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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Official Based Questions & Solutions

Date: 31 January, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

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SUBJECT: MATHEMATICS

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PART : MATHEMATICS

61. If the domain of the function $f(x)=\frac{[x]}{1+x^2}$, where [x] is greatest integer $\le$ x, is [2,6), then its range is

(1) $\left(\frac{5}{37},\frac{2}{5}\right]$ (2) $\left(\frac{5}{26},\frac{2}{5}\right]-\left\{\frac{9}{29},\frac{27}{109},\frac{18}{89},\frac{9}{53}\right\}$

(3) $\left(\frac{5}{26},\frac{2}{5}\right]$ (4) $\left(\frac{5}{37},\frac{2}{5}\right]-\left\{\frac{9}{29},\frac{27}{109},\frac{18}{89},\frac{9}{53}\right\}$

NTA Ans. (1)
Reso Ans. (1)

Sol. $$y=\frac{[x]}{1+x^2}=\begin{cases}\frac{2}{1+x^2} & ;\ 2\le x<3\\ \frac{3}{1+x^2} & ;\ 3\le x<4\\ \frac{4}{1+x^2} & ;\ 4\le x<5\\ \frac{5}{1+x^2} & ;\ 5\le x<6\end{cases}$$

Since, $y=\frac{[x]}{1+x^2}$ is decreasing.

for range, $f(2)=\frac{2}{5}=0.4$

$f(3)=\frac{3}{10}=0.3$

$f(4)=\frac{4}{17}=0.23$

$f(5)=\frac{5}{26}=0.19$

$f(6)=\frac{5}{37}=0.13$

So range of $y\in\left(\frac{5}{37},\frac{2}{5}\right]$

62. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(1) $\frac{2}{7}$ (2) $\frac{3}{7}$ (3) $\frac{5}{7}$ (4) $\frac{5}{6}$

NTA Ans. (3)
Reso Ans. (3)

Sol. Total possibility = 2B + 4 others
or
= 3B +3 others
or
4B + 2 others
or
5B + 1 others

or
6B + 0 others

Now

$$\text{required probability} = \frac{\frac{^5C_2}{^6C_2}+\frac{^6C_2}{^6C_2}}{\frac{^2C_2}{^6C_2}+\frac{^3C_2}{^6C_2}+\frac{^4C_2}{^6C_2}+\frac{^5C_2}{^6C_2}+\frac{^6C_2}{^6C_2}}$$

$$= \frac{10+15}{1+3+6+10+15} = \frac{25}{35} = \frac{5}{7}$$

63. If the sum and product of four positive consecutive terms of a G,.P. are 126 and 1296, respectively, then the sum of common ratios of all such GPs is

(1) $\frac{9}{2}$ (2) 3 (3) 7 (4) 14

NTA Ans. (3)
Reso Ans. (3)

Sol. G.P.$\Rightarrow a/r^3$, a/r , ar, ar^3

given : $a/r^3 . a/r$. ar. ar^3 = 1296 $\Rightarrow a^4$ = 1296 $\Rightarrow$ a = 6

and $a/r^3 + a/r$ + ar + ar^3 = 126 $\Rightarrow \frac{1}{r^3}+\frac{1}{r}$ + r + r^3 = 21

$(r^3 + \frac{1}{r^3}) + (r + \frac{1}{r}) = 21$

$\Rightarrow$ $(r + \frac{1}{r})^3 - 3r.\frac{1}{r}(r + \frac{1}{r}) + (r + \frac{1}{r}) = 21$

$t^3 - 2t - 21 = 0$

$(t - 3)(t^2 + 3t + 7) = 0$,

$t = 3$

$r + \frac{1}{r} = 3$

$r^2 - 3r + 1 = 0 \left\langle \begin{matrix} r_1 \\ r_2 \end{matrix} \right.$

$\Rightarrow r_1^2 + r_2^2 = 9 - 2 = 7$

64. Let $y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right)\right)\right)$. Then, at x = 1.

(1) $2y'+\sqrt{3}\pi^2y=0$ (2) $\sqrt{2}y'-3\pi^2y=0$ (3) $2y' + 3\pi^2y = 0$ (4) $y' + 3\pi^2y = 0$

NTA Ans. (3)
Reso Ans. (3)

