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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)
Questions & Solutions

Date: 30 January, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

61. Let x = $\left(8\sqrt{3}+13\right)^{13}$ and y = $\left(7\sqrt{2}+9\right)^{9}$. If [t] denotes the greatest integer $\leq$ t, then

(1) [x] is even but [y] is odd
(2) [x] +[y] is even
(3) [x] and [y] are both odd
(4) [x] is odd but [y] is even

NTA Ans. (2)
Reso. Ans. (2)

Sol. I + f = x = $\left(8\sqrt{3}+13\right)^{13} = {}^{13}C_0\left(8\sqrt{3}\right)^{13} + {}^{13}C_1\left(8\sqrt{3}\right)^{13}.13 + ... + {}^{13}C_{13}13^{13}$

$\therefore \left(8\sqrt{3}-13\right)^{13} \in (0, 1)$

So let $f' = \left(8\sqrt{3}-13\right)^{13} = {}^{13}C_0\left(8\sqrt{3}\right)^{13} - {}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1 + ... - {}^{13}C_{13}(13)^{13}$

Now I + $f - f'$ = 2 (Positive Integer)

$\therefore$ 0 < f < 1

o < f' < 1

$\Rightarrow$ $-1 < f - f' < 1$

$\Rightarrow$ $f - f' = 0$ as L.H.S. in also an integer

$\Rightarrow$ I = even integer = [x]

Similarly

y = I + f = $\left(7\sqrt{2}+9\right)^{9} = {}^{9}C_0\left(7\sqrt{2}\right)^{9} + {}^{9}C_1\left(7\sqrt{2}\right)^{8}.9 + ... + {}^{9}C_9 9^9$

Let $f' = \left(7\sqrt{2}-9\right)^{9} = {}^{9}C_0\left(7\sqrt{2}\right)^{9} - {}^{9}C_1\left(7\sqrt{2}\right)^{8}.9 + ... - {}^{9}C_9 9^9$

$\Rightarrow$ I + $f - f'$ = 2 (Positive integer)

$\therefore$ $-1 < f - f' < 1$ $\Rightarrow$ $f - f' = 0$

$\Rightarrow$ I = [y] = even integer

$\Rightarrow$ [x] + [y] = even

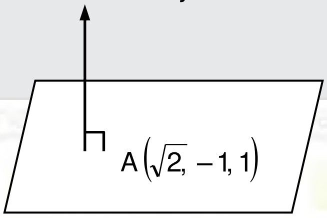
62. A vector $\vec{v}$ in the first octant is inclined to the x – axis at 60º, to the y-axis at 45º and to the z-axis at an acute angle. If a plane passing through the points $\left(\sqrt{2},-1,1\right)$ and (a,b,c) is normal to $\vec{v}$, then

(1) $\sqrt{2}a - b + c = 1$
(2) $a + \sqrt{2}a + c = 1$
(3) $\sqrt{2}a + b + c = 1$
(4) $a + b + \sqrt{2}a = 1$

NTA Ans. (2)
Reso. Ans. (2)

Sol. $\therefore$ $l = \cos 60° = \frac{1}{2}$

Let $\vec{v} = l\hat{i} + m\hat{j} + n\hat{k}$



m = $\cos 45° = \frac{1}{\sqrt{2}}$

$\therefore$ $l^2 + m^2 + n^2 = 1$

$\Rightarrow$ $n^2 = \frac{1}{4}$

$\Rightarrow$ n = $\frac{1}{2}, -\frac{1}{2}$ reject as $\vec{v}$ is in first octant

$\Rightarrow$ $\vec{n} = \frac{1}{2}\left(\hat{i} + \sqrt{2}\hat{j} + \hat{k}\right)$

So, dr's of normal to the plane can be taken as $1, \sqrt{2}, 1$

Hence

$1\left(a-\sqrt{2}\right) + \sqrt{2}(b+1) + 1(c-1) = 0 \Rightarrow a + \sqrt{2}b + c = 1$

