ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
AMTI – NMTC – 2023 November - PRIMARY – FINAL

INSTRUCTIONS

1. Answer all questions. Each question carries 10 marks.
2. Elegant and innovative solutions will get extra marks.
3. Diagrams and justification should be given wherever necessary.
4. Before starting to answer, fill in the **FACE SLIP** completely.
5. Your 'rough work' should be done in the answer sheet itself.
6. Maximum time allowed is **THREE hours.**

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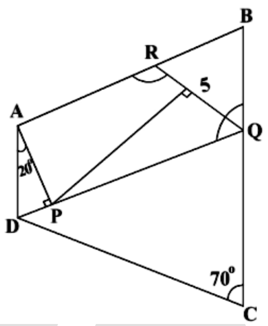
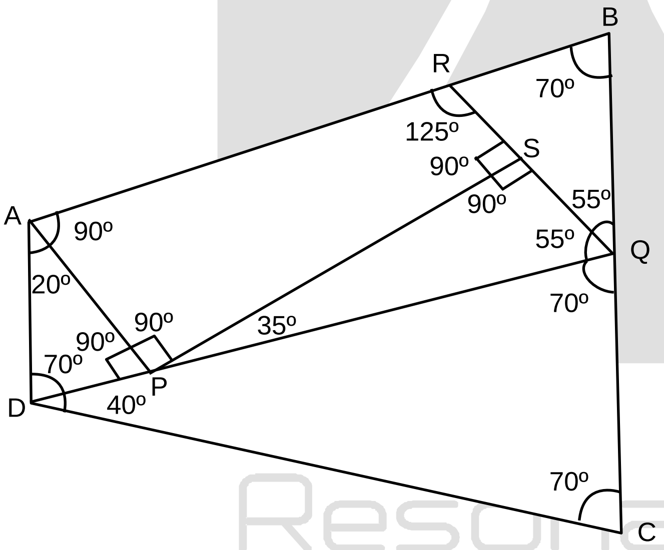
1. ABCD is an isosceles trapezium as shown in the figure, in which AB = DC ; $\angle DAP = 20^o$; DP is perpendicular to AP; $\angle C = 70^o$; QR is the bisector of $\angle BQD$ and PS $\perp$ QR. Calculate $\angle SPQ$ and $\angle SRA$. Justify each of the steps in calculation.



Sol.



ABCD is an isosceles trapezium so

$\angle B = \angle C = 70^o$

also $\angle A + \angle B = 180^o$ {sum of adjscent angle

So $20° + \angle RAP + 70° = 180^o$}

$\angle RAP = 180^o – 90^o = 90^o$ ....(1)

In $\Delta ADP \Rightarrow 20^o + 90^o + \angle ADP = 180^o$

So ADP = 70^o .... (2)

Now ADP = $\angle DQC$ (alternate interior angle)

So $\angle DQC = 70^o$ ....(3)

Now $\angle DQC + \angle DQB = 180^o$ (Linear pair)

$70^o + \angle DQB = 180^o$

$\angle DQB = 110^o$ ....(4)

Now RQ is angle bisertor so

$\angle PQS = \angle SQB = \dfrac{110}{2} = 55^\circ$ ....(5)

Now in $\Delta PSQ \rightarrow \angle SPQ + \angle PQS + \angle QSP = 180^\circ$

$= \angle SPQ + 55^\circ + 90^\circ = 180^\circ$

$\angle SPQ = 35^\circ$ ....(6)

In quad APQR $\rightarrow \angle RAP + \angle APQ + \angle PQR + \angle QRA = 360^\circ$

$90^\circ + 90^\circ + 55^\circ + \angle QRA = 360^\circ$

$\Rightarrow \angle QRA = 125^\circ$

2. Ramanujan is a sixth grade student. His mathematics teacher gave a problem sheet in maths as home task for the Puja holidays. Ramanujan wants to complete it in 4 days and wants to enjoy the holidays for the remaining 6 days.
On the first day, he worked out one-fifth the number of problems plus 12 more problems.
On the second day, he worked out one-fourth the remaining problems plus 15 more problems.
On the third day, he solved one-third of the remaining problems plus 20 more problems.
The fourth day, he worked out successfully the remaining 60 problems and completed the work.
Find the total number of problems given by the teacher and the number of problems solved by Ramanujan on each day.

