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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 31 January, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

Resonance Eduventures Ltd.

Reg. Office & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.) - 324005

Ph. No.: +91-744-2777777, 2777700 | **FAX No. :** +91-022-39167222

To Know more : sms **RESO** at **56677** | **Website :** www.resonance.ac.in | **E-mail :** contact@resonance.ac.in | **CIN :** U80302RJ2007PLC024029

Toll Free : 1800 258 5555 7340010333 facebook.com/ResonanceEdu twitter.com/ResonanceEdu www.youtube.com/resowatch blog.resonance.ac.in

PART : MATHEMATICS

61. If $\phi(x) = \frac{1}{\sqrt{x}}\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t - 3\phi'(t)\right)dt$, x > 0, then $\phi'\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{4}{6+\sqrt{\pi}}$ (2) $\frac{8}{6+\sqrt{\pi}}$ (3) $\frac{8}{\sqrt{\pi}}$ (4) $\frac{4}{6-\sqrt{\pi}}$

NTA. (2)
Reso. (2)

Sol. $\phi(x) = \frac{1}{\sqrt{x}}\int_{\pi/4}^{x}\left(4\sqrt{2}\sin t - 3\phi'(t)\right)dt$ ___(1)

$\sqrt{x}\,\phi(x) = \int_{\pi/4}^{x}\left(4\sqrt{2}\sin t - 3\phi'(t)\right)dt$

Differentiate both side w.r.t. x

$\frac{1}{2\sqrt{x}}\phi(x) + \sqrt{x}\phi'(x) = (4\sqrt{2}\sin x - 3\phi'(x))(1) - 0$ ___(2)

Put x = π/4 ⇒ φ(π/4) = 0 (from equation 1)

Now when x = π/4 ⇒ $\sqrt{\frac{\pi}{4}}\phi'(\frac{\pi}{4}) = 4\sqrt{2}.\frac{1}{\sqrt{2}} - 3\phi'(\frac{\pi}{4})$ (from equation 2)

$\Rightarrow \phi'(\frac{\pi}{4})\left(3+\sqrt{\frac{\pi}{4}}\right) = 4$

$\Rightarrow \phi'(\frac{\pi}{4}) = \frac{8}{6+\sqrt{\pi}}$

62. Let y = y(x) be the solution of the differential equation $(3y^2 - 5x^2)ydx + 2x(x^2 - y^2)dy = 0$ such that y(1) = 1. Then $|(y(2))^3 - 12y(2)|$ is equal to :

(1) 64 (2) $32\sqrt{2}$ (3) 32 (4) $16\sqrt{2}$

NTA. (2)
Reso. (2)
Sol. $(3y^2 - 5x^2)\, ydx + 2x\,(x^2-y^2)\, dy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y\left(5x^2-3y^2\right)}{2x\left(x^2-y^2\right)}$

This is homogeneous

Let y = tx ⇒ $\frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + x\frac{dt}{dx} = \frac{(tx)(5x^2-3t^2x^2)}{2x\left(x^2-t^2x^2\right)} = \frac{t\left(5-3t^2\right)}{2\left(1-t^2\right)}$

$x\frac{dt}{dx} = \frac{5t-3t^3}{2-2t^2} - t = \frac{3t-t^3}{2-2t^2}$

$\int\frac{2(1-t^2)}{3t-t^3}dt = \int\frac{1}{x}dx$

Let $3t - t^3 = z \Rightarrow = 3(1-t^2)\,dt = dz$

$(1-t^2)\,dt = \frac{1}{3}\,dz$

$\frac{2}{3}\int \frac{dz}{z} = \int \frac{1}{x}dx$

$\frac{2}{3}\,\ell n(z) = \ln x + \ln c$

$z^{2/3} = cx \Rightarrow z = c^{3/2}\,x^{3/2}$

$\Rightarrow 3t - t^3 = c^{3/2}\,x^{3/2}$

Now $t = y/x \Rightarrow \frac{3y}{x} - \frac{y^3}{x^3} = c^{3/2}x^{3/2}$

When x=1, y = 1 $\Rightarrow c^{3/2} = 2$

$3\frac{y}{x} - \frac{y^3}{x^3} = 2\,x^{3/2}$

$3x^2y - y^3 = 2x^{9/2}$

Put x = 2 $12y(2) - y(2)^3 = 2(2)^{9/2} = 32\sqrt{2}$

Ans. (2)

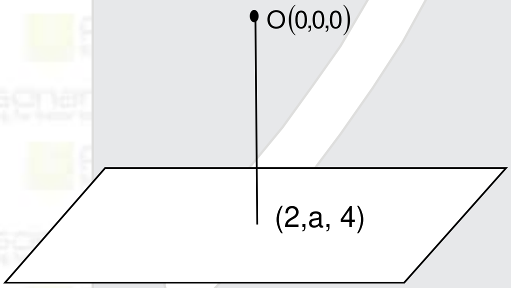
63. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the point A, B, C is (2, a, 4),a $\in$ N. If the volume of the tetrahedron OABC is 144 unit^3, then which of the following points is NOT on P ?