63. If P is a 3× 3 real matrix such that $P^T = aP + (a - 1)I$, Where a > I, then

(1) |Adj P| > 1 (2) |Adj P| = $\frac{1}{2}$ (3) P is a singular matrix (4) |Adj P| = 1

NTA Ans. (4)
Reso. Ans. (4)

Sol. $(P^T)^T = (aP + (a-1)I)^T$

$\Rightarrow \quad P = aP^T + (a-1)I$

$\Rightarrow \quad P = a(ap + (a-1)I) + (a-1)I$

$\Rightarrow \quad P = a^2P + a(a-1)I + (a-1)I$

$\Rightarrow \quad P = a^2P + (a^2-1)I$

$\Rightarrow \quad (a^2-1)P + (a^2-1)I = 0$

$\Rightarrow \quad (a^2-1)(P + I) = 0$

$\therefore \quad a > 1$

$\Rightarrow \quad P + I = 0$

$\Rightarrow \quad P = -I$

adj P = adj (–I)

$\Rightarrow \quad |adj P| = |adj(-I)| = |-I|^2$

$\Rightarrow \quad |adj P| = 1$

64. Let q be the maximum integral value of p in [0, 10] for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region {(x, y): 0 ≤ y ≤ (x –q)², 0 ≤ x ≤ q) is

(1) 164 (2) 243 (3) $\frac{125}{3}$ (4) 25

NTA Ans. (2)
Reso. Ans. (2)

Sol. D = p²– 5p = Perfect square of a rational number

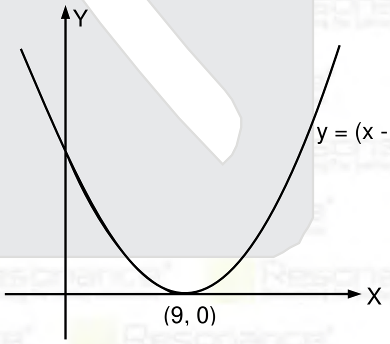
$\Rightarrow D = p(p-5)$

Now put values in reverse order as q is maximum value of p

p = 10 reject

p = 9 select

$\Rightarrow q = 9$



So area of shaded region = $\int_0^9 (x-9)^2 dx$

$= \left[\frac{(x-9)^3}{3}\right]_0^9$

$\Rightarrow \quad$ 243 sq. unit

65. Let f, g and h be the real valued function defined on R as

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 1, & x = 0 \end{cases}, \quad g(x) = \begin{cases} \frac{\sin(x+1)}{x+1}, & x \neq -1 \\ 1, & x = -1 \end{cases}$$

and h(x) = 2[x] – f(x) ; where [.] is the greatest integer ≤ x. Then the value of $\lim\limits_{x\to 1} g(h(x-1))$ is

(1) 1 (2) 0 (3) sin (1) (4) -1

NTA Ans. (1)

Reso. Ans. (1)

Sol. L.H.L

$\lim\limits_{h\to 0^+} g(h(1-h-1))$

$\lim\limits_{h\to 0^+} g\left(2[-h]-\frac{-h}{|-h|}\right)$

$\lim\limits_{h\to 0^+} g(2(-1)+1)$

$\lim\limits_{h\to 0^+} g(-1) = 1$

R.H.L

$= \lim\limits_{h\to 0^+} g(h(1+h-1))$

$= \lim\limits_{h\to 0^+} g(2[h]-f(h))$

$= \lim\limits_{h\to 0^+} g(0-1)$

$= \lim\limits_{h\to 0^+} 1$

$= 1$

66. Let $a_1 = 1, a_2, a_3, a_4, \ldots\ldots$ be consecutive natural number, then

$$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+\ldots\ldots+\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right) \text{ is equal to}$$

(1) $\cot^{-1}(2022)-\frac{\pi}{4}$ (2) $\frac{\pi}{4}-\cot^{-1}(2022)$

(3) $\tan^{-1}(2022)-\frac{\pi}{4}$ (4) $\frac{\pi}{4}-\tan^{-1}(2022)$

NTA Ans. (2)

