GAUSS CONTEST - FINAL - PRIMARY

CLASS- V & VI

AMTI - SATURDAY, 07TH JANUARY, 2023

Instructions:

1. ***Answer all questions. Each question carries 10 marks.***
2. ***Elegant and innovative solutions will get extra marks.***
3. ***Diagram and justification should be given wherever necessary.***
4. ***Before start answering, fill in the FACE SLIP completely.***
5. ***Your 'rough work' should be in the answer sheet itself.***
6. ***Maximum time allowed is THREE hours.***

1. Three skilled workers Akbar, Baskar and Charles are employed by a person to do three different jobs. After completion of the work the total fee the person gave to the three workers is Rs 6000. It is found that Rs 400 more than $\frac{2}{5}$ of Akbar's share, Rs 200 more than $\frac{2}{7}$ of Baskar's share and Rs 100 more than $\frac{9}{17}$ of Charles' share are all equal. How much did each get?

Sol. Let Akbar's share = Rs. x

Baskar's share = Rs. y

Charles' share = Rs. z

ATQ. $\frac{2}{5}x + 400 = \frac{2}{7}y + 200 = \frac{9}{17}z + 100 = k$

Now, $\frac{2}{5}x + 400 = k$ $\frac{2}{7}y + 200 = k$

$x = \frac{5}{2}(k-400)$ $y = \frac{7}{2}(k-200)$

$\frac{9}{17}z + 100 = k$ $z = \frac{17}{9}(k-100)$

Now, x + y + z = 6000

$\frac{5}{2}(k-100) + \frac{7}{2}(k-200) + \frac{17}{9}(k-100) = 6000$

$\frac{45(k-400)+63(k-200)+34(k-100)}{18} = 6000$

45 k –18000 + 63 k – 12600 + 34k – 3400 = 108000

142k – 34000 = 108000

142k = 108000 + 34000

142 k = 142000

K = 1000

So, Akbar's Share = $x = \frac{5}{2}(1000-400) = Rs.1500$

Baskar's = Rs. $y = \frac{7}{2}(1000-200) = Rs.2800$

Charles' = Rs. $z = \frac{17}{9}(1000-100) = Rs.1700$

2. A teacher of a primary class asked his students to calculate

$$2\frac{3}{7} \text{ of } \frac{\left(13\frac{1}{2}-9\frac{2}{3}\right)}{\left(15\frac{1}{5}-11\frac{7}{30}\right)}=A$$

The teacher has 49A chocolates with him. He distributed equal number of chocolates (more than one chocolate) to each student of his class irrespective of whether the students got the correct answer or not. After the distribution the teacher is left with only one chocolate and he took it. Find the maximum strength of the class.

Sol. $$A=2\frac{3}{7} \text{ of } \frac{\left(13\frac{1}{2}-9\frac{2}{3}\right)}{\left(15\frac{1}{5}-11\frac{7}{30}\right)}$$

$$=\frac{17}{7}\times\frac{\left(\frac{27}{2}-\frac{29}{3}\right)}{\left(\frac{76}{5}-\frac{337}{30}\right)}$$

$$=\frac{17}{7}\times\frac{\left(\frac{3\times27-2\times29}{6}\right)}{\left(\frac{6\times76-337}{30}\right)}$$

$$A=\frac{17}{7}\times\frac{\left(\frac{80-58}{6}\right)}{\left(\frac{456-337}{30}\right)}$$

$$=\frac{17}{7}\times\frac{\left(\frac{23}{6}\right)}{\left(\frac{119}{30}\right)}$$

$$=\frac{17}{7}\times\frac{23}{6}\times\frac{30}{119}$$

$$A=\frac{23\times5}{7\times7}$$

$$A=\frac{115}{49}$$

Now, Number of chocolates = 49A

So, 49A = $49\times\frac{115}{49}=115$

So total chocolates = 115

According to the given condition, one chocolates is for teacher

So total chocolates for students = 115–1 = 114

Now, For the maximum strength of the class by distributing equal number of chocolates (more than one chocolate) we have to distribute 2 chocolates to each student.

So, maximum strength of the class = $\frac{114}{2}$ = 57.

3. In a forest, Foxes always tell the truth and jackals always lie. When seen in poor light, they are indistinguishable. A person meets three of them A, B and C, in such a poor light. He asks A, "Are you a jackal?" Although A answers his question, he could not hear it clearly. B tells him that A denied being a Jackal and C says that A really is a jackal. Among the three how many are jackals?

Sol. A B C

He asks A "Are you a jackal"

A definitely says 'No'

Because if A is jackal then he lies always and give his answer as 'No' if A is a fox then He tell the truth always and give answer as 'No'

It means A says 'No'

Now B hear As answer and says A denied being a jackal (which means A said 'No') So here B is saying a truth which means B is a Fox.

Now C said A really is a jackal.

Here we can take two cases.

Case-1 : C is a jackal then he lies about A and now A is Fox.

So	A	B	C
	Fox	Fox	jackal

Case-2 : C is a fox then he saying truth about A and now A is a jackal.