(1) (0,4,4) (2) (3,0,4) (3) (0,6,3) (4) (2,2,4)

NTA. (2)

Reso. (2)

Sol.



Equation of P : $2(x-2) + a(y-a) + 4(z-4) = 0$

$\Rightarrow 2x + ay + 4z = 20 + a^2$

$\Rightarrow \frac{x}{\frac{20+a^2}{2}} + \frac{y}{\frac{20+a^2}{a}} + \frac{z}{\frac{20+a^2}{4}} = 1$

Volume of tetrahedron = $\frac{1}{6}\left(\frac{20+a^2}{2}\right)\left(\frac{20+a^2}{a}\right)\left(\frac{20+a^2}{4}\right) = 144$

$\Rightarrow \left(\frac{20+a^2}{12}\right)^3 = 4a$

Let $a = 2k^3$ $\Rightarrow \left(\frac{20+4k^6}{12}\right)^3 = 8k^3$

$\Rightarrow \left(\frac{20+4k^6}{12}\right) = 2k$

$\Rightarrow k^6 + 5 = 6k$

$\Rightarrow k = 1 \Rightarrow a = 2$

Equation of plane 2x + 2y + 4z = 24

x + y + 2z = 12

all three point (0, 4, 4), (0, 6, 3), (2, 2, 4) satisfy equation of plane but (3, 0, 4) does not satisfy plance

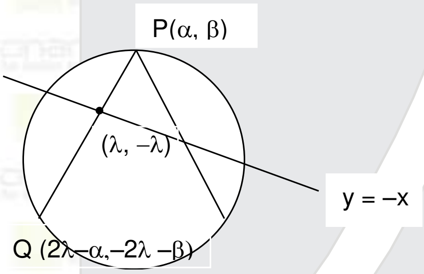
64. The set of all values of a^2 for which the line x + y = 0 bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1 + a)x - (1 - a)y = 0$, is equal to :

(1) $(8, \infty)$ (2) $(0, 4]$ (3) $(4, \infty)$ (4) $(2,12]$

NTA. (1)

Reso. (1)

Sol.



$x^2 + y^2 - \left(\frac{1+a}{2}\right)x - \left(\frac{1-a}{2}\right)y = 0$

Let $\frac{1+a}{2} = \alpha$, $\frac{1-a}{2} = \beta$

circle will be $x^2 + y^2 - \alpha x - \beta y = 0$

Point Q will lie on circle

$(2\lambda - \alpha)^2 + (-2\lambda - \beta)^2 - \alpha(2\lambda - \alpha) - \beta(-2\lambda - \beta) = 0$

$\Rightarrow 4\lambda^2 + 3\lambda(\beta - \alpha) + (\alpha^2 + \beta^2) = 0$

Now D > 0 $\Rightarrow 9(\beta - \alpha)^2 - 4(4)(\alpha^2 + \beta^2) > 0$

$\Rightarrow 9a^2 - 16\left(\frac{1+a^2}{2}\right) > 0$

$\Rightarrow a^2 > 8$

65. Let $\alpha > 0$. If $\int_0^\alpha \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$, then α is equal to :

(1) 2 (2) $2\sqrt{2}$ (3) 4 (4) $\sqrt{2}$

NTA. **(1)**
Reso. **(1)**

Sol. $\int_0^\alpha \frac{x\left(\sqrt{\alpha+x}+\sqrt{x}\right)}{\alpha}dx = \frac{1}{\alpha}\left[\int_0^\alpha x\sqrt{\alpha+x}\,dx + \int_0^\alpha x^{3/2}dx\right]$

Put $\alpha + x = t^2 \Rightarrow dx = 2tdt$

$$\frac{1}{\alpha}\int_{\sqrt{\alpha}}^{\sqrt{2\alpha}}\left(t^2-\alpha\right)2t^2dt + \frac{2\alpha^{3/2}}{5}$$

$$\frac{1}{\alpha}\left[\frac{2t^5}{5}-\frac{2\alpha t^3}{3}\right]_{\sqrt{\alpha}}^{\sqrt{2\alpha}} + \frac{2\alpha^{3/2}}{5}$$

$$\frac{4\alpha\sqrt{2\alpha}+10\alpha\sqrt{\alpha}}{15} = \frac{16+20\sqrt{2}}{15} \Rightarrow \alpha = 2$$

66. Let $a_1, a_2, a_3, \ldots\ldots.$ be an A.P. If $a_7 = 3$, the product (a_1a_4) is minimum and the sum of its first n terms is zero then $n! - 4a_{n(n+2)}$ is equal to