Reso. Ans. (2,3)

Sol. $a_1, a_2, a_3, \ldots\ldots\ldots$ are consecutive natural number

$$\therefore \tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)+\ldots\ldots+\tan^{-1}\left(\frac{a_{2022}-a_{2021}}{1+a_{2021}a_{2022}}\right)$$

$= \tan^{-1} a_2 - \tan^{-1} a_1 + \tan^{-1} a_3 - \tan^{-1} a_2 + \ldots\ldots + \tan^{-1} a_{2022} - \tan^{-1} a_{2021}$

$= \tan^{-1} a_{2022} - \tan^{-1} 1$

$= \tan^{-1} 2022 - \frac{\pi}{4}$

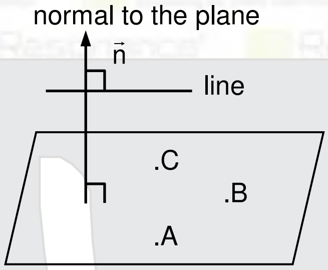
$= \frac{\pi}{2} - \cot^{-1} 2022 - \frac{\pi}{4} = \frac{\pi}{4} - \cot^{-1} 2022$

67. If a plane passes through the point (-1,k,0), (2,k,–1) , (1,1,2) and is parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is

(1) $\frac{13}{6}$ (2) $\frac{5}{17}$ (3) $\frac{17}{5}$ (4) $\frac{6}{13}$

NTA Ans. (1)
Reso. Ans. (1)

Sol. Line is parallel to to the vector $\hat{i}+\hat{j}-\hat{k}$



Let A (–1, k, 0), B(2, k, –1), C(1, 1, 2)

So $\vec{n}=\vec{AB}\times\vec{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 0 & -1\\ 2 & 1-k & 2\end{vmatrix}$

$\Rightarrow$ $\vec{n}=(1-k)\hat{i}-8\hat{j}+(3-3k)\hat{k}$

$\therefore$ $\hat{n}.(\hat{i}+\hat{j}-\hat{k})=0$

$\Rightarrow$ $1-k-8-3+3k=0$

$\Rightarrow$ $k=5$

hence $\frac{k^2+1}{(k-1)\,(k-2)}=\frac{26}{(4)\,(3)}=\frac{13}{6}$

68. If the function f(x) = $\frac{x^3}{3}+2bx+\frac{ax^2}{2}$ and g (x) = $\frac{x^3}{3}+ax+bx^2, a\neq 2b$ have a common extreme point, then a + 2b +7 is equal to:

(1) $\frac{3}{2}$ (2) 3 (3) 6 (4) 4

NTA Ans. (3)
Reso. Ans. (3)

Sol. $f'(x)=0$ and $g'(x)=0$ have a common root

$x^2+ax+2b=0$

$x^2+2bx+a=0$

$\Rightarrow \frac{x^2}{a^2-4b^2}=\frac{x}{2b-a}=\frac{1}{2b-a}$

$\Rightarrow x=1$ or $-x=a+2b$

$\Rightarrow a+2b=-1$

$\Rightarrow a+2b+7=6$

69. The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line y = 1. If a, b, c, d, e, f are positive real numbers and a, b c are in G.P. , then

(1) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P (2) d, e, f are in A.P. (3) d, e, f are in G.P (4) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P

NTA Ans. (4)
Reso. Ans. (4)

Sol. y = 1 is common to both

$\Rightarrow ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$

$\because b^2 = ac$

$\Rightarrow ax^2 + 2\sqrt{a}\sqrt{c}\,x + c = 0$

$\Rightarrow \left(\sqrt{a}\,x + \sqrt{c}\right)^2 = 0$

$\Rightarrow x = -\sqrt{\frac{c}{a}}$

Put this value in second equation

$d\left(\frac{c}{a}\right) - 2e\sqrt{\frac{c}{a}} + f = 0$

$\Rightarrow \frac{cd}{a} + f = 2e\frac{\sqrt{c}}{\sqrt{a}}$

$\Rightarrow cd + af = 2e\sqrt{ac}$

$\Rightarrow cd + af = 2eb$

dir. by ac

$\frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

70. Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}| = 1; |\vec{b}| = 4$ and $\vec{a}.\vec{b} = 2$. If $\vec{c} = \left(2\vec{a} \times \vec{b}\right) - 3\vec{b}$, then the value of $\vec{b}.\vec{c}$ is

