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ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Screening Test – Kaprekar Contest

(NMTC --SUB-JUNIOR LEVEL – VII and VIII GRADES)

Saturday, the 31 August, 2024

INSTRUCTIONS

Time : 2 Hrs.

M.M. 30

1. Fill in the Response sheet with your Name, Class and the Institution through which you appear, in the specified places.

2. Diagrams are only Visual guides; they are not drawn to scale.

3. You may use separate sheets to do rough work.

4. Use of Electronic gadgets such as Calculator, Mobile Phone or Computer is not permitted.

5. Duration of Test: 10 am to 12 Noon (Two hours)

6. For each correct response you get 1 mark ; for each incorrect response, you lose ½ mark.

SECTION-A

1. There is a 6-digit number in which the first and the fourth digit from the first are the same, the second and the fifth digit from the first are the same and the third and the sixth digit from the first are the same. Then the number is always

a) A square number
b) Divisible by 5
c) Divisible by 11
d) An odd number.

Ans. **(c)**

Sol. 6 digit number = abcabc

expended form = 100000 a + 10000 b + 1000 c + 100 a + 10 b + c

= 100100 a + 10010 b + 1001 c

abcabc = 1001 (100 a + 10 b + c)

one of the factor of number = 1001 which is divisible by 11.

2. Starting from the number 1, Ritu generates a series of numbers as 1, 3, 6, 11, 18, 29, 42, … such that the differences of the consecutive numbers from the beginning give consecutive primes. In this series she came across a perfect square for the first time. The Square root of this perfect square is

a) 14
b) 19
c) 23
d) 21

Ans. **(d)**

Sol. 1, 3, 6, 11, 18, 29, 42, 59, 78, 101, 130, 161, 198, 239, 282, 329, 382, 441 → $(21)^2$

(differences:) 2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59

3. The expression

$$\frac{x\left(\frac{\sqrt{x}+\sqrt{y}}