Sol. First day work = $\frac{1}{5}x + 12$

First day remaining work = $x - \left(\frac{1}{5}x + 12\right) = x - \frac{x}{5} - 12$

$= \left(\frac{4x}{5} - 12\right)$

Second day work = $\frac{1}{4}\left(\frac{4x}{5} - 12\right) + 15 = \frac{x}{5} - 3 + 15$

$= \left(\frac{x}{5} + 12\right)$

Remaining work = $\left(\frac{4x}{5} - 12\right) - \left(\frac{x}{5} + 12\right) = \frac{4x}{5} - \frac{x}{5} - 24$

$= \left(\frac{3x}{5} - 24\right)$

Third day work = $\frac{1}{3}\left(\frac{3x}{5} - 24\right) + 20 = \left(\frac{x}{5} - 8 + 20\right) = \left(\frac{x}{5} + 12\right)$

Remaining = $\left(\frac{3x}{5} - 24\right) - \left(\frac{x}{5} + 12\right) = \left(\frac{2x}{5} - 36\right)$

Fourth day work = $\frac{2x}{5} - 36 = 60$

$\frac{2x}{5} = 96$

$x = \frac{96 \times 5}{2} = 240$

total problem = 240

3. There are 4 cards and on each card a whole number is written. All the numbers are different from one another. Two girls of grade six, Deepa and Dilruba play a game. Deepa takes 3 cards at a time leaving a card behind. She multiplies the numbers and gets an answer. In the same way, again, she leaves one different card and selects the other three and multiplies the numbers. She got the answers 480, 560, 420 and 336. Now, Dilruba has to find the numbers in each card. Dilruba worked out and found the correct numbers.

What are they? Work out systematically and find the numbers.

Sol. $A \times B \times C = 480$ ....(1)

$A \times B \times D = 560$ ....(2)

$B \times C \times D = 420$ ....(3)

$A \times C \times D = 336$ ....(4)

$$\frac{Eq.1}{Eq.2} \Rightarrow \frac{C}{D} = \frac{6}{7} \Rightarrow D = \frac{7}{6}C$$

$$\frac{Eq.2}{Eq.3} \Rightarrow \frac{A}{C} = \frac{28}{21} \Rightarrow A = \frac{28}{21}C$$

$$\frac{Eq.4}{Eq.1} \Rightarrow \frac{D}{B} = \frac{7}{10} \Rightarrow B = \frac{10}{7}D \Rightarrow \frac{10}{7} \times \frac{7}{6} \times C = \frac{5}{3}C \Rightarrow B = \frac{5}{3}C$$

$A \times B \times C = 480$

$$\frac{28}{21} \times C \times \frac{5}{3} \times C \times C = 480$$

$$C^3 = \frac{480 \times 21 \times 3}{28 \times 5} = 8 \times 3 \times 3 \times 3$$

$C = 2 \times 3 = 6$

$$A = \frac{28}{21} \times 6 = 8$$

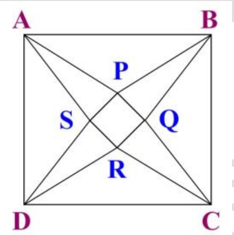
$$D = \frac{7}{6} \times 6 = 7$$

$$B = \frac{5}{3} \times 6 = 10$$

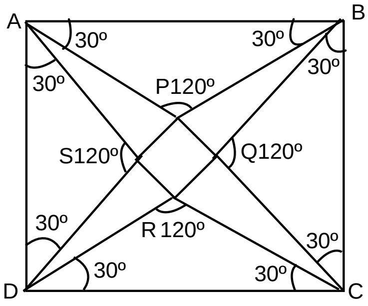
A, B, C, D = 8, 10, 6, 7

4. An angle is divided into 3 equal parts by two straight lines; such lines are called trisectors. ABCD is a square. The lines (AP, AS) are trisectors of $\angle BAD$. Similarly, we have the trisectors (BP,BQ), (CQ,CR) and (DR,DS).

Prove that PQRS is a square.



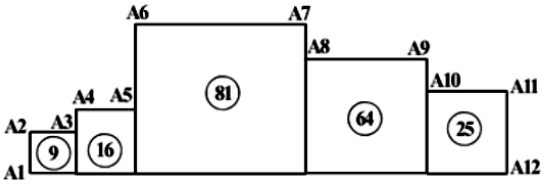
Sol.