Sol. $Y= f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{3/2}\right)\right)\right)$

$$\frac{dy}{dx} = 3\sin^2\left(\pi_3\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)\right)^{3/2}\right) x\ \frac{\pi}{3}\left(-\sin\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x+1)\right)\right)^{3/2}$$

$$\left(\cos\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}(-4x^3+5x^2+1)^{3/2}\right) x\ \frac{\pi}{3\sqrt{2}}(3/2\ (-4x^3+5x^2+1)^{1/2})\ (-12x^2+10x)\right.\right.$$

at x = 1

$$y = f(1) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}.2\sqrt{2}\right)\right)\right)$$

$$= \sin^3\left(\frac{\pi}{3}(-1/2)\right)$$

$$= \sin^3\left(-\frac{\pi}{6}\right) = \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$$

$$\left(\frac{dy}{dx}\right)_{x=1} = 3\left(-\frac{1}{2}\right)^2 . \left(-\frac{\pi}{3}\right)\left(\frac{\sqrt{3}}{2}\right).\left(\frac{\sqrt{3}}{2}\right). \frac{\pi}{3\sqrt{2}}\left(\frac{3}{2}\sqrt{2}\right)(-2)$$

$$= \frac{3}{4}.\frac{\pi}{3}.\frac{3}{4} = \frac{3\pi^2}{16}$$

Now (3) $\quad 2\ y^1 + 3\ \pi^2 y = 2\left(\frac{3\pi^2}{16}\right) + 3\ \pi^2\left(-\frac{1}{8}\right) = \frac{3\pi^2}{8} - \frac{3\pi^2}{8} = 0$

65. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is :

(1) 2 (2) 3 (3) 1 (4) 0

NTA Ans. (3)

Reso Ans. (3)

Sol. $\sqrt{(x-1)(x-3)} + \sqrt{(x-3)(x+3)} = \sqrt{2(2x-1)(x-3)}$

$x-3=0$ and $\sqrt{x-1}+\sqrt{x+3}=\sqrt{2(2x-1)}$ ....(1)

Squaring both side ; $x - 1 + x + 3 + 2\sqrt{(x-1)(x+3)} = 4x - 2$

$2\sqrt{(x-1)(x+3)} = 2x - 4$

$\sqrt{x^2+3x-x-3} = x - 2$........(2)

Squaring both side ; $x^2+2x-3 = x^2-4x+4$

$6x = 7 \Rightarrow x = \frac{7}{6}$

At $x = \frac{7}{6}$ equation (2) positive = negative

Rejected

$\therefore$ real root x = 3

66. (S1) $(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology
(S2) $((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.
Then
(1) both (S1) and (S2) are correct
(2) Only (S2) is correct
(3) Only (S1) is correct
(4) both (S1) and (S2) are wrong

NTA Ans. (1)
Reso Ans. (3)
Sol.

q	q	p⇒q	~q	p∧ ~q	(p⇒q) ∧ (p ∧ ~q)
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

~p	~q	~p⇒~q	~p∨q	((~p)⇒ (~q)) ∧(~p) ∨q)
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

67. Let the shortest distance between the lines $L : \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$ and L1 : x + 1 = y – 1 = 4 – z be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible?
(1) $\alpha + 2\gamma = 24$
(2) $2\alpha + \gamma = 7$
(3) $\alpha - 2\gamma = 19$
(4) $2\alpha - \gamma = 9$

NTA Ans. (1)
Reso Ans. (1)

Sol. $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -\hat{i} - \hat{j} - 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 6\hat{i} + (\lambda - 1)\hat{j} + (-\lambda - 4)\hat{k}$

$2\sqrt{6} = \left|\frac{-6-\lambda+1+2\lambda+8}{\sqrt{1+1+4}}\right|$

$|\lambda+3| = 12 \quad \Rightarrow \lambda = 9, -15$ but $\lambda \geq 0 \therefore \lambda = 9$

$\alpha = -2k + 5, \gamma = k - \lambda$ where $k \in R$

$\Rightarrow \alpha + 2\gamma = 5 - 2\lambda = -13.$

68. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a}+\vec{b}+\vec{c}| = |\vec{a}+\vec{b}-\vec{c}|$ and $\vec{b}\cdot\vec{c} = 0$.