(1) $\frac{381}{4}$ (2) 9 (3) $\frac{33}{4}$ (4) 24

NTA. **(4)**
Reso. **(4)**

Sol. $a_7 = 3 = a + 6d$

$\Rightarrow a = 3 - 6d$

$a_1a_4 = a(a + 3d)$

$= (3 - 6d)(3 - 3d)$

$= 18d^2 - 27d + 9$

$d = \frac{27}{2\times 18} = \frac{3}{4}$

$a = 3 - \frac{9}{2} = \frac{-3}{2}$

$S_n = \frac{n}{2}[2a + (n - 1)d] = 0$

$-3 + (n-1)\frac{3}{4} = 0$

$\Rightarrow n = 5$

Now $n! - 4a_{n(n+2)} = 5! - 4a_{35}$

$= 120 - 4(a+34d)$

$= 120 - 4\left(\frac{-3}{2} + 34\times\frac{3}{4}\right)$

$= 120 + 6 - 102 = 24$

67. $\lim\limits_{x\to\infty} \dfrac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$

(1) is equal to $\frac{27}{2}$ (2) is equal to 9

(3) does not exist (4) is equal to 27

NTA. **(4)**

Reso. **(4)**

Sol. $\lim\limits_{x\to\infty} x^3 \dfrac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{(x+\sqrt{x^2-1})^6+(x-\sqrt{x^2-1})^6}$

$(a + b)^6 + (a–b)^6 = 2\ (T_1 + T_3 + T_5 + T_7)$

$= 2\ ({}^6C_0\, a^6 + {}^6C_2\, a^4b^2 + {}^6c_4\, a^2\, b^4 + {}^6C_6\, b^6)$

$\lim\limits_{x\to\infty} \dfrac{(2(3x+1)^3+30(3x+1)^2(3x-1)+30(3x+1)(3x-1)^2+2(3x-1)^3)x^3}{2x^6+30x^4(x^2-1)+30x^2(x^2-1)^2+(x^2-1)^3}$

$\lim\limits_{x\to\infty} \dfrac{x^6[2(3+1/x)^3+30(3+1/x)^2(3-1/x)+30(3+1/x)(3-1/x)^2+2(3-1/x)^3}{x^6[2+30(1-1/x^2+30(1-1/x^2)^2+2(1-1/x^2)^3]}$

$= \dfrac{2(3)^3+30(3)^3+30(3)^3+2(3)^3}{2+30+30+2} = (3)^3 = 27$

68. Let H be the hyperbola , whose foci are $\left(1\pm\sqrt{2},0\right)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is :

(1) 3 (2) $\frac{5}{2}$ (3) 2 (4) $\frac{3}{2}$

NTA. **(3)**

Reso. **(3)**

Sol. $S(1 + \sqrt{2}, 0)\ S'(1 - \sqrt{2}, 0)$

$SS' = 2ae = 2\sqrt{2} \Rightarrow ae = \sqrt{2} \quad \Rightarrow a\sqrt{2} = \sqrt{2} \Rightarrow a = 1$

$L.R. = \dfrac{2b^2}{a} = \dfrac{2a^2(e^2-1)}{a} = 2a(e^2 - 1) = 2$

69. The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which $((p\wedge q)\Rightarrow(r\vee q)\wedge((p\wedge r)\Rightarrow q)$ is a tautology, is :

(1) 1 (2) 2 (3) 4 (4) 3

NTA. **(2)**

Reso. **(2)**

Sol. $p \wedge q \to (r \vee q) \wedge ((p \wedge r) \to q))$

we know $p \to q \equiv \sim p \vee q$

Hence $(p \wedge r) \to q \equiv \sim(p \wedge r) \vee q$

$\equiv (\sim p \vee \sim r) \vee q$

$\Rightarrow p \wedge q \to (r \vee q) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow \sim(p \wedge q) \vee (r \vee q) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow (\sim p \vee \sim q) \vee (r \vee q) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow (\sim p \vee (\sim q \vee r \vee q) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow (\sim p \vee t) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow \sim p \vee \sim r \vee q$

When $r = \sim p$ or $r = q \quad \Rightarrow \sim p \vee \sim r \vee q = t$

70. Among the relations

$S = \{(a,b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\}$ and $T = \{(a,b) : a, b \in R, a^2 - b^2 \in Z\}$,

(1) S is transitive but T is not
(2) both S and T are symmetric
(3) neither S not T is transitive
(4) T is symmetric but S is not

NTA. **(4)**
Reso. **(4)**
Sol. $a \; T \; b \Rightarrow a^2 - b^2 \in I$ always true Hence equivalence

$a \; S \; b \Rightarrow 2 + \frac{a}{b} > 0$ not symmetric when $a = 2, b = -6$

When $a = -8$, $b = 6$, $c = 3$

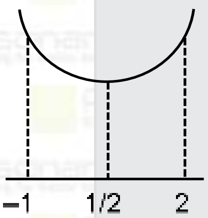
Clearly $(-8, 6) \in S$, $(6, 3) \in S$ but $(-8, 3) \notin S$

Hence not transitive.

71. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where [t] denotes the greatest integer function, in the interval [–1, 2], is

(1) $\frac{3}{2}$
(2) $\frac{1}{4}$
(3) $\frac{5}{4}$
(4) $\frac{3}{4}$

NTA. **(4)**
Reso. **(4)**
Sol. $f(x) = [x^2 - x + 1] + |x^2 - x + 1| \quad x \in [-1, 2]$
Let $t = x^2 - x + 1 \quad x \in [-1, 2]$



$t \in [(1/2)^2 - 1/2 + 1, (2)^2 - 2 + 1]$
$t \in [3/4, 3]$

$y = f(x) = [t] + |t| \; t \in [3/4, 3]$

$$y = \begin{cases} t & \frac{3}{4} \le t < 1 \\ t+1 & 1 \le t < 2 \\ t+2 & 2 \le t < 3 \\ 6 & t = 3 \end{cases}$$

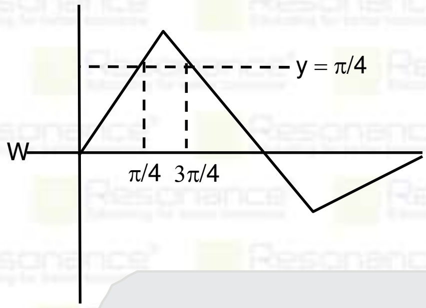
$f(x)_{min} = 3/4$ Ans. (4)

72. Let $(a, b) \subset (0,2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0$, $\theta \in (0,2\pi)$, holds . If $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$ and $\alpha - \beta = b - a$, then α is equal to;

(1) $\frac{\pi}{8}$
(2) $\frac{\pi}{48}$
(3) $\frac{\pi}{16}$
(4) $\frac{\pi}{12}$

NTA. **(4)**
Reso. **(4)**
Sol. $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \theta \in (0, 2\pi)$
$\sin^{-1}(\sin\theta) - (\pi/2 - \sin^{-1}(\sin \theta)) > 0$

$\sin^{-1}(\sin\theta) > \pi/4 \Rightarrow \theta \in (\pi/4, 3\pi/4)$



$(a, b) = (\frac{\pi}{4}, \frac{3\pi}{4}) \Rightarrow b - a = \frac{\pi}{2} \Rightarrow \alpha - \beta = \frac{\pi}{2}$ _____(1)

Now $\alpha x^2 + \beta x + \sin^{-1}(x^2-6x + 10) + \cos^{-1})(x^2-6x + 10) = 0$

$x^2 - 6x + 10 = 1 + (x-3)^2 \geq 1$

Hence x = 3 is only possible sol.

$9\alpha + 3\beta + \pi/2 = 0$ _____(2)

On solving 1 and 2 we get $\alpha = \frac{\pi}{12}$

73. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha(> 0)$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30 - \alpha$. If the mean and variance of the marks of the combined class of 100 + n students are respectively 50 and 350, then the sum of variances of classes A and B is

(1) 500 (2) 450 (3) 650 (4) 900

NTA. (1)

Reso. (1)

Sol.

A	B	A + B
$\bar{x}_1 = 40$	$\bar{x}_2 = 55$	$\bar{x} = 50$
$\sigma_1 = \alpha$	$\sigma_2 = 30 - \alpha$	$\sigma_2 = 350$
$n_1 = 100$	$n_2 = n$	$100 + n$

$\bar{x} = \frac{100 \times 40 + 55n}{100 + n} = 50$

$\Rightarrow n = 200$

$\sigma_1^2 = \frac{\sum x_i^2}{100} - 40^2 = \alpha^2$

$\sigma_2^2 = \frac{\sum y_i^2}{200} - 55^2 = (30 - \alpha)^2$

$\sigma^2 = \frac{\sum x_i^2 + \sum y_i^2}{300} - 50^2 = 350^2$

$350^2 = \frac{(1600 + \alpha^2) \times 100 + [3025 + (30 - \alpha)^2] \times 200}{300} - 50^2$

on solving $\alpha^2 - 40\alpha + 300 = 0$

$\Rightarrow \alpha = 10$ or 30

$\alpha = 30$ is not possible

$\sigma_1^2 + \sigma_2^2 = 10^2 + 20^2 = 500$

74. Let $f : R - \{2,6\} \rightarrow R$ be real valued function defined as $f(x) = \dfrac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

(1) $\left(-\infty, -\dfrac{21}{4}\right] \cup \left[\dfrac{21}{4}, \infty\right)$ (2) $\left(-\infty, -\dfrac{21}{4}\right] \cup [0, \infty)$

(3) $\left(-\infty, -\dfrac{21}{4}\right) \cup (0, \infty)$ (4) $\left(-\infty, -\dfrac{21}{4}\right] \cup [1, \infty)$

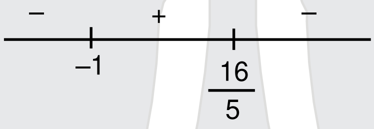
NTA. **(2)**

Reso. **(2)**

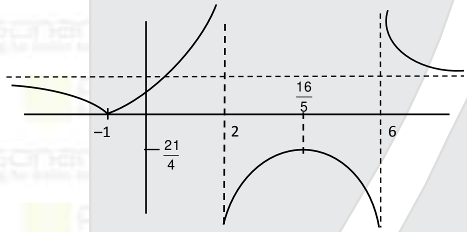
Sol. $y = \dfrac{x^2+2x+1}{x^2-8x+12} = \dfrac{(x+1)^2}{(x-2)(x-6)}$ ... (1)

