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ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
Screening Test – Bhaskara Contest

NMTC JUNIOR LEVEL – IX & X GRADES

Saturday, the 07 October 2023

INSTRUCTIONS

Time : 2 Hrs. **M. M. : 30**

1. Fill in the Response sheet with your Name, Class and the Institution through which you appear, in the specified places.

2. Diagrams are only Visual guides; they are not drawn to scale.

3. You may use separate sheets to do rough work.

4. Use of Electronic gadgets such as Calculator, Mobile Phone or Computer is not permitted.

5. Duration of Test: 10 am to 12 Noon (Two hours)

6. For each correct response you get 1 mark ; for each incorrect response, you lose ½ mark.

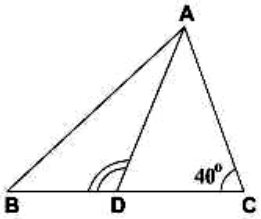
SECTION-A

1. If a, b, c are real numbers such that the polynomial $x^3 + 6x^2 + ax + b$ is the cube of $(x + c)$ then

a) $(a + b + c)$ is divisible by 13 b) $a + b = 11c$

c) $a > b$ and $b < c$ d) $(a + b + c)$ is divisible by 11.

2. In the adjoining figure, AB = 9 cm, AC = 7 cm, BC = 8 cm, AD is the median and $\angle C = 40°$. Then measure of $\angle ADB$ (in degrees) is



a) 100 b) 140 c) 45 d) 120

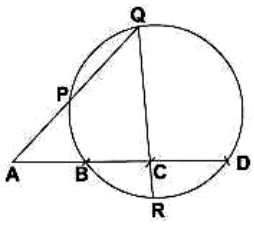
3. If $x^2 + 6x + 1 = 0$ and $\frac{x^4 + kx^2 + 1}{3x^3 + kx^2 + 3x} = 2$ then the value of k is

a) 68 b) 72 c) 65 d) 70

4. If $x = \sqrt[3]{49} + \sqrt[3]{42} + \sqrt[3]{36}$, then the value of $x - \frac{1}{x^2}$ is

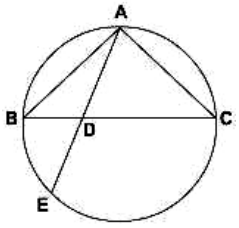
a) $2\sqrt[3]{42}$ b) $3\sqrt[3]{42}$ c) $\sqrt[3]{42}$ d) $4\sqrt[3]{42}$

5. In the adjoining figure, AB = BC = CD. P is the midpoint of AQ. If CR = 4, QC = 12, then PQ is equal to



a) $4\sqrt{3}$ b) $6\sqrt{3}$ c) $8\sqrt{3}$ d) $2\sqrt{3}$

6. In the adjoining figure, A is the midpoint of the arc BAC. Given that AB = 15 and AD = 10. Then the value of AE is



a) 22 b) 23 c) 22.5 d) 23.5

7. The number of real numbers x which satisfy the equation $\frac{8^x + 27^x}{12^x + 18^x} = \frac{7}{6}$ is

a) 1 b) 2 c) 0 d) 4

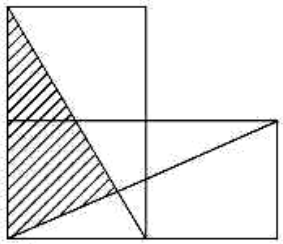
8. a, b are real numbers such that $2a^2 + 5b^2 = 20$. Then the maximum value of a^4b^6 is

a) 256 b) 1024 c) 1262 d) 16

9. The number of ordered pairs (x, y) of integers such that $x - y^2 = 4$ and $x^2 + y^4 = 26$ is

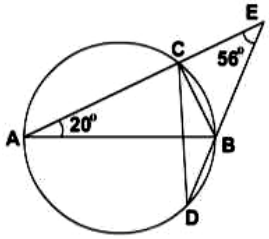
a) 4 b) 3 c) 2 d) 1

10. In the adjoining figure, three equal squares are placed. The squares are unit squares. The area of the shaded region (in cm^2) is



a) $\frac{5}{4}$ b) $\frac{4}{5}$ c) $\frac{3}{2}$ d) $\frac{3}{4}$

11. In the adjoining figure, AB is a diameter of the circle. Given $\angle BAC = 20°$, $\angle AEB = 56°$. Then the measure (in degrees) of $\angle BCD$ is