Consider the following two statements :

(A) $|\vec{a}+\lambda\vec{c}| \geq |\vec{a}|$ for $\lambda \in R$

(B) $\vec{a}$ and $\vec{c}$ are always parallel

Then

(1) Both (A) and (B) are correct
(2) Only (A) is correct
(3) Only (B) is correct
(4) Neither (A) nor (B) is correct

NTA Ans. (2)

Reso Ans. (2)

Sol. $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}+\vec{b}-\vec{c}|^2$

$4\vec{b}.\vec{c} + 4\vec{a}.\vec{c} = 0 \Rightarrow \vec{a}.\vec{c} = 0$

Statement -1 $|\vec{a}+\lambda\vec{c}|^2 \geq |\vec{a}|^2$

$a^2 + \lambda^2c^2 + 2\vec{a}.\vec{c}\lambda \geq a^2$

$\lambda^2c^2 \geq 0$ which is true

Statement -1 is true

Statement -2 $\vec{a}$ & $\vec{c}$ are perpendicular. So, statement 2 is false

69. Let $\alpha \in (0,1)$ and $\beta = \log_e(1-\alpha)$. Let $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^n}{n}, x \in (0,1)$.

Then the integral $\int_0^\alpha \frac{t^{50}}{1-t}dt$ is equal to

(1) $P_{50}(\alpha) - \beta$
(2) $-(\beta + P_{50}(\alpha))$
(3) $\beta + P_{50}(\alpha)$
(4) $\beta - P_{50}(\alpha)$

NTA Ans. (2)

Reso Ans. (2)

Sol. $\int_0^\alpha t^{50}(1-t)^{-1}dt$

$= \int_0^\alpha t^{50}\left(1+t+t^2+.......\infty\right)dt = \left(\frac{t^{51}}{51}+\frac{t^{52}}{52}+......\right)_0^\alpha = \frac{\alpha^{51}}{51}+\frac{\alpha^{52}}{52}+......$

$-\ln(1-\alpha) - P_{50}(\alpha) = -(\beta + P_{50}(\alpha))$

70. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to :

(1) 2050 (2) 4094 (3) 6144 (4) 4097

NTA Ans. (4)
Reso Ans. (4)

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$

Now $(A + I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 + {}^{11}C_1 A^{10} + \ldots\ldots + {}^{11}C_{11} I$

$= A({}^{11}C_0 + {}^{11}C_1 + \ldots\ldots\ldots + {}^{11}C_{10}) + I$

$= A(2^{11} - 1) + I$

Trace of $(A + I)^{11}$

$= 2^{11} + 4(2^{11} - 1) + 1 + (-3)(2^{11} - 1) + 1$

$= 2 \times 2^{11} + 1$

$= 2^{12} + 1$

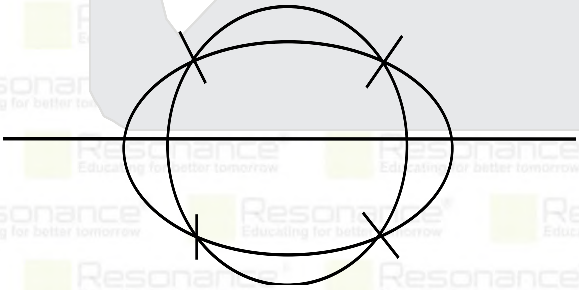
71. For all $z \in C$ on the curve $C_1 : |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then :

(1) The curve C_1 lies inside C_2
(2) the curves C_1 and C_2 intersect at 4 points
(3) the curve C_2 lies inside C_1
(4) the curves C_1 and C_2 intersect at 2 points

NTA Ans. (2)
Reso Ans. (2)

Sol. $|z| = 4$ is a circle $x^2 + y^2 = 16$ and $z + \frac{1}{z}$ is an ellipse $\frac{x^2}{\left(\frac{17}{4}\right)^2} + \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1$

$\therefore$ the curves C_1 and C_2 intersect at 4 points



72. If $\sin^{-1}\frac{\alpha}{17} + \cos^{-1}\frac{4}{5} - \tan^{-1}\frac{77}{36} = 0$, $0 < \alpha < 13$, then $\sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha)$ is equal to

(1) $16 - 5\pi$ (2) 16 (3) 0 (4) π

NTA Ans. (4)
Reso Ans. (4)

Sol. $\sin^{-1}\frac{\alpha}{17} = \tan^{-1}\frac{77}{36} - \cos^{-1}\frac{4}{5}$

$\sin^{-1}\frac{\alpha}{17} = \tan^{-1}\frac{77}{36} - \sin^{-1}\frac{3}{5}$

$\frac{\alpha}{17} = \sin\left(\tan^{-1}\frac{77}{36}\right)\cos\sin^{-1}\left(\frac{3}{5}\right) - \cos\tan^{-1}\left(\frac{77}{36}\right)\sin\sin^{-1}\left(\frac{3}{5}\right)$