(1) – 60 (2) – 48 (3) – 84 (4) – 24

NTA Ans. (2)
Reso. Ans. (2)

Sol. $\vec{c} = \left(2\vec{a} \times \vec{b}\right) - 3\vec{b}$ ....(1)

$\vec{b}$..(1)

$\Rightarrow \vec{b}.\vec{c} = 0 - 3\vec{b}.\vec{b} = -48$

71. $\lim_{n\to\infty} \frac{3}{n}\left[4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \ldots\ldots + \left(3 - \frac{1}{n}\right)^2\right]$ is equal to

(1) $\frac{19}{3}$ (2) 12 (3) 0 (4) 19

NTA Ans. (4)
Reso. Ans. (4)

Sol. $\lim_{n\to\infty}\frac{3}{n}\left[4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+.....+\left(3-\frac{1}{n}\right)^2\right]$

$\lim_{n\to\infty}\frac{3}{n}\left[\left(2+\frac{0}{n}\right)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+.....+\left(2+\frac{n-1}{n}\right)^2\right]$

$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{3}{n}\left(2+\frac{r}{n}\right)^2$

Let $\frac{r}{n}=x$

$\therefore \frac{1}{n}=dx$

$= 3\int_0^1(2+x)^2dx$

$=3\left(\frac{(2+x)^3}{3}\right)_0^1$

$= 3^3 - 2^3$

$= 27-8 = 19$

72. Let a, b, c > 1, a^3, b^3 and c^3 be in A.P and $\log_a b$, $\log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first terms is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is – 444, then abc is equal to:

(1) 343 (2) 216 (3) $\frac{343}{8}$ (4) $\frac{125}{8}$

NTA Ans. (2)

Reso. Ans. (2)

Sol. $2b^3 = a^3 + c^3$

$\left(\frac{\log a}{\log c}\right)^2=\frac{\log b}{\log a}.\frac{\log c}{\log b}$

$\Rightarrow (\log a)^3 = (\log c)^3$

$\Rightarrow a = c$

$\Rightarrow 2b^3 = 2c^3 \quad \Rightarrow \quad b = c$

$\Rightarrow a = b = c$

$T_1 = 2a, \quad d = -\frac{3a}{5}$

$\therefore -444 = 10\left(4a-\frac{57a}{5}\right)$

$-444 = -\frac{370a}{5} \Rightarrow a=\frac{444}{74}$

$\Rightarrow \quad a = 6$

$\Rightarrow \quad abc = 6.6.6 = 216$

73. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is :

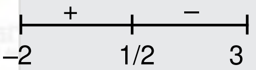
(1) $[2\sqrt{2}, \sqrt{11}]$ (2) $[\sqrt{5}, \sqrt{10}]$ (3) $[\sqrt{5}, \sqrt{13}]$ (4) $[\sqrt{2}, \sqrt{7}]$

NTA Ans. (2)
Reso. Ans. (2)

Sol. $f'(x) = \frac{1}{2\sqrt{2+x}} - \frac{1}{2\sqrt{3-x}}$

$\Rightarrow f'(x) = \frac{\sqrt{3-x} - \sqrt{2+x}}{2\sqrt{2+x}\sqrt{3-x}} = 0$

$\Rightarrow \quad x = \frac{1}{2}$ (domain is $-2 \le x \le 3$)



So at $x = \frac{1}{2}$, maxima

$f(-2) = \sqrt{5}$, $f\left(\frac{1}{2}\right) = \sqrt{10}$, $f(3) = \sqrt{5}$

range $\in \left[\sqrt{5}, \sqrt{10}\right]$

74. Let S be the set of all value of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots, a_{100}$ is 25. Then S is

(1) {99} (2) ϕ (3) N (4) {9}

NTA Ans. (3)
Reso. Ans. (3)

