : S → P(S), where P(S) denote the power set of S, such that $f(n) \subset f(m)$ where n < m is ________ .

Ans. **(3240)**

Sol. Case I

f(1) has only 1 element in {1}, {2} ,{3}, {4}, {5}, {6}

f(2) has 2 elements in which 1 is same as f(1)

and so on

$^6C_1 . {}^5C_1 . {}^4C_1 . {}^3C_1 . {}^2C_1 .1 = 720$

Case II

f(1) = ϕ

f(2)	f(3)	f(4)	f(5)	f(6)	
1	2	3	4	5	$^6C_1 . {}^5C_1 . {}^4C_1 . {}^3C_1 . {}^2C_1 = 720$
1	2	3	4	6	$^6C_1 . {}^5C_1 . {}^4C_1 . {}^3C_1 . {}^1C_1 = 360$
1	2	3	5	6	4 × 360
1	2	4	5	6	
1	3	4	5	6	
2	3	4	5	6	

Total 720 + 720 + 360 + 4 × 360 =

86. If the equation of the plane passing through the point (1, 1, 2) and perpendicular to the line x – 3y + 2z – 1 = 0 = 4x –y + z is Ax + By + Cz = 1, then 140 (C–B+A) is equal to _______ .

Ans. **(15)**

Sol. $\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix} = -\hat{i} + 7\hat{j} + 11\hat{k}$

Eq^n. Of plane: –1(x–1) + 7(y –1) + 11 (z – 2) = 0

–x + 7y + 11z = 28

$A = -\frac{1}{28}, B = \frac{7}{28}, C = \frac{11}{28}$

$140(C - B + A) = 140\left(\frac{11-7-1}{28}\right) = 15$ Ans.

87. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then a + 3b – 5 is equal to ______ .

Ans. **(37)**

Sol. $\sum_{i=i}^{7} x_i = 7 \times 8 \quad = 56$

$\Sigma x_i^2 = 7(16+64) = 560$

Where 14 is removed then

Now man a $\Rightarrow$ 6a = $\sum_{i=i}^{7} x_i - 14$

$\Rightarrow$ 6a + 14 = 56

$\Rightarrow$ a = 7

New variance b $\Rightarrow$ 6b = $\left(\sum_{i=i}^{7} x_1^2 - 196\right) - (7)^2.6$

$\Rightarrow$ 6b = 560 – 196 – 294

$\Rightarrow$ 6b = 560 – 490 = 70

$\Rightarrow$ 3b = 35

Hence a + 3b – 5 = 7 + 35 – 5

= 37

88. If $\lambda_1 < \lambda_2$ are two values of λ such that the angle between the planes P_1 : $\vec{r}.(3\hat{i} - 5\hat{j} + \hat{k}) = 7$ and P_2 : $\vec{r}.(\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1, 10\lambda_2, 2)$ to the plane P_1 is ______ .

Ans. **(315)**

Sol. θ = angle between planes P_1 & P_2

$\sin\theta = \frac{2\sqrt{6}}{5}$, $\cos\theta = \frac{1}{5}$

$\vec{n}_1.\vec{n}_2 = |\vec{n}_1||\vec{n}_2|\cos\theta$

$3\lambda - 5 - 3 = \sqrt{35}\sqrt{\lambda^2 + 10} \times \frac{1}{5}$

$19\lambda^2 - 120\lambda + 125 = 0$

$\lambda_1 = \frac{25}{19}, \lambda_2 = 5$

$(38\lambda_1, 10\lambda_2, 2) = (50, 50, 2)$

and $P_1 : 3x - 5y + z = 7$

$d^2 = \left(\frac{3\times 50 - 5\times 50 + 2 - 7}{\sqrt{9+25+1}}\right)^2$

$= 315$

89. Let z = 1 + i and $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}$ arg(z_1) is equal to ______ .

Ans. **(9)**

Sol. $z_1 = \frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$, z = 1 + i

$$= \frac{1+i(1-i)}{(1-i)[1-(1+i)]+\frac{1}{1+i}} = \frac{1+i+1}{-i-1+\frac{1-i}{2}} = \frac{(i+2)2}{-2-2i+1-i}$$

$$= \frac{2(i+2)(1-3i)}{-(1+3i)(1-3i)}$$

$$= \frac{(i+3+2-6i)}{-5}$$

$$= \frac{5-5i}{-5} = -1 + i$$

arg (z_1) = $\pi - \tan^{-1}\left|\frac{y}{x}\right|$

$= \pi - \tan^{-1}(1)$

$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\frac{12}{\pi}\arg(z_1) = \frac{12}{\pi}\times\frac{3\pi}{4} = 9$

90. Let $f^1(x) = \frac{3x+2}{2x+3}$, $x \in R - \left\{\frac{-3}{2}\right\}$

For n ≥ 2, define $f^n(x) = f^1 o f^{n-1}(x)$.

If $f^5(x) = \frac{ax+b}{bx+a}$, gcd (a, b) = 1, then a + b is equal to _____ .

Ans. **(3125)**

Sol. $f^1(x) = \frac{3x+2}{2x+3} = \frac{a_1x+b_1}{b_1x+a_1}$, $a_1 + b_1 = 3 + 2 = 5$

$$f^2(x) = f^1(f^1x) = \frac{3\left(\frac{3x+2}{2x+3}\right)+2}{2\left(\frac{3x+2}{2x+3}\right)+3} = \frac{(3^2+2^2)x+2.3.2}{(3^2+2^2)+2.3.2x} = \frac{a_2x+b_2}{b_2x+a_2}$$

$a_2 + b_2 = (3+2)^2 = 5^2$

Similarly $f^3(x) = \frac{a_3x+b_3}{b_3x+a_3} \Rightarrow a_3 + b_3 = (3+2)^3 = 5^3$

$$f^5(x) = \frac{ax+b}{bx+a} \Rightarrow a+b = (3+2)^5 = 3125$$

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