$\frac{\alpha}{17} = \frac{77}{85}\times\frac{4}{5} - \frac{36}{85}\cdot\frac{3}{5}$

$= \frac{308-108}{425} = \frac{200}{425} = \frac{8}{17}$

$\alpha = 8$

Now $\sin^{-1}\sin\alpha + \cos^{-1}\cos\alpha$

$= \sin^{-1}\sin 8 + \cos^{-1}\cos 8$

$- 8 + 3\pi + 8 - 2\pi$

$= \pi$

73. For the system of linear equations

$x + y + z = 6$

$\alpha x + \beta y + 7z = 3$

$x + 2y + 3z = 14,$

which of the following is NOT true?

(1) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution

(2) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

(3) For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

(4) If $\alpha = \beta = 7$, then the system has no solution

NTA Ans. (3)

Reso Ans. (3)

Sol. $x + y + z = 6$

$\alpha x + \beta y + 7z = 3$

$x + 2y + 3z = 14$

$\Rightarrow \quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & 7 \\ 1 & 2 & 3 \end{vmatrix} = 2\beta - \alpha - 7$

Clearly if $\alpha = \beta$ and $\alpha \neq 7$ then $|A| \neq 0$ so system has a unique solution $\Rightarrow$ (1) is true

and $\alpha = \beta = 7 \Rightarrow |A| = 0$ and system of equations is

$x + y + z = 6$ ... (1)

$x + y + z = \frac{3}{7}$ ... (2)

$x + 2y + 3z = 14$

equation (1) and (2) contract each other so no solution $\Rightarrow$ (4) is true

and for every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, $|A| \neq 0$ $\therefore$ (3) is false

74. Let R be a relation on N × N defined by (a, b) R (c, d) if and only if ad(b – c) = bc(a – d). Then R is

(1) transitive but neither reflexive nor symmetric
(2) symmetric but neither reflexive nor transitive
(3) symmetric and transitive but not reflexive
(4) reflexive and symmetric but not transitive

NTA Ans. (2)
Reso Ans. (2)

Sol. (a, b) R (c, d) $\Leftrightarrow \frac{b-c}{bc} = \frac{a-d}{ad}$

$\frac{1}{c} - \frac{1}{b} = \frac{1}{d} - \frac{1}{a}$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{d} - \frac{1}{c}$

Reflexive : (a, b) R (a, b) $\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}$ false $\therefore$ not Reflexive

Symmetric : (a, b) R (c, d) $\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{d} - \frac{1}{c} \Rightarrow \frac{1}{c} - \frac{1}{d} = \frac{1}{b} - \frac{1}{a} \Rightarrow$ (c, d) R (a, b) $\therefore$ Symmetric

Transitive : (a, b) R (c, d) $\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{d} - \frac{1}{c}$

(c, d) R (e, f) $\Rightarrow \frac{1}{c} - \frac{1}{d} = \frac{1}{f} - \frac{1}{e}$

$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{e} - \frac{1}{f} \neq$ (a, b) R (e, f) $\therefore$ not Transitive

75. The value of $\int_{\pi/3}^{\pi/2} \frac{(2+3\sin x)}{\sin x(1+\cos x)} dx$ is equal to

(1) $\frac{7}{2} - \sqrt{3} - \log_e \sqrt{3}$
(2) $\frac{10}{3} - \sqrt{3} - \log_e \sqrt{3}$
(3) $\frac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$
(4) $-2 + 3\sqrt{3} + \log_e \sqrt{3}$

NTA Ans. (3)
Reso Ans. (3)

Sol. $= \int_{\pi/3}^{\pi/2} \frac{2(1-\cos x)}{\sin x(1-\cos^2 x)} dx + \int_{\pi/3}^{\pi/2} \frac{3}{1+\cos x} dx$

$= \int_{\pi/3}^{\pi/2} \frac{2}{\sin^3 x} dx - 2\int_{\pi/3}^{\pi/2} \cot x.\operatorname{cosec}^2 x dx + \frac{3}{2}\int_{\pi/3}^{\pi/2} \sec^2 \frac{x}{2} dx$

$$= 2\int_{\pi/3}^{\pi/2} \sqrt{1+\cot^2 x}.\text{cosec}^2 x dx - 2\int_{\pi/3}^{\pi/2} \cot x.\text{cosec}^2 x ds + 3\left[\tan\frac{x}{2}\right]_{\pi/3}^{\pi/2}$$