a) 12 b) 10 c) 14 d) 16

12. The number of ordered pairs (m, n) of integers such that $1 \le m, n \le 100$ and $m^n n^m$ leaves a remainder 1 when divided by 4 is

a) 2250 b) 1000 c) 1125 d) 1250

13. The number of ordered pairs of positive integers (x, y) satisfying the equation $x^2 + 4y = 3x + 16$ is

a) 1 b) 2 c) 3 d) 4

14. The algebraic expression $(a + b + ab + 2)^2 + (a - ab + 2 - b)^2 - 2b^2(1 + a^2)$ reduces to

a) $4(a + 2)^2$ b) $2(a + 2)^2 + 4ab^2$ c) $(a - 2)^2$ d) $2(a - 2)^2 + 4ab^2$

15. The sum of (1×4) + (2×7) + (3×10) + (4×13) + … 49 terms is equal to

a) 122500 b) 116800 c) 11800 d) 117600

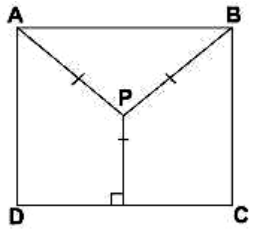
SECTION-B

16. If the equations $x^3 + ax + 1 = 0$ and $x^4 - ax^2 + 1 = 0$ have a common root, then the value of a^2 is _____.

17. If a, b, c, d are positive reals such that abcd = 1 then the maximum value of $a^2 + b^2 + c^2 + d^2 + ab + ac + ad + bc + bd + cd$ is _____.

18. The sum of all natural numbers 'n' for which n(n + 1) is a perfect square is _____.

19. P is a point inside the square ABCD such that PA = PB = Distance of P from CD.

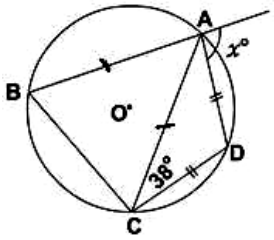


The ratio of the areas of the triangle PAB to the area of the square ABCD is $\frac{m}{n}$ where m, n are relatively prime integers. Then the value of m + n = _____.

20. The sum of roots of the simultaneous equations
$\sqrt[y]{4^x} = 32\sqrt[x]{8^y}$, $\sqrt[y]{3^x} = 3\sqrt[y]{9^{1-y}}$ is _____.

21. If $2\sqrt{3+\sqrt{5-\sqrt{13+\sqrt{48}}}} = \sqrt{a}+\sqrt{b}$ where a, b are natural numbers, then the value of a + b is _____.

22. In the adjoining figure, $\angle ACD = 38°$. Then the measure (in degrees) of angle x is _____



23. If $\frac{a}{b+c} + \frac{c}{a+b} = \frac{2b}{c+a}$ (where a + b, b + c, c + a, a + b + c are all not zero), then the numerical value of $\frac{a^2+c^2}{b^2}$ is _____.

24. The geometric and arithmetic means of two positive numbers are respectively 8 and 17. The larger among the two numbers is _____.

25. The number of two-digit numbers in which the tens and the units digit are different and odd is _____.

26. The value of $\left(5\sqrt[3]{4} - 3\sqrt[3]{1/2}\right)\left(12\sqrt[3]{2} + \sqrt[3]{16} - 2\sqrt[3]{2}\right)$ is equal to _____.

27. If $\frac{xy}{x+y} = 1$, $\frac{yz}{y+z} = 2$, $\frac{zx}{z+x} = 3$, then the numerical value of 15x – 7y – z is _____.

28. The sum of all natural numbers which satisfy the simultaneous inequations $x + 3 < 4 + 2x$ and $5x - 3 < 4x - 1$ is _____.

29. In an increasing geometric progression (with 1st term a and n^{th} term t_n), the difference between the fourth and the first term is 52 and the sum of the first three terms is 26. Then the numerical value of $\frac{t_{2024}}{t_{2023}} + \frac{a^{2024}}{a^{2023}}$ is _____

30. The base of a triangle is 4 units less than the altitude drawn to it. The area of the triangle is 96 (unit^2). The ratio of the base to height is $\frac{p}{q}$ where p, q are relatively prime to each other. Then the value of p + q is _____.