Sol. X : $a_1, a_1 + 1, a_1 + 2, \ldots, a_1 + 99$

$\bar{x} = \frac{a_1 + (a_1+1) + \ldots + (a_1+99)}{100}$

$\bar{x} = a_1 + 49.5$

$\therefore$ M.D. $= \frac{\sum |x_i - \bar{x}|}{n} = 25$

$\Rightarrow \quad 2(49.5 + 48.5 + \ldots + 0.5) = 2500$

$\Rightarrow \quad 2500 = 2500$

$\Rightarrow \quad$ It is true for all $a_i \in N$

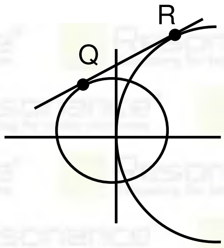
75. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(1) 76 (2) 81 (3) 72 (4) 64

NTA Ans. (3)
Reso. Ans. (3)

Sol. Any tangent for $y^2 = 16x$ is $y = mx + \frac{4}{m}$

it is also a tangent to $x^2 + y^2 = 8$



if $\left(\frac{4}{m}\right)^2 = 8(1+m^2)$

$\frac{16}{m^2} = 8(1+m^2)$

$\Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow (m^2 + 2)(m^2 - 1) = 0 \Rightarrow m = \pm 1$

$\therefore$ common tangent is $y = x + 4$

$\Rightarrow x - y + 4 = 0$

tangent to $x^2 + y^2 = 8$ is $xx_1 + yy_1 - 8 = 0$

$\therefore \frac{x_1}{1} = \frac{y_1}{-1} = \frac{-8}{4}$ $\quad \therefore Q(-2, 2)$

tangent to $y^2 = 16x$ is $yy_1 = 8(x + x_1)$

$8x - yy_1 + 8x_1 = 0$

$\frac{8}{1} = \frac{-y_1}{-1} = \frac{8x_1}{4}$

$y_1 = 8, x_1 = 4,$

$\therefore P(4, 8)$

$\therefore$ distance = PQ = $\sqrt{(4+2)^2 + (8-2)^2} = \sqrt{36+36} = 6\sqrt{2}$

$\therefore PQ^2 = 72$

76. Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest.

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to :

(1) $(P \vee Q) \vee ((\sim P) \vee R)$ (2) $(P \vee \sim Q) \wedge (P \vee \sim R)$

(3) $(\sim P) \vee \sim Q \wedge ((\sim P) \vee \sim R)$ (4) $((\sim P) \vee (\sim Q)) \wedge ((\sim P) \vee R)$

NTA Ans. (4)

Reso. Ans. (4)

Sol. Given $P \rightarrow (\sim Q \wedge R)$

$= \sim P \vee (\sim Q \wedge R)$

$= ((\sim P) \vee \sim Q) \wedge (\sim P \vee R)$

77. The number of ways of selecting two numbers a and b, $a \in \{2, 4, 6, \ldots, 100\}$ and $b \in \{1, 3, 5, \ldots, 99\}$ such that 2 is the remainder when a + b is divided by 23 is

(1) 54 (2) 108 (3) 268 (4) 186

NTA Ans. (2)
Reso. Ans. (2)

Sol. Clearly $\frac{a+b}{23} = \lambda + \frac{2}{23}, \lambda \in N$

$\Rightarrow$ $a + b = 23\lambda + 2$

$\Rightarrow$ a + b can be 25, 71, 117, 163

When a + b = 25 $\Rightarrow$ Number of ways = 12

When a + b = 71 $\Rightarrow$ Number of ways = 35

When a + b = 117 $\Rightarrow$ Number of ways = 42

When a + b = 163 $\Rightarrow$ Number of ways = 19

So total number of ways = 108

78. The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2+3y^2}{3x^2+y^2}\right)$, y (1) = 0 is

(1) $\log_e |x+y| - \frac{xy}{(x+y)^2} = 0$ (2) $\log_e |x+y| - \frac{2xy}{(x+y)^2} = 0$