Let cot x = t

$$= -2\int_{1/\sqrt{3}}^{0} \sqrt{1+t^2}dt + 2\int_{1/\sqrt{3}}^{0} tdt + 3\left[1-\frac{1}{\sqrt{3}}\right]$$

$$= 2\left[\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\log\left(t+\sqrt{1+t^2}\right)\right]_0^{1/\sqrt{3}} - \left[t^2\right]_0^{1/\sqrt{3}} + 3 - \sqrt{3}$$

$$= \frac{2}{3} + \log\sqrt{3} - \frac{1}{3} + 3 - \sqrt{3} = \frac{10}{3} - \sqrt{3} + \frac{1}{2}\log 3$$

76. A wire of length 20 m is to be cut into two pieces. A piece of length ℓ_1 is bent to make a square of area A_1 and the other piece of length ℓ_2 is made into a circle of area 42. If $2A_1 + 3A_2$ is minimum then $(\pi\ell_1) : \ell_2$ is equal to :

(1) 6:1 (2) 3:1 (3) 4:1 (4) 1:6

NTA Ans. (1)

Reso Ans. (1)

Sol. $\ell = 4x + 2\pi r$, where x = side of square and r = radius of circle

now let $2a_1 + 3a_2 = A$

$A = 2x^2 + 3\pi r^2$

$$= 2x^2 + 3\pi\left(\frac{\ell-4x}{2\pi}\right)^2 = 2x^2 + \frac{3}{4\pi}(\ell-4x)^2$$

$$\frac{dA}{dx} = 4x - \frac{2\times3\times4}{4\pi}(\ell-4x)$$

for maxima minima

$$\frac{dA}{dx} = 0$$

$$\therefore x = \frac{2\times3}{4\pi}(\ell-4\pi)$$

$4\pi x = 6\ell - 24x$

$$x = \frac{6\ell}{4\pi+24}$$

now $4x = \ell_1 = \dfrac{6\ell}{\pi+6}$ also $\ell_2 = 2\pi r$

$$= \ell - 4x = \ell - \frac{6\ell}{\pi+6} = \frac{\pi\ell + 6\ell - 6\ell}{\pi+6} = \frac{\pi\ell}{\pi+6}$$

$$\text{now } \frac{\pi\ell_1}{\ell_2} = \frac{\frac{\pi(6\ell)}{\pi+6}}{\frac{\pi\ell}{\pi+6}} = 6$$

77. If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, b<2$, from the origin is 1, then the eccentricity of the ellipse is :

(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

NTA Ans. (2)

Reso Ans. (2)

Sol. Normal at (2cos θ, b sin θ) is

$2x \sec\theta - by \operatorname{cosec}\theta = 4 - b^2$

Now, $1 = \left(\frac{4-b^2}{\sqrt{4\sec^2\theta + b^2\operatorname{cosec}^2\theta}}\right)_{max}$.

$1 = \frac{4-b^2}{2+b}$

$2 + b = 4 - b^2$

$b^2 + b - 2 = 0$

$b = -2$ or 1

so $b = 1$

Now $e = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

78. Let a differentiable function f satisfy $f(x)+\int_3^x \frac{f(t)}{t}dt = \sqrt{x+1}, x \geq 3$. Then 12 f (8) is equal to :

(1) 19 (2) 17 (3) 1 (4) 34

NTA Ans. (2)

Reso Ans. (2)

Sol. $f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$

$\frac{dy}{dx}+\frac{y}{x} = \frac{1}{2\sqrt{x+1}}$

$I.F = e^{\int \frac{1}{x}dx} = x$

$y.x = \int x\frac{1}{2\sqrt{x+1}}dx + c$ put $x + 1 = t^2$, $dx = 2tdt$

$y.x = \frac{1}{2}\int\frac{(t^2-1)2tdt}{t} + c$

$y.x = \frac{1}{2}\left(\frac{2}{3}t^3 - 2t\right) + c$

$y.x = \frac{\left(\sqrt{x+1}\right)^3}{3} - \sqrt{x+1} + c$

now f(3) = 2

$6 = \frac{(2)^3}{3} - 2 + c$

$8 - \frac{8}{3} = c$

$\frac{16}{3} = c$

$\therefore f(x) = \frac{1}{x}\left(\frac{(\sqrt{x+1})^3}{3} - (\sqrt{x+1}) + \frac{16}{3}\right)$