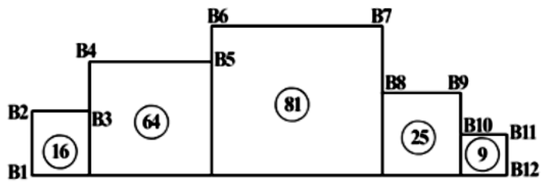


Angle divided into 3 equal parts = $\frac{90^\circ}{3}$ = 30º

So, $\angle$SAD = $\angle$SDA = $\angle$RDC = $\angle$RCD = $\angle$QCB = $\angle$QBC = $\angle$PBA = $\angle$PAB = 30º
and also $\angle$SAP = $\angle$PBQ = $\angle$QCR = $\angle$SDR = 30º
ΔASD is isosceles $\angle$A = $\angle$D = 30º
So, $\angle$ASD = 120º
Also ΔAPB , ΔBQC and ΔCRD isosceles
In ΔASD and ΔAPB
AD = AB (Square sides)
$\angle$SAD = $\angle$PAB
$\angle$SDD = $\angle$PBA (Both are 30º)
By ASA both triangle are congruent
ΔASD $\cong$ ΔAPB
So, by CPCT
$\begin{bmatrix} AS = AP \\ DS = BD \end{bmatrix}$ like that ΔASD , ΔDRC and ΔCRB also congruent.
Now , ΔASP, ΔBPQ, ΔCQR and SDR are isosceles triangle
In ΔASP $\Rightarrow$ $\angle$A + $\angle$S + $\angle$P = 180º
$\angle$S = $\angle$P
30º + $\angle$S + $\angle$S = 180
2$\angle$S = 180º – 30º
$\angle$S = 75º = $\angle$P
By same in all triangle we can prove same thing we know that the complete angle is = 360º
Now in quadrilateral PQRS $\rightarrow$ SPQ = 360º – (75° + 120° + 75°) = 90º
by same $\angle$PQR , $\angle$QRS are $\angle$RSP all are equal to 90º
Now, IN ΔASP and ΔDSR
AS = DS
$\angle$A = $\angle$D = 30º
AD = 6DR
by SAS both triangle congruent
ΔASP $\cong$ ΔDSR
by CPCT
[SP = SR]
by same SP = PQ = QR = RS
Quadrilateral all side are equal and all angle 90º
So PQRS is a square

5. Five squares of different dimensions are arranged in two ways as shown in the following diagrams. The numbers inside each square represents its area in square units.





Calculate the perimeter $A_1A_2A_3A_4A_5A_6A_7A_8A_9A_{10}A_{11}A_{12}A_1$ and the corresponding perimeter of figure 2. Are they same? If they are different, which is greater?

Sol.

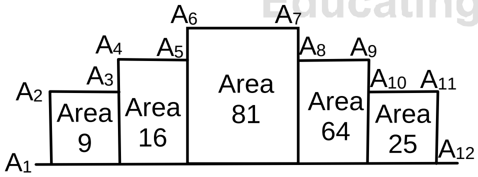


Figure -1

Total perimeter $A_1A_2A_3A_4A_5A_6A_7A_8A_9A_{10}A_{11}A_{12}A_1$
= 3 + 3 + 1 + 4 + 5 + 9 + 1 + 8 + 3 + 5 + 5 + 29
= 76

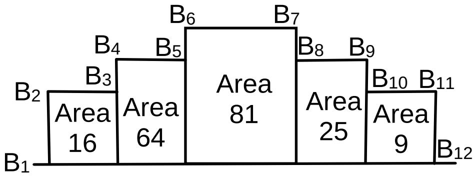


Figure -2

Total perimeter = 4 + 4 + 4 + 8 + 1 + 9 + 4 + 5 + 2 + 3 + 3 + 29 = 76

Both perimeter are same.

6. (i) In a book, a problem on fractions is given as

$$\frac{1}{3\frac{1}{5}} - \frac{2\frac{1}{4}}{9} + \frac{3\frac{5}{8}}{} + \frac{\frac{4}{7}}{4\frac{4}{7}}$$

The denominator of the third term is not printed. The answer is given to be 2. What is the missing denominator? Let it be a.

