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ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
Screening Test – Bhaskara Contest

NMTC JUNIOR LEVEL – IX & X GRADES

Saturday, the 31 August 2024

INSTRUCTIONS

Time : 2 Hrs. **M. M. : 30**

1. Fill in the Response sheet with your Name, Class and the Institution through which you appear, in the specified places.

2. Diagrams are only Visual guides; they are not drawn to scale.

3. You may use separate sheets to do rough work.

4. Use of Electronic gadgets such as Calculator, Mobile Phone or Computer is not permitted.

5. Duration of Test: 10 am to 12 Noon (Two hours)

6. For each correct response you get 1 mark ; for each incorrect response, you lose ½ mark.

SECTION-A

1. If $x^2 + x = 1$, then the value of $\frac{x^7 + 34}{x + 2}$ is equal to

a) 7 b) 1 c) 13 d) 17

Ans. **(c)**

Sol. $x^2 + x = 1$

$x^2 = 1 - x$ …(1)

squaring both sides

$x^4 = x^2 + 1 - 2x$

$= 1 - x + 1 - 2x = 2 - 3x$ …(2)

From (1) and (2)

$x^4 \times x^2 = (2 - 3x)(1 - x)$

$x^6 = 2 - 5x + 3x^2$

$x^6 = 2 - 5x + 3(1 - x)$

$x^6 = 5 - 8x$

$x^7 = 5x - 8x^2$

$= 5x - 8(1 - x)$

$= 13x - 8$

So $\frac{x^7 + 34}{x + 2}$

$= \frac{13x - 8 + 34}{x + 2}$

$= \frac{13x + 26}{x + 2}$

$= \frac{13(x + 2)}{x + 2} = 13$

2. The angle between the hour hand and the minute hand of a clock at the time 9:38 pm is

a) 60° b) 61° c) 59° d) 62°

Ans. **(b)**

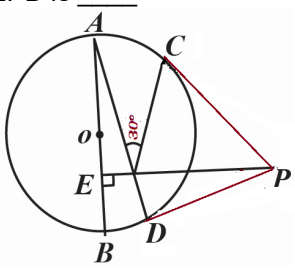
Sol. $Q = \left|\frac{11}{2}M - 30H\right|$

$= \left|\frac{11}{2} \times 38 - 30 \times 9\right|$

$= |209 - 270|$

$= 61$

3. In the adjoining figure, AOB is a diameter of the circle with centre O. PC and PD are two tangents. Then the measure of $\angle EPD$ is ____



a) 15° b) 10° c) 12° d) 20°

Ans. **(b)**

4. The value of x satisfying $4^x - 3^{x-1/2} = 3^{x+1/2} - 2^{2x-1}$ is of the form $\frac{a}{b}$ where gcd(a , b) = 1. Then the value of $\left(\frac{a+b}{a-b}\right)$ is equal to

a) 7 b) –5 c) 4 d) 5

Ans. **(d)**

Sol. $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

$2^{2x} - \frac{3^x}{\sqrt{3}} = 3^x \times \sqrt{3} - \frac{2^{2x}}{2}$

$2^{2x} \times \frac{3}{2} = 3^x\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$

$2^{2x} \times \frac{3}{2} = 3^x \times \frac{4}{\sqrt{3}}$

$2^{2x} \times 3^{3/2} = 3^x \times 2^3$

On comparing

$x = \frac{3}{2} = \frac{a}{b}$

$\frac{a+b}{a-b} = \frac{3+2}{3-2} = 3$

5. The number of polynomials of the form $(x^3 + ax^2 + bx + c)$ which are divisible by $x^2 + 1$ where a, b, c $\in$ {1,2,3,4,...,12} is