$\therefore 12f(8) = \frac{12}{8}\left(\frac{27}{3} - 3 + \frac{16}{3}\right)$

$= \frac{3}{2}\left(6 + \frac{16}{3}\right) = 17$

79. Let y = f (x) represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$. Then $S = \left\{x \in R : \tan^{-1}\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}\right\}$:

(1) is an empty set
(2) contains exactly one element
(3) contains exactly two elements
(4) is an infinite set

NTA Ans. (3)
Reso Ans. (3)

Sol. Equation of parabola $\sqrt{\left(x+\frac{1}{2}\right)^2 + y^2} = \left|y + \frac{1}{2}\right|$

$x^2 + \frac{1}{4} + x + y^2 = y^2 + \frac{1}{4} + y$

$x^2 + x = y = f(x)$

Now $\tan^{-1}\sqrt{f(x)} + \sin^{-1}\sqrt{f(x)+1} = \frac{\pi}{2}$

$\cos^{-1}\frac{1}{\sqrt{1+f(x)}} + \sin^{-1}\sqrt{f(x)+1} = \frac{\pi}{2}$

So $\sqrt{f(x)+1} = \frac{1}{\sqrt{1+f(x)}}$

$f(x) + 1 = 1$

$f(x) = 0 \Rightarrow x = 0, -1$

80. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point (3, 2). Let C_2 be the image of C_1 in T. Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is:

(1) $2(2+\sqrt{2})$
(2) $2(1+\sqrt{2})$
(3) $4(1+\sqrt{2})$
(4) $3+2\sqrt{2}$

NTA Ans. (3)
Reso Ans. (3)

Sol. Given circle $x^2 + y^2 - 4x - 6y + 11 = 0$

tangent at the point (3,2) is

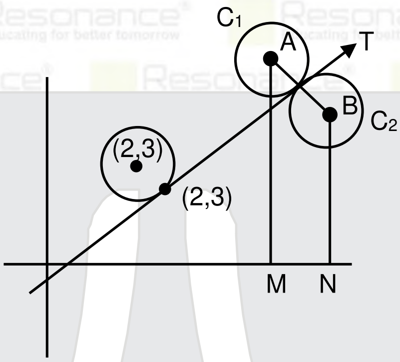
$3x + 2y - 2(x + 3) - 3(y + 2) + 11 = 0$

$\Rightarrow x - y = 1$

On rolling the given circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 unit an the tangent T : x – y – 1 = 0 centre of the circle also moving upward 4 unit on T) Hence centre of new circle C_1 is $\left(2+2\sqrt{2},3+2\sqrt{2}\right)$ and radius remains same $\sqrt{2}$

$\Rightarrow$ $C_1 : (x - 2 - 2\sqrt{2})^2 + (y - 3 - 2\sqrt{2})^2 = 2$

C_2 is imaged C_1 in T $\Rightarrow$ centre of C_2 is $\left(4+2\sqrt{2},1+2\sqrt{2}\right)$



$\Rightarrow$ $A\left(2+2\sqrt{2},3+2\sqrt{2}\right)$

$B\left(4+2\sqrt{2},1+2\sqrt{2}\right)$

Feet of perpendicular of from A and B are

$M\left(2+2\sqrt{2},0\right)$

$N\left(4+2\sqrt{2},0\right)$

$\Rightarrow$ Area of trapezium AMNB

$$= \frac{1}{2}\left(3+2\sqrt{2}+1+2\sqrt{2}\right)(2) = \left(4+4\sqrt{2}\right)$$

81. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}$, $\beta \in IN$. Then α is equal to ______

NTA Ans. (2)

Reso Ans. (2)

Sol. $\alpha > 0$, $\left(x^{2/3} + \frac{2}{x^3}\right)^{30}$

$T_{r+1} = {}^{30}C_r\,(x^{2/3})^{30-r}\left(\frac{2}{x^3}\right)^r$

$= {}^{30}C_r\, x^{20-\frac{2r}{3}-3r}\,(2)^r$

$= {}^{30}C_r(2)^r\; x^{\frac{60-11r}{3}}$ $\quad 0 \le r \le 30$

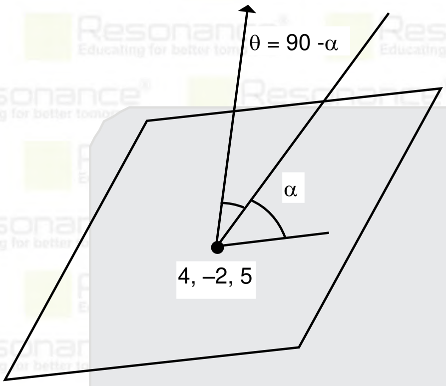
For $\beta x^{-\alpha}$, $\alpha > 0$ the smallest value of α is 2(for r = 6)

$T_7 = {}^{30}C_6\,(2)^6 x^{-2}$

$\alpha = 2$

82. Let θ be the angle between the planes $P_1 : \vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k}) = 9$ and $P_2 : \vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k}) = 15$. Let L be the line that meets P_2 at the point (4, –2, 5) and makes and angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to ______.