So	A	B	C
	Jackal	Fox	Fox

So, in each case there is only one jackal.

4. Consider a natural number n. If n is less than 10 times the product of the digits, then n is called a *dwarf* number. Find the number of dwarf numbers between 10 and 200.

Sol. Let x, y be the digits

$\therefore$ 10x + y < 10xy

If any digit is zero

Then 10x + y > 10xy

Eg. 20 $\rightarrow$ 20 > 10 (2) (0)

And if unit digit is '1'

Then 10x + y > 10xy]

Eg. 91 $\rightarrow$ 91 > 10(9) (1)

91 > 90

So we can only take

12, 13 ………….., 19

22, 23 ………….., 29

32, 33 ………….., 39

42, 43 ………….., 49

52, 53 ………….., 59

22, 63 ………….., 69

72, 73 ………….., 79

82, 93 ………….., 99

Total = 72

For three digit numbers x, y, z

100 x + 10y + z < 10(xyz)

So, we can skip numbers from 100–110. As it contain a 'zero' in it.

So, by observation, we can see.

127, 128, 129 $\rightarrow$ 3

135, 136 ………….., 139 $\rightarrow$ 5

144, 145 ………….., 149 $\rightarrow$ 6

154, 155 ………….., 159 $\rightarrow$ 6

163, 164 ………….., 169 $\rightarrow$ 7

173, 174 ………….., 179 $\rightarrow$ 7

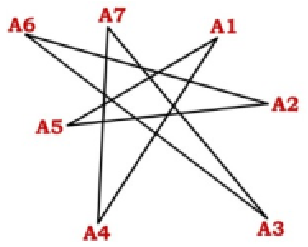
183, 184 ………….., 189 $\rightarrow$ 7

193, 194 ………….., 199 $\rightarrow$ 7

48

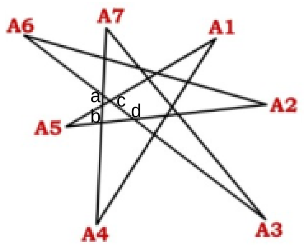
So, value of n = 72 + 48 = 120

5. In the given figure, $A_1 A_2 A_3 A_4 A_5 A_6 A_7$ is a 7-pointed star.



Find the value of $\angle A_1 + \angle A_2 + \angle A_3 + \angle A_4 + \angle A_5 + \angle A_6 + \angle A_7$

Sol.



In ΔA_1A_4b

$\angle A_1 bA_7 = A_1 + A_4$ (Exterior angle)

In ΔA_3A_7a

$\angle A_3aA_4 = A_3 + A_7$ (Exterior angle)

In ΔA_2A_6d

$\angle A_5dA_6 = A_2 + A_6 = \angle A_5dc$ (Exterior angle)

Now In ΔA_5cd

$\angle A_5cd = \angle A_1bA_7 + \angle A_3aA_4$ (Exterior angle)

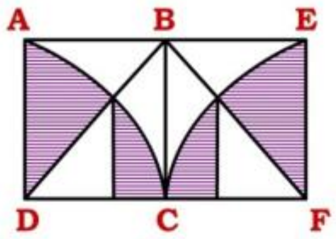
$= A_1 + A_4 + A_3 + A_7$

Now $A_5 + \angle A_5cd + \angle A_5dc = 180^o$

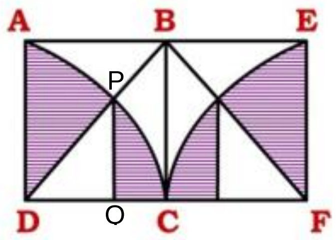
$A_5 + A_1 + A_4 + A_3 + A_7 + A_2 + A_6 = 180^o$

$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 = 180$

6. Two squares of side length 20 cm are joined together as in the diagram. With *D*, *F* as centres, quadrants are drawn. Taking $\pi = 3.14$, find the area of the shaded portion. Let A be the area in cm^2. Find A.



Sol.



AD = 20 = DP (radius of the circle)

Area of quadrant ADC = $\frac{\pi(20)^2}{4} = \frac{3.14 \times 20 \times 20}{4} = 3.14 \times 100 = 314 cm^2$

Area of quadrant ADC = area of quadrant EFC = 314 cm^2

Now In ΔPDQ PD = 20

$\angle PDQ = 45^o$

So $\angle DPQ = 45$ because $\angle DQP$ is taken 90^o

$\therefore$ DQ = PQ (side opposite to equal angle are equal)

So $(DP)^2 = (DQ)^2 + (QP)^2$

$(20)^2 = 2(DQ)^2$ $\because$ DQ = QP

$400 = 2(DQ)^2$

$200 = (DQ)^2$

$10\sqrt{2} = DQ$

So area of ΔDPQ

$= \frac{1}{2} \times DQ \times PQ$

$= \frac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2}$

$= 100 \ cm^2$

Area of shaded region = 2{area of quadrant ADC – area of ΔDPQ}

= 2 {314 – 100} = 2 × 214 = 428 cm^2