a) 12^3 b) 12^2 c) 12 d) 1

Ans. **(c)**

6. The number of real solutions of the equation $\frac{(x+2)(x+3)(x+4)(x+5)}{(x-2)(x-3)(x-4)(x-5)} = 1$ is

a) 1 b) 2 c) 3 d) 0

Ans. **(a)**

Sol. $\frac{(x+2)(x+3)(x+4)(x+5)}{(x-2)(x-3)(x-4)(x-5)} = 1$

$\frac{(x^2+7x+10)(x^2+7x+12)}{(x^2-7x+10)(x^2-7x+12)} = 1$

$\frac{x^2+7x+10}{x^2-7x+10} = \frac{x^2-7x+12}{x^2+7x+12}$

componendo & divide

$\frac{2(x^2+10)}{14x} = \frac{2(x^2+12)}{-14x}$

$x(x^2 + 10) = -x(x^2 + 12)$

$x[x^2 + 10 + x^2 + 12] = 0$

$x(2x^2 + 22) = 0$

$x(x^2 + 11) = 0$

$\Rightarrow x(x^2 + 11) = 0$

$\Rightarrow x = 0$ or $x^2 = -11$

(Not real solution)

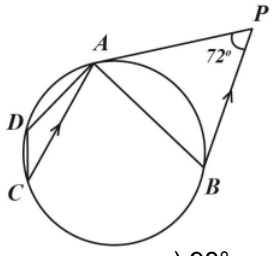
Only 1 solution

7. If $a = \sqrt{23a+b}$, $b = \sqrt{23b+a}$, $a \neq b$ then the value of $\sqrt{a^2+b^2+48}$ is

a) 30 b) 25 c) 24 d) 23

Ans. **(c)**

8. In the adjoining figure, PA and PB are tangents to the circle. AC is parallel to PB. Then measure of $\angle CDA$ is



a) 118° b) 108° c) 98° d) 88°

Ans. **(b)**

9. If $\sqrt{\frac{19^8+19^x}{19^x+1}} = 361$, then x satisfies the equation

a) $4x^2 - 7x - 15 = 0$ b) $2x^2 - 9x - 5 = 0$ c) $3x^2 + 11x - 4 = 0$ d) $3x^2 - 11x - 4 = 0$

Ans. **(d)**

10. If $S = 4^2 + 2.5^2 + 3.6^2 + \ldots + 25.28^2$, then the value of $\frac{S}{325}$ is equal to

a) 436 b) 326 c) 346 d) 324

Ans. **(a)**

11. A sequence $\{a_n\}$, $n \geq 1$ with $a_1 = \frac{1}{2}$ and $a_n = \frac{a_{n-1}}{2na_{n-1}+1}$ is given. Then the value of $a_1 + a_2 + a_3 + \ldots + a_{2024}$ is equal to

a) $\frac{2025}{2024}$ b) $\frac{2024}{2025}$ c) 2025 d) $\frac{1}{2025}$

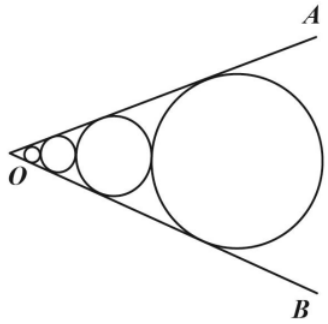
Ans. **(b)**

12. If a and b ($\alpha > \beta$) satisfy the equation $x^{1+\log_{10}x} = 10x$ then the value of $\alpha + \frac{1}{\beta}$ is equal to

a) 100 b) 20 c) 10 d) $\frac{1}{100}$

Ans. (b)

13. In the adjoining figure, four successively touching circles are placed in the interior of $\angle AOB$. The first (smallest) has a radius 7 cm. The third circle has a radius 28 cm. Then the radius of the largest circle (in cm) is



a) 42 b) 48 c) 52 d) 56

Ans. (d)

14. The coefficient of x in the equation $x^2 + px + q = 0$ was taken as 17, in place of 13 and its roots were found to be -2 and –15. If α , β are the roots of the original equation, then the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

a) $30x^2 + 109x + 30 = 0$ b) $20x^2 – 107x + 20 = 0$

c) $30x^2 – 109x + 30 = 0$ d) $20x^2 + 107x + 20 = 0$

Ans. (c)

15. If $(1 + xy + x + y)^2 – (1– xy + x – y)^2 = ky(1 + x)^2$, then k equals to

a) 1 b) 2 c) 3 d) 4

Ans. (d)

Section- B (Fill in the Blanks)