NTA Ans. (9)
Reso Ans. (9)
Sol.



$$\cos\theta = \frac{(\hat{i}+\hat{j}+2\hat{k}).(2\hat{i}-\hat{j}+\hat{k})}{6} = \frac{2-1+2}{6} = \frac{1}{2}$$

$\theta = \pi/3$ $\alpha = \pi/6$

$\therefore (\tan^2\theta)(\cot^2\alpha) = (3)(3) = 9$

83. The remainder on dividing 5^{99} by 11 is ________

NTA Ans. (9)
Reso Ans. (9)
Sol. $5^{99} = 625 \times 5^{95}$

$= 625 \times (3124 + 1)^{19}$

$= 625 \times (11\lambda + 1)^{19}$

Remainder = 625 or 9

84. Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$$12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \ldots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$$ is equal to _____.

NTA Ans. (8)
Reso Ans. (8)
Sol. Given a + 4d = 2(a + 6d)

a + 8d = 0 ....(1)

and $a_{11} = 18 \Rightarrow a + 10d = 18$ ....(2)

From (1) and (2)

$-8d + 10d = 18 \Rightarrow d = 9$ and $a = -72$

$a_{10} = a + 9d = -72 + 81 = 9$

$a_{18} = a + 17d = -72 + 153 = 81$

Now

$$12\left[\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}}+\frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}}+......+\frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right]$$

$$12\left[\frac{\sqrt{a_{10}}-\sqrt{a_{11}}}{-d}+\frac{\sqrt{a_{11}}-\sqrt{a_{12}}}{-d}+......+\frac{\sqrt{a_{17}}-\sqrt{a_{18}}}{-d}\right]$$

$$12\left[\frac{\sqrt{a_{10}}-\sqrt{a_{18}}}{-d}\right]=12\left[\frac{3-9}{-9}\right]=12\times\frac{6}{9}=8$$

85. If the variance of the frequency distribution.

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3, then α is equal to ______.

NTA Ans. (5)
Reso Ans. (5)
Sol.

x_i	f_i	$d_i = x_i - 5$	$f_i d_i^2$	$f_i d_i$
2	3	–3	27	-9
3	6	–2	24	-12
4	16	–1	16	–16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma_\alpha^2=\sigma_d^2=\frac{\sum f_i d_i^2}{\sum f_i}-\left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$$

$$=\frac{150}{45+\alpha}-0=3$$

$\Rightarrow 150 = 135 + 3\alpha$

$\Rightarrow 3\alpha = 15 \Rightarrow \alpha = 5$

86. Let $\vec{a}$ and $\vec{b}$ be two vector such that $|\vec{a}|=\sqrt{14}, |\vec{b}|=\sqrt{6}$ and $|\vec{a}\times\vec{b}|=\sqrt{48}$. Then $(\vec{a}\cdot\vec{b})^2$ is equal to ______.

NTA Ans. (36)
Reso Ans. (36)

Sol. $(\vec{a}.\vec{b})^2+(\vec{a}\times\vec{b})^2=|a|^2|b|^2$

$(\vec{a}.\vec{b})^2+48=14\times 6$

$(\vec{a}.\vec{b})^2=84-48=36$

87. Let the line $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane 2x + y + 3z = 16 at the point P. Let the point Q be the foot of perpendicular from the point R(1,–1,–3) on the line L. If α is the area of triangle PQR, then α^2 is equal to

NTA Ans. (180)

Reso Ans. (180)

Sol. Any point on L $((2\lambda + 1), (-\lambda - 1), (\lambda + 3))$ lies on plane