(3) $\log_e |x+y| + \frac{xy}{(x+y)^2} = 0$ (4) $\log_e |x+y| + \frac{2xy}{(x+y)^2} = 0$

NTA Ans. (4)
Reso. Ans. (4)

Sol. Let y = vx

$$v + x\frac{dv}{dx} = -\left(\frac{1+3v^2}{3+v^2}\right)$$

$$\Rightarrow \frac{v^2+3}{(v+1)^3}dv + \frac{1}{x}dx = 0$$

$$\Rightarrow \int \frac{4}{(v+1)^3}dv + \int \frac{1}{v+1}dv - \int \frac{2}{(v+1)^2}dv + \int \frac{1}{x}dx = 0$$

$$\Rightarrow \frac{-2}{(v+1)^2} + \ln|v+1| + \frac{2}{v+1} + \ln|x| = c$$

$$\Rightarrow \frac{-2x^2}{(x+y)^2} + \ln\left|\frac{x+y}{x}\right| + \frac{2x}{x+y} + \ln|x| = c$$

$$\Rightarrow \frac{2xy}{(x+y)^2} + \ln|x+y| = c$$

$\because$ y(1) = 0

$\Rightarrow$ c = 0

$$\Rightarrow \frac{2xy}{(x+y)^2} + \ln|x+y| = 0$$

79. Let $\lambda \in R$, $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$,

If $\left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right)\times(\vec{a}-\vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then $\left|\lambda(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|^2$ is equal to

(1) 136 (2) 132 (3) 140 (4) 144

NTA Ans. (3)
Reso. Ans. (3)

Sol. $(\vec{b}-\vec{a})\times\left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

This is vector triple product

$\Rightarrow \left((\vec{b}-\vec{a}).(\vec{a}\times\vec{b})\right)(\vec{a}+\vec{b}) - \left((\vec{b}-\vec{a}).(\vec{a}+\vec{b})\right)(\vec{a}\times\vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow 0 + \left((\vec{a}-\vec{b}).(\vec{a}+\vec{b})\right)(\vec{a}\times\vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow \left((\lambda^2+13) - (\lambda^2+5)\right)(\vec{a}\times\vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

Now $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix} = (4-3\lambda)\hat{i} - (2\lambda+3)\hat{j} - (\lambda^2+2)\hat{k}$

$\Rightarrow (4-3\lambda)\hat{i} - (2\lambda+3)\hat{j} - (2+\lambda^2)\hat{k} = \hat{i} - 5\hat{j} - 3\hat{k}$

$\Rightarrow 4 - 3\lambda = 1$ and $2\lambda + 3 = 5$ and $2 - \lambda^2 = -3$

$\Rightarrow \lambda = 1$ $\Rightarrow \vec{a}\times\vec{b} = \hat{i} - 5\hat{j} - 3\hat{k}$

Therefore $\left|\lambda(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|^2 = 2^2\left|\vec{a}\times\vec{b}\right|^2 = 140$

80. For $\alpha, \beta \in R$, suppose the system of linear equations

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

has infinitely many solutions. Then α and β are the roots of

(1) $x^2 - 18x + 56 = 0$ (2) $x^2 + 14x + 24 = 0$

(3) $x^2 - 10x + 16 = 0$ (4) $x^2 + 18x + 56 = 0$

NTA Ans. (1)
Reso. Ans. (1)

Sol. $\Delta = \Delta x = \Delta y = \Delta z = 0$

$\Rightarrow \Delta = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$ $\Rightarrow 8 + \alpha + 8 - 3\alpha - 8 = 0$

$\Rightarrow \alpha = 4$

$\Rightarrow \Delta_z = \begin{vmatrix} 1 & -1 & 5 \\ 2 & 2 & 8 \\ 3 & -1 & \beta \end{vmatrix} = 0$ $\Rightarrow 3(-18) + 1(-2) + \beta(4) = 0$