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-2(x+1)(5x-16)}{(x-2)^2(x-6)^2}$

Number line of $\dfrac{dy}{dx}$ =



So Graph of $y = \dfrac{(x+1)^2}{(x-2(x-6)}$ for $x \in (2, 6)$



from graph range is

Range $\in \left(-\infty, \dfrac{21}{4}\right] \cup [0, \infty)$

75. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0$, $x \in R$ has :

(1) four solutions two of which are negative

(2) two solutions and both are negative

(3) no solution

(4) two solutions and only one of them is negative

NTA. **(2)**

Reso. **(2)**

Sol. Let $e^x = t$

$t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$

divide complete equation by t^2

$t^2 + \dfrac{1}{t^2} + 8(t - \dfrac{1}{t}) + 13 = 0$

$(t - \dfrac{1}{t})^2 + 8(t - \dfrac{1}{t}) + 15 = 0$

$t - \dfrac{1}{t} = z$

$z^2 + 8z + 15 = 0$

$z = -3, -5$

$t - \frac{1}{t} = -3$ or $t - \frac{1}{t} = -5$

$t^2 + 3t - 1 = 0$ or $t^2 + 5t - 1 = 0$

$t = \frac{-3 \pm \sqrt{13}}{2}, \frac{-5 \pm \sqrt{29}}{2}$

$e^x = \frac{-3+\sqrt{13}}{2}, \frac{-5+\sqrt{29}}{2} = \alpha, \beta$

both $\frac{-3+\sqrt{13}}{2}$ and $\frac{-5+\sqrt{29}}{2} \in (0, 1)$

$x = \ell n(\alpha), \ell n(\beta)$ both negative

76. The complex number $z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ is equal to :

(1) $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

(2) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

(3) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

(4) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

NTA. (4)
RESO. (4)

Sol. $Z = \dfrac{i-1}{\cos\pi/3 + i\sin\pi/3} = \dfrac{\sqrt{2}e^{i3\pi/4}}{e^{i\pi/3}} = \sqrt{2}\,e^{i5\pi/12} = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

77. If a point P (α, β, γ) satisfying

$(\alpha\ \beta\ \gamma)\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0\ 0\ 0)$

lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to

(1) $\frac{5}{4}$ (2) -1 (3) 11 (4) $\frac{11}{5}$

NTA. (3)
RESO. (3)

Sol. $(\alpha\ \beta\ \gamma)\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0\ 0\ 0)$

$2\alpha + 2\beta + 8\gamma = 0$ ______(1)
$10\alpha + 3\beta + 4\gamma = 0$ ______(2)
$8\alpha + 8\beta + 8\gamma = 0$ ______(3)
from (1) & (3) by cross multiplication

$\frac{\alpha}{1} = \frac{\beta}{6} = \frac{\gamma}{-7} = k \Rightarrow \alpha = k, \beta = 6k, \gamma = -7k$

Point P (α, β, γ) lie on the plane $2x + 4y + 3z = 5 \Rightarrow 2\alpha + 4\beta + 3\gamma = 5$
$\Rightarrow 2k + 24k - 21k = 5$
$\Rightarrow k = 1$

Hence $6\alpha + 9\beta + 7\gamma = 6k + 54k - 49k = 11k = 11$

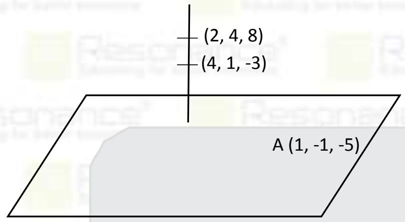
78. Let P be the plane, passing through the point (1, – 1, – 5) and perpendicular to the line joining the points (4,1, – 3) and (2,4, 3). Then the distance of P from the point (3, – 2, 2) is

(1) 6 (2) 4 (3) 5 (4) 7

NTA. (3)

RESO. (3)

Sol.



D'r of Plane Normal : 2, -3, -6

Equation of Plane : 2 (x-1) – 3 (j + 1) – 6 (z + 5) = 0

2x – 3y – 6z – 35 = 0

Hence $\perp$ distance of plane from (3, -2, 2)

$$\perp \text{ distance} = \left|\frac{6+6-12-35}{\sqrt{2^2+(-3)^2+(-6)^2}}\right| = 5$$

79. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=5\hat{i}-3\hat{j}+3\hat{k}$, be there(three) vector. If $\vec{r}$ is a vector such that, $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$, then $25|\vec{r}|^2$ is equal to

(1) 560 (2) 339 (3) 449 (4) 336

NTA. (2)

RESO. (2)