$\therefore 2(2\lambda + 1) + (-\lambda + 1) + 3(\lambda + 3)(\lambda + 3) = 16$

$6\lambda + 10 = 16 \Rightarrow \lambda = 1$

$\therefore P = (3, -2, 4)$

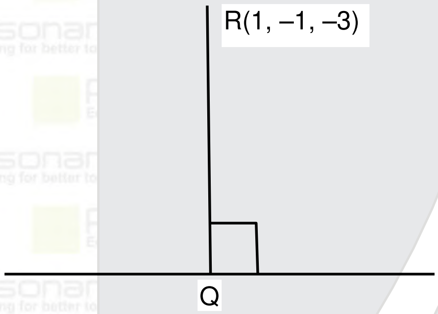
DR of QR $(2\lambda, -\lambda, \lambda + 6)$

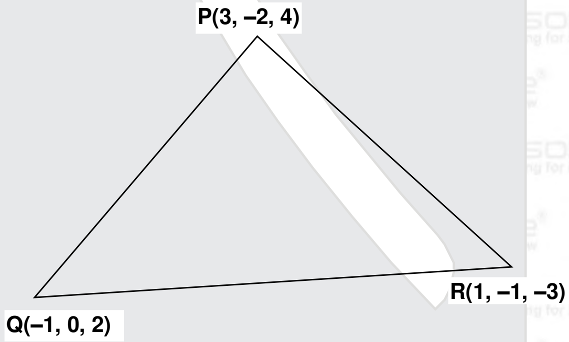
DR of L = (2, – 1, 1}

$\because QR \perp L \therefore 4\lambda + \lambda + \lambda + 6 = 0$

$6\lambda + 6 = 0 \Rightarrow \lambda = -1$

Q = (–1, 0, 2)





$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 5\hat{k}$

$\overrightarrow{QP} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$\overrightarrow{QR} \times \overrightarrow{QP} = -12\hat{i} - 24\hat{j}$

Area $\alpha = \frac{1}{2}\left|\overrightarrow{QR} \times \overrightarrow{QP}\right|$

$\alpha = \frac{1}{2}\sqrt{144 + 576}$

$\alpha^2 = 180$

88. Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is_________

NTA Ans. (2997)

Reso Ans. (2997)

Sol. Number starting with 2 &3 = 2×6^4

Number starting with 40 = 6^3

Number starting with 420, 422, 423, 424, 427 = 5×6^2

Similarly starting with 4290 = 6

Total = $2 \times 6^4 + 6^3 + 5 \times 6^2 + 6 + 3 = 2997$

89. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to__________

NTA Ans. (710)

Reso Ans. (710)

Sol. Number divisible by 3: 1002,1005.......... 2799

$T_n = 1002 + (n - 1)\, 3 = 2799$

$n(3) = 600$

Number Divisible by 11: $n(11) = 164$

Number Divisible by 33: $n(33) = 54$

Total number = $n(3) + n(11) - n(33) = 600 + 164 - 54 = 710$

90. Let for $x \in R$,

$$f(x) = \frac{x + |x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

Then area bounded by the curve y = (fog) (x) and the line y = 0, 2y – x = 15 is equal to ______.

NTA Ans. (72)

Reso Ans. (72)

Sol. $f(x) = \frac{x + |x|}{2} = \begin{bmatrix} x & x \geq 0 \\ 0 & x < 0 \end{bmatrix}$ s

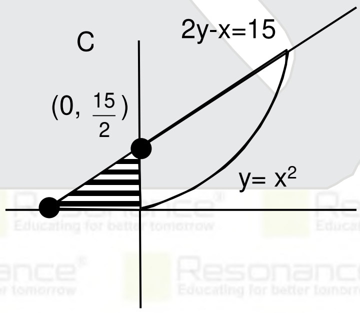
$$g(x) = \begin{bmatrix} x^2 & x \geq 0 \\ x & x < 0 \end{bmatrix}$$

$$fog(x) = f[g(x)] = \begin{bmatrix} g(x) & g(x) \geq 0 \\ 0 & g(x) < 0 \end{bmatrix}$$

curve $fog(x) = \begin{bmatrix} x^2 & x \geq 0 \\ 0 & x < 0 \end{bmatrix}$

line $2y - x = 15$

bounded area $A = \int_0^1 \left(\frac{x + 15}{2} - x^2 \right) dx + \frac{1}{2} \times \frac{15}{2} \times 15$



$$\left. \frac{x^2}{4} + \frac{15x}{2} - \frac{x^3}{3} \right|_{10}^{8} + \frac{225}{4}$$

$$\frac{9}{4} + \frac{45}{2} - 9 + \frac{225}{4} = \frac{99 - 36 + 225}{4}$$

$$= \frac{288}{4} = 72$$

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