$\Rightarrow \beta = 14$

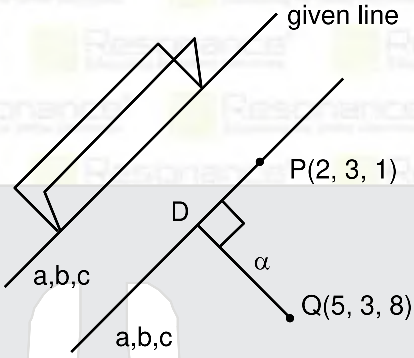
So equation is $x^2 - 18x + 56 = 0$

81. Let a line L pass through the point P(2, 3, 1) and be parallel to the line x + 3y – 2z – 2 = 0 = x – y + 2z. If the distance of L from the point (5, 3, 8) is α, then $3\alpha^2$ is equal to _____.

NTA Ans. (158)
Reso. Ans. (158)

Sol. $a + 3b - 2c = 0$
$a - b + 2c = 0$

$\frac{a}{4} = \frac{b}{-4} = \frac{c}{-4}$



So dr's of L can be –1, 1 , 1

$L : \frac{x-2}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

Let D $(2 - \lambda, 3 + \lambda, 1 + \lambda)$

Dr's of DQ : $3 + \lambda, -\lambda, 7 - \lambda$

$\therefore$ DQ⊥L

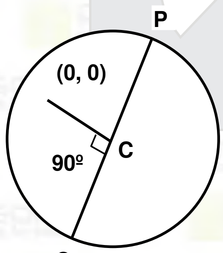
$\Rightarrow -3 - \lambda - \lambda + 7 - \lambda = 0 \quad \Rightarrow \lambda = \frac{4}{3}$

$\Rightarrow \alpha^2 = \frac{169}{9} + \frac{16}{9} + \frac{289}{9} = \frac{474}{9} \Rightarrow 3\alpha^2 = 158$

82. Let P(a_1, b_1) and Q (a_2, b_2) be two distinct points on a circle with center C $\left(\sqrt{2}, \sqrt{3}\right)$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ______ .

NTA Ans. (24)
Reso. Ans.(24)

Sol. $\frac{1}{2} \times PC \times \sqrt{5} = \frac{\sqrt{35}}{2} ; PC = \sqrt{7}$



$a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2$
= 2 (5 + 7) = 24

83. The 8th common term of the series
$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots\ldots,$
$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots\ldots$
is ______ .

NTA Ans. (151)
Reso. Ans. (151)

Sol. For common series a = 11, d = LCM of (4, 5) = 20
$T_8 = 11 + (8 - 1) \times 20$
$= 11 + 140 = 151$

84. 50^{th} root of a number x is 12 and 50^{th} root of another number y is 18. Then the remainder obtained on dividing (x + y) by 25 is _______ .

NTA Ans. (23)
Reso. Ans. (23)

Sol. $(x)^{\frac{1}{50}} = 12$ and $(y)^{\frac{1}{50}} = 18$
$\Rightarrow x = 12^{50} \quad \Rightarrow \quad y = 18^{50}$
$x + y = 12^{50} + 18^{50}$
$= (2^2 \times 3^1)^{50} + (3^2 \times 2)^{50}$
$= 6^{50}(2^{50} + 3^{50})$
$=(5+1)^{50}\ (4^{25}+9^{25})$
$= (25\lambda+1)[(5-1)^{25} + (10-1)^{25}]$
$= (25\lambda+1)\ (25k-1+25\mu-1)$
$= (25\lambda+1)\ (25t + 23)$
$= 25\ell + 23$
so remainder is 23, when x+y is divided by 25.

85. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q. If p : q = m : n, where m and n are co-prime, then m + n is equal to ______ .