16. When $x^{10} +1$ is divided by $x^2 +1$, we get

$$ax^8 + bx^7 + cx^6 + dx^5 + ex^4 + fx^3 + gx^2 + hx + k$$

as quotient. Then the value of

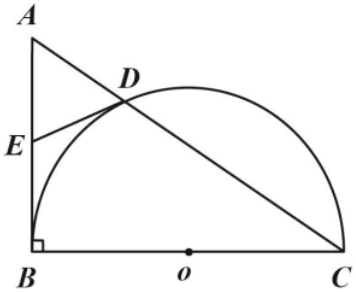
$$a^{2024} + b^{2024} + c^{2024} + d^{2024} + e^{2024} + f^{2024} + g^{2024} + h^{2024} + k^{2024}$$ is _____

Ans. (05)

17. The equation $x^4 – 4x^3 + ax^2 + bx + 1 = 0$ has 4 positive roots. Then a + b = _____

Ans. (02)

18. In the adjoining figure, BOC is the diameter of the semicircle with centre O. DE is the tangent at D. If AB = k(AE), then the numerical value of k is _____.



Ans. (02)

19. In triangle ABC, tanA : tanB : tanC = 1 : 2: 3.

If $\frac{AC}{AB}=\frac{p\sqrt{q}}{r}$, where q is Square free and gcd(p, r) = 1 then the value of p + q + r is _____.

Ans. **(07)**

20. Simon was given a number and asked to divide it by 120. He divided the number by 5, 6 and 7 and got 3, 2 and 2 as remainders respectively. The remainder when the number is divided by 120 is _____.

Ans. **(43)***

21. The greatest number that leaves the same remainder when it divides 30, 53 and 99 is _____.

Ans. **(23)**

22. If $f(x + 1) = x^2 - 3x + 2$ and if the roots of the equation f(x) = 0 are α and β, then the value of $\alpha^2 + \beta^2$ is _____.

Ans. **(13)**

Sol. $f(x + 1) = x^2 - 3x + 2$

roots of $f(x) = 0 \begin{cases} \alpha \\ \beta \end{cases}$

let $y = x + 1$ then $x = y - 1$

$f(y) = (y - 1)^2 - 3(y - 1) + 2$

$= y^2 + 1 - 2y - 3y + 3 + 2$

$f(y) = y^2 - 5y + 6$

$f(x) = x^2 - 5y + 6$

$f(x) = 0$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2, 3$

$a = 2$

$b = 3$

$\alpha^2 + \beta^2 = 2^2 + 3^2 = 4 + 9 = 13$

23. The maximum volume of a cylinder is cut from a cube of edge α. The volume of the remaining solid is ka^3, where $k = \frac{p}{q}$, gcd(p, q) = 1. Taking $\pi = \frac{22}{7}$, the value of p + q is _____.

Ans. **(17)**

24. If the irreducible quadratic factor of $5x^4 + 9x^3 - 2x^2 - 4x - 8$ is $ax^2 + bx + c$, then the value of $a^2 + b^2 - c^2$ is _____.

Ans. **(25)**

Sol. $p(x) = 5x^4 + 9x^3 - 2x^2 - 4x - 8$

$P(1) = 0$

so $p(x) = (x - 1)(5x^2 + 14x^2 + 12x + 8)$

$P(-2) = 0$

$P(x) = (x - 1)(x + 2)(5x^2 + 4x + 4)$

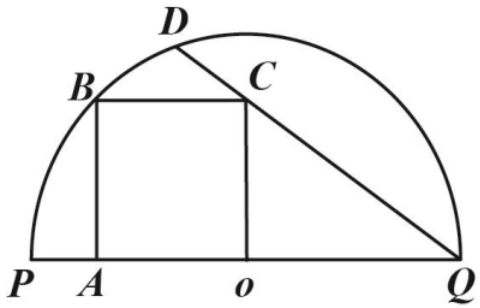
$\Rightarrow$ $a = 5, b = 4, c = 4$

so $a^2 + b^2 - c^2 = 5^2 + 4^2 - 4^2 = 25$

25. In the adjoining figure, POQ is the diameter of the semicircle with centre O.

OABC is a square whose area is 36 cm^2. If QD = x cm, the value of $x\sqrt{3}$ is _____.