(ii) Simplify: $\dfrac{1}{3-\dfrac{1}{2-\dfrac{1}{\left(\frac{5}{7}\right)}}}$. Let the answer be of the form $\frac{p}{q}$ where p, q have no common factors. Let $b = \frac{p}{q}$

(iii) Find the value of $\left(\frac{1}{a^2}+b\right)$

Sol. **(i)** $\dfrac{1}{\frac{16}{5}} - \dfrac{\frac{9}{4}}{9} + \dfrac{\frac{28}{8}}{a} + \dfrac{\frac{4}{7}}{\frac{32}{7}} = 2$

$\frac{5}{16} - \frac{1}{4} + \frac{29}{8a} + \frac{1}{8} = 2$

$\frac{5-4+2}{16} + \frac{29}{8a} = 2$

$\frac{3}{16} + \frac{29}{8a} = 2$

$\frac{29}{8a} = 2 - \frac{3}{16}$

$\frac{29}{8a} = \frac{32-3}{16}$

$\frac{29}{8a} = \frac{29}{16}$

8a = 16

a = 2

(ii) $\dfrac{1}{3-\dfrac{1}{2-\dfrac{7}{5}}}=\dfrac{1}{3-\dfrac{1}{\dfrac{3}{5}}}$

$=\dfrac{1}{3-\dfrac{5}{3}}$

$=\dfrac{1}{\dfrac{9-5}{3}}=\dfrac{3}{4}$

$\left[b=\dfrac{p}{q}=\dfrac{3}{4}\right]$

(iii) $\dfrac{1}{a^2}+b$

$=\dfrac{1}{2^2}+\dfrac{3}{4}$

$=\dfrac{1}{4}+\dfrac{3}{4}$

$=\dfrac{1+3}{4}$ =1 Ans.

7. a, b are two integers. Find all pairs a and b such that $\frac{1}{a}+\frac{1}{b}=\frac{1}{2}$. Arrive at your result logically.

Sol. Given a, b are two integers

$\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{2}$

$\dfrac{1}{a}=\dfrac{1}{2}-\dfrac{1}{b}$

$\dfrac{1}{a}=\dfrac{b-2}{2b}$

$a=\dfrac{2b}{b-2}$ … (i)

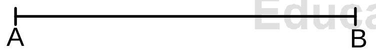
here b – 2 is factor of 2b for get integer, there are only four pairs which satisfy eq. (i)

b = 3, b = 4, b = – 2, b = 6

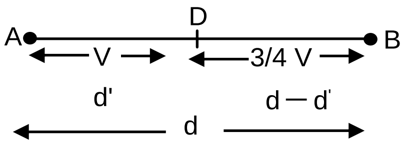
a = 6, a = 4, a = 1, a = 3

8. A train starts from a station A and travels with constant speed up to 100 kms/hr. After some time, there appeared a problem in the engine and so the train proceeds with $\frac{3}{4}$ th of the original speed and arrives at Station B, late by 90 minutes. Had the problem in the engine occurred 60 kms further on, then the train would have reached 15 minutes sooner. Find the original speed of the train and distance between stations A and B.

Sol.



Let Total distance between AB = d
and speed of train = V
and total time = t



Time taken A to D

$\Rightarrow\ t_1 = \frac{d'}{v}$

and time taken D to B

$\Rightarrow\ t_2 = \frac{d-d'}{3/4V}$

Now $t_1 + t_2 = t + \frac{90}{60}$

$\Rightarrow \frac{d'}{v} + \frac{4(d-d')}{3v} = \frac{d}{v} + \frac{3}{2}$

$\Rightarrow \frac{d'}{v} + \frac{4d}{3v} - \frac{4d'}{3v} = \frac{d}{v} + \frac{3}{2}$

$\Rightarrow \frac{1}{3}\frac{d}{v} - \frac{d'}{3v} = \frac{3}{2}$

$\Rightarrow\ d - d' = \frac{9}{2}V$

Now from second condition

$\Rightarrow \frac{d'+60}{V} + \frac{4(d-d-60)}{3V} = \frac{d}{v} + \frac{75}{60}$

$\Rightarrow \frac{d'}{v} + \frac{60}{v} + \frac{4d}{3v} - \frac{4d'}{3v} - \frac{4}{3}\times\frac{60}{v} = \frac{d}{v} + \frac{5}{4}$

$\Rightarrow \frac{d}{3v} - \frac{d'}{3v} - \frac{20}{v} = \frac{5}{4}$

$\Rightarrow \frac{1}{3v}(d-d') - \frac{20}{v} = \frac{5}{4}$

$\Rightarrow \frac{1}{3v}\times\frac{9v}{2} - \frac{20}{v} = \frac{5}{4}$

$\Rightarrow \frac{3}{2} - \frac{20}{v} = \frac{5}{4}$

$\Rightarrow \frac{20}{v} = \frac{3}{2} - \frac{5}{4} = \frac{1}{4}$

V = 80 km/hr.

Now $d - d' = \frac{9}{2} \times 80$

$\Rightarrow d - d' = 360$ km.