NTA Ans. (14)
Reso. Ans. (14)

Sol. $p = \frac{^6C_1}{6 \times 6} = \frac{1}{6}$

$q = \frac{^6C_1 \times {}^5C_1 \times 4}{6 \times 6 \times 6 \times 6} = \frac{5}{54}$

$\therefore p : q = 9 : 5 \Rightarrow m + n = 14$

86. If $\int \sqrt{\sec 2x - 1}\, dx = \alpha \log_e \left| \cos 2x + \beta + \sqrt{\cos 2x\left(1 + \cos\frac{1}{\beta}x\right)} \right|$ + constant, then $\beta - \alpha$ is equal to _____ .

NTA Ans. (1)
Reso. Ans. (1)

Sol. $\int \sqrt{\sec 2x - 1}dx = \int \sqrt{\frac{1-\cos 2x}{\cos 2x}}dx \; = \sqrt{2}\int \frac{\sin x}{\sqrt{2\cos^2 x - 1}}$

put cos x = t $\Rightarrow$ – sin x dx = dt

$= -\sqrt{2}\int \frac{dt}{\sqrt{2t^2 - 1}} = -\ln|\sqrt{2}\cos x + \sqrt{\cos 2x}\,| + c \; = -\frac{1}{2}\ln|\cos 2x + \frac{1}{2} + \sqrt{\cos 2x}.\sqrt{1+\cos 2x}\,| + c$

$\because \beta = \frac{1}{2}, \alpha = -\frac{1}{2} \Rightarrow \beta - \alpha = 1$

87. The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _____ .

NTA Ans. (240)
Reso. Ans. (240)

Sol. $- - - - - - - \underline{1} = \frac{6!}{2!3!} = 60$

$- - - - - - - \underline{3} = \frac{6!}{3!} = 120$

$- - - - - - - \underline{5} = \frac{6!}{3!2!} = 60$

Total = 60 + 120 + 60 = 240

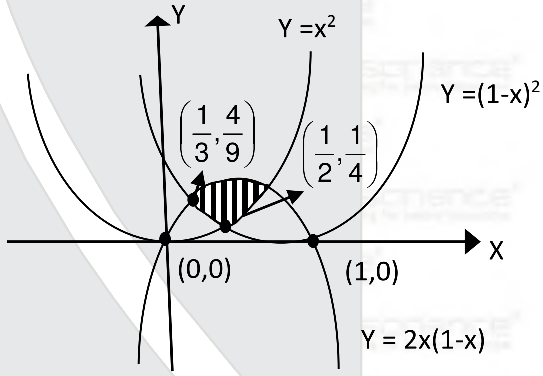
88. Let A = {1, 2, 3, 5, 8, 9}. Then the number of possible functions f : A → A such that f (m.n) = f(m). f(n) for every m, n ∈ A with m. n ∈ A is equal to _____ .

NTA Ans. (432)
Reso. Ans. (432)

Sol. f(1) = 1
f(9) = f(3) × f(3)
⇒ f(3) = 1 or 3
Total number of such functions = 1 × 6 × 2 × 6 × 6 × 1 = 432

89. Let A be the area of the region $\{(x,y) : y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$. Then 540 A is equal to ______ .

NTA Ans. (25)
Reso. Ans. (25)
Sol.



$$A = 2\int_{\frac{1}{3}}^{\frac{1}{2}} (2x - 2x^2 - (1-x)^2)\, dx$$

$$= 2\left[2x^2 - x^3 - x\right]_{1/3}^{1/2}$$

So $A = \frac{5}{108}$ $\Rightarrow 540\ A = \frac{5}{108} \times 540 = 25$

90. If the value of real number a > 0 for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to _____ .

NTA Ans. (13)
Reso. Ans. (13)

Sol. Let α be the common root

So $\alpha^2 - 5a\alpha + 1 = 0$

$\alpha^2 - a\alpha - 5 = 0$

$$\Rightarrow \frac{\alpha^2}{26a} = \frac{\alpha}{6} = \frac{1}{4a}$$

$$\Rightarrow \alpha^2 = \frac{13}{2} \Rightarrow \alpha = \sqrt{\frac{13}{2}} \Rightarrow a = \frac{3}{2\alpha} = \frac{3}{\sqrt{2\beta}}$$

$\Rightarrow \beta = 13$

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