Sol. Given $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$

$\Rightarrow \vec{r}\times\vec{b}-\vec{c}\times\vec{b}=0$

$\left(\vec{r}-\vec{c}\right)\times\vec{b}=0$

$\vec{r}-\vec{c} \,||\, \vec{b}$

$\vec{r}=\vec{c}+\lambda\vec{b}$

$\because \vec{r}.\vec{a}=0 \Rightarrow \vec{c}.\vec{a}+\lambda\left(\vec{b}.\vec{a}\right)=0$

$$\lambda=\frac{\vec{c}.\vec{a}}{\vec{b}.\vec{a}}=\frac{-8}{5}$$

$$\vec{r}=\vec{c}-\frac{8}{5}\vec{b}=\left(5\hat{i}-3\hat{j}+3\hat{k}\right)-\frac{8}{5}\left(\hat{i}-\hat{j}+2\hat{k}\right)$$

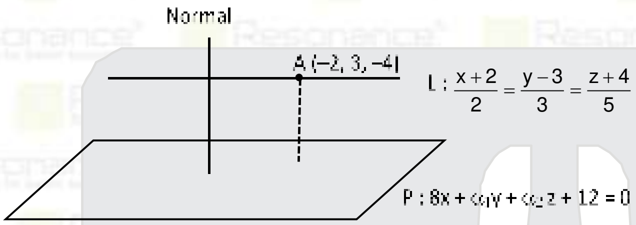
$5\vec{r}=17\hat{i}-7\hat{j}-\hat{k}$

$25|\vec{r}|^2=(17)^2+(-7)^2+(-1)^2=289+49+1=339$

80. Let the plane P : $8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line $L : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1, then the distance between P and L is

(1) $\sqrt{\frac{2}{7}}$ (2) $\frac{6}{\sqrt{14}}$ (3) $\sqrt{\frac{7}{2}}$ (4) $\sqrt{14}$

NTA. (4)
RESO. (4)
Sol.



Normal ⊥ L ⇒ $8 \times 2 + 3\alpha_1 + 5\alpha_2 = 0$ ______(1)
For Y intercept of plane put x = z = 0 in equation of plane

$$\Rightarrow y = \frac{-12}{\alpha_1} = 1 \Rightarrow \alpha_1 = -12$$

Put α_1 in equation (1) we get $\alpha_2 = 4$
Hence equation of plane P : 2x – 3y + z + 3 = 0

Now distance of point A (–2, 3, –4) from the plane P is = $\left|\frac{2(-2)-3(3)-4+3}{\sqrt{(2)^2+(-3)^2+(1)^2}}\right| = \sqrt{14}$

81. Let A = $[a_{ij}]$. $a_{ij} \in Z \cap [0, 4]$, $1 \le i, j \le 2$. The number of matrices A such that the sum of all entries is a prime number p ∈ (2, 13) is ________ .

NTA. (196)
RESO. (204)

Sol. Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $a_{11}, a_{12}, a_{21}, a_{22} \in \{0, 1, 2, 3, 4\}$

Now $a_{11} + a_{12} + a_{23} + a_{22}$ = prime number p = 3, 5, 7, 11

$\Rightarrow (x^0 + x^1 + x^2 + x^3 + x^4)^4$

$\Rightarrow \left(\frac{1-x^5}{1-x}\right)^4 = (1 - x^5)^4 (1 - x)^{-4}$

$= \left({}^4C_{r_1}(-x^5)^{r_1}\right)\left({}^{4+r_2-1}C_{r_2} x^{r_2}\right)$

$= {}^4C_{r_1} \, {}^{3+r_2}C_{r_2} (-1)^{r_1} (x)^{5r_1+r_2}$

$5r_1 + r_2 = 3, 5, 7, 11$

when $5r_1 + r_2 = 3 \Rightarrow r_1 = 0, r_2 = 3$

when $5r_1 + r_2 = 5 \Rightarrow r_1 = 0, r_2 = 5$ or $r_1 = 1, r_2 = 0$

when $5r_1 + r_2 = 7 \Rightarrow r_1 = 1, r_2 = 2$ or $r_1 = 0, r_2 = 7$

when $5r_1 + r_2 = 11 \Rightarrow r_1 = 0, r_2 = 11$ or $r_1 = 1, r_2 = 6$ or $r_1 = 2, r_2 = 1$

sum of all coefficient $= {}^4C_0 \times {}^6C_3 + {}^4C_0\,{}^8C_5 - {}^4C_1\,{}^3C_0 + {}^4C_0\,{}^{10}C_7 - {}^4C_1\,{}^5C_2 + {}^4C_0\,{}^{14}C_{11} - {}^4C_1\,{}^9C_6 + {}^4C_2\,{}^4C_1$

= 204

82. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a}\times\vec{b}) = 3(\vec{c}\times\vec{a})$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}.\vec{b}}\right)^2$ is equal to ________ .