Ans. **(24)**



26. If a = $\sqrt{2024}$, b = $\sqrt{2025}$, the value of $2(ab)^{1/2}(a+b)^{-1}\left\{1+\frac{1}{4}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2\right\}^{1/2}$ is _____.

Ans. **(01)**

Sol. $2(ab)^{1/2}\,(a+b)^{-1}\left\{1+\frac{1}{4}\left(\frac{\sqrt{a}}{b}-\sqrt{\frac{b}{a}^2}\right)\right\}^{1/2}$

$$\frac{2\sqrt{ab}}{(a+b)}\left\{1+\frac{1}{4}\frac{(a-b)^2}{(\sqrt{ab})^2}\right\}^{1/2}$$

$$\frac{2\sqrt{ab}}{a+b}\left\{1+\frac{(a-b)^2}{4ab}\right\}^{1/2}$$

$$\frac{2\sqrt{ab}}{(a+b)}\frac{(a+b)}{2\sqrt{ab}} = 1$$

27. In a decreasing geometric progression, the 2nd term is 6. The sum of all infinite terms of the progression is one-eighth of the sum to infinity of the squares of the terms. The sum of the 1st and the 4th terms is $\frac{p}{q}$ where p, q are relatively prime to each other. Then the value of $\left[\frac{p}{q}\right]$, where [x] represents the greatest integer not exceeding x is _____

Ans. **(13)**

28. The value of $\left(\frac{\sqrt{10}}{10}\right)^{(\log_{10} 9)-2}$ is of the form $\frac{a}{b}$, where a, b are relatively prime to each other. Then a – b is equal to _____.

Ans. **(07)**

Sol. $\left(\frac{\sqrt{10}}{10}\right)^{(\log_{10} 9)-2}$

$=\left(\frac{1}{\sqrt{10}}\right)^{\log_{10} 9-\log_{10} 10^2}$

$=\left(\frac{1}{\sqrt{10}}\right)^{\log_{10}\left(\frac{9}{100}\right)}$

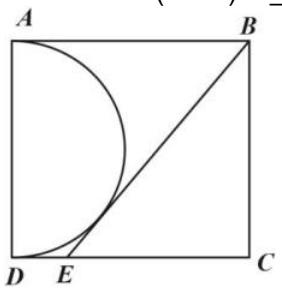
$= 10^{-\frac{1}{2}\left[\log_{10}\left(\frac{3}{10}\right)^2\right]}$

$= 10^{\log_{10}\left(\frac{3}{10}\right)^{-1}}$

$=\left(\frac{3}{10}\right)^{-1}=\frac{10}{3}=\frac{a}{b}$

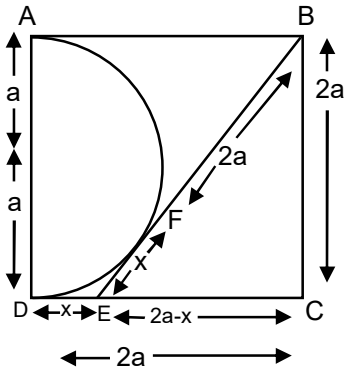
a – b = 10 – 3 = 7

29. ABCD is a square. BE is the tangent to the semicircle on AD as diameter. The area of the triangle BCE is 216 cm^2. The radius of the semicircle (in cm) is _____



Ans. **(12)**

Sol.



AD = 2a

DE = x = FE

CE = 2a – x

BF = AB = 2a

in ΔBEC By Pythagoras theorem

$(2a + x)^2 = (2a)^2 + (2a - x)^2$
$4a^2 + x^2 + 4ax = 4a^2 + 4a^2 + x^2 - 4ax$
$a = 2x$

$x = \frac{a}{2}$

$CE = 2a - x = 2a - \frac{a}{2}$

$CE = 2a - \frac{a}{2}$

$CE = \frac{3a}{2}$

$ar(\Delta BEC) = \frac{1}{2} \times BC \times CE = \frac{1}{2} \times 2a \times \frac{3a}{2}$

$3a^2 = 216 \times 2$
$a = 12$

30. a, b, c, d are real constants in a $f(x) = ax^{2025} + bx^{2023} + cx^{2021} + dx^{2019}$ and $f(-4) = 18$. Then the maximum value of $|f(4)| + |2\cos x|$ is _____.

Ans. **(20)**