NTA. **(3)**
RESO. **(3)**

Sol. Given $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$

$2(\vec{a}\times\vec{b}) = 3(\vec{c}\times\vec{a})$ $\quad \vec{b} \wedge \vec{c} = 2\pi/3$

$3(\vec{c}\times\vec{a}) + 2(\vec{b}\times\vec{a}) = 0$

$(3\vec{c} \times 2\vec{b}) \times \vec{a} = 0$

$3\vec{c} \times 2\vec{b} = \lambda\vec{a}$

Squaring both sides

$9|\vec{c}|^2 + 4|\vec{b}|^2 + 12(\vec{b}.\vec{c}) = \lambda^2|\vec{a}|^2$

$36 + 1 + 12 \times \frac{1}{2} \times 2(\cos(2\pi/3)) = \lambda^2(31)$

$\lambda^2 = 1$

$\lambda = \pm 1$

$3\vec{c} + 2\vec{b} = \pm\vec{a}$ ....(1)

Dot with $\vec{b} = 3(\vec{b}.\vec{c}) + 2(\vec{b}.\vec{b}) = \pm\,\vec{a}.\vec{b}$

$\vec{a}.\vec{b} = \pm\left(-\frac{3}{2} + \frac{1}{2}\right) = \pm(-1)$

$(\vec{a}.\vec{b})^2 = 1$

$3(\vec{c}\times\vec{a}) = 2(\vec{a}\times\vec{b})$

$(\vec{c}\times\vec{a})^2 = \frac{4}{9}(\vec{a}\times\vec{b})^2$

$= \frac{4}{9}[|\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2]$

$= \frac{4}{9}\left[\frac{31}{4} - (1)\right]$

$= \frac{4}{9} \times \frac{27}{4} = 3$

83. The coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is ________ .

NTA. **(5040.00)**
Reso. **(5040.00)**

Sol. $T_{r+1} = {}^9C_r \left(\frac{5}{2x^2}\right)^r \cdot \left(\frac{4x}{5}\right)^{9-r} = {}^9C_r \frac{2^{18-3r}}{5^{9-2r}} x^{9-3r}$

for coefficient of $x^{-6} \Rightarrow 9 - 3r = -6 \Rightarrow r = 5$

coff. of $x^{-6} = \frac{{}^9C_5 \cdot 2^3}{5^{-1}} = \frac{9.8.7.5}{4.3.2.1}.(8 \times 5)$

$= 5040$

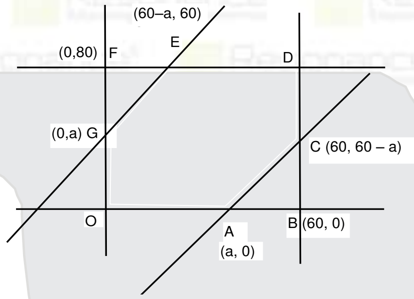
84. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space [0, 60] is less than or equal to a. If P(A) = $\frac{11}{36}$, then a is equal to _______ .

NTA. **(10)**

Reso. **(10)**

Sol. $|x - y| < a \Rightarrow -a < x - y < a$

$\Rightarrow \quad x - y < a$ and $x - y > -a$



$$P(A) = \frac{ar(OACDEG)}{(OBDF)}$$

$$= \frac{ar(OBDF) - ar(ABC) - ar(EFG)}{ar(OBDF)}$$

$$\Rightarrow \quad \frac{11}{36} = \frac{(60)^2 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600}$$

$\Rightarrow \quad 1100 = 3600 - (60 - a)^2$

$\Rightarrow \quad (60 - a)^2 = 2500 \Rightarrow 60 - a = 50$

$\Rightarrow \quad a = 10$

85. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^{\ell}}\right)^9$ is –84 and the coefficient of $x^{-3\ell}$ is $2^{\alpha}\beta$ where $\beta < 0$ is an odd number, then $|\alpha\ell - \beta|$ is equal to _______ .

NTA. **(98)**

Reso. **()**

Sol. General Term $T_{r+1} = {}^9C_r \left(\frac{x^{\frac{5}{2}}}{2}\right)^{9-r}\left(\frac{-4}{x^{\ell}}\right)^r = (-1)^r\, {}^9C_r\, x^{\frac{45}{2}-\frac{5r}{2}-lr}\, 2^{3r-9}$

for constant term $45 - 5r - 2lr = 0 \qquad \Rightarrow r = \frac{45}{5+2l}$

and coefficient of constant term = $(-1)^r\, {}^9C_r\, 2^{3r-9} = -84 \quad \Rightarrow r = 3$

$\Rightarrow l = 5$

Now coefficient of $x^{-3l} = x^{-15} \Rightarrow \frac{45}{2} - \frac{5r}{2} - lr = -15 \Rightarrow r = 5$ when $l = 5$

Now coefficient of $x^{-15} = (-1)^5\ {}^9C_5\ 2^6 = -63\ (2^7)$

Hence $\alpha = 7$, $\beta = -63$, $l = 5$

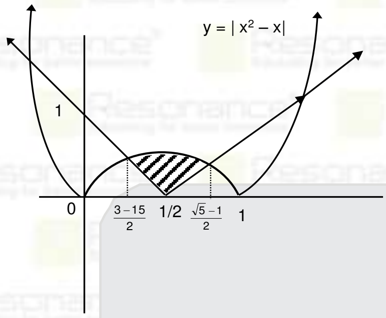
Hence $|\alpha l - \beta| = 98$

86. Let the area of the region $\{(x,y): |2x-1| \le y \le |x^2-x|, 0 \le x \le 1\}$ be A. Then $(6A + 11)^2$ is equal to _____.

NTA. (125)

Reso. (125)

Sol. $y \ge |2x-1|$ $\qquad$ $y \le |x^2 - x|$



Both curve are symmetric about $x = \frac{1}{2}$ Hence

$$A = 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}} \left(\left(x-x^2\right)-\left(1-2x\right)\right)dx$$

$$A = 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}} \left(-x^2+3x-1\right)dx = 2\left(\frac{-x^3}{3}+\frac{3}{2}x^2-x\right)_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}$$

on solving $\qquad 6A + = 5\sqrt{5}$

$\qquad (6A + 11)^2 = 125$

87. Let A be a n × n matrix such that |A| = 2. If the determinant of the matrix Adj $(2. \text{Adj}(2A^{-1}))$ is 2^{84}, then n is equal to _____ .

NTA. (5)

Reso. (5)

Sol. Given |A| = 2

Hence Adj $(2. \text{Adj}(2A^{-1})) = |2. \text{Adj}(2A^{-1})|^{n-1}$

$= (2^n |\text{Adj}(2A^{-1})|)^{n-1}$

$= (2^n |2A^{-1}|^{n-1})^{n-1}$

$= 2^{n(n-1)} ((2^n |A^{-1}|)^{(n-1)})^{(n-1)}$ $\qquad \therefore |A^{-1}| = \frac{1}{|A|} = \frac{1}{2}$

$= 2^{n(n-1)} ((2^{(n-1)})^{(n-1)})^{(n-1)}$

$= 2^{n(n-1)+(n-1)^3} = 2^{84}$

$\Rightarrow n = 5$

88. The sum $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \ldots + 15.29^2$ is _____ .

NTA. **(6952)**

Reso. **(6952)**

Sol. $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + \ldots + 15.29^2 =$

$$1 + \sum_{r=1}^{7} (2r+1)\ (4r+1)^2 - 2r(4r-1)^2$$

$$1 + \sum_{r=1}^{7} 2r(16r + (4r+1)^2$$

$$1 + 48\sum_{r=1}^{7} r^2 + 8\sum r + \sum 1$$

$$1 + 48\times \frac{7\times8\times15}{6} + 8\times\frac{7\times8}{2} + 7$$

$1 + 56 \times 120 + 56 \times 4 + 7$

$8 + 56 \times 124 = 6944 + 8 = 6952$

89. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to :

NTA. **(45.00)**

Reso. **(45.00)**

Sol.
$$\frac{\left(\frac{(2n+1)!}{(n+2)!}\right)}{\frac{(2n-1)!}{(n-1)!}} = \frac{11}{21}$$

$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$$

$\Rightarrow 42(2n + 1) = 11\ (n + 1)\ (n + 2)$

$\Rightarrow 11n^2 - 51n - 20 = 0$

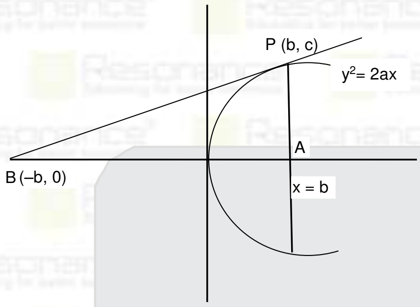
$\Rightarrow (n - 5)\ (11n + 4) = 0$

$\Rightarrow n = 5$

$\Rightarrow n^2 + n + 15 = 45$

90. Let S be the set of all $a \in N$ such that the area of the triangle formed by the tangent at the point P(b, c), b, c $\in$ N, on the parabola $y^2 = 2ax$ and the lines x = b, y = 0 is 16 unit^2, then $\sum_{a \in S} a$ is equal to _____.

NTA. (146.00)
Reso. (146.00)
Sol.



Tangent at P (b, c) will be $\Rightarrow ax - cy + ab = 0$
it meets x -axis at (–b, 0)

Area of triangle (PAB) = $\frac{1}{2}$ (2b) (c) = 16 $\Rightarrow$ bc = 16

$\Rightarrow$ (b,c) = (1, 16), (2, 8), (4, 4), (8, 2), (16, 1)
Now (b,c) lie on parabola hence satisfy it

$c^2 = 2ab \Rightarrow c^2 = 2a\left(\frac{16}{c}\right) \Rightarrow a = \frac{c^3}{32}$

when possible value of c is c = 4, 8, 16 $\Rightarrow$ a = 2, 16, 128

$\sum_{a \in S} a = 2 + 16 + 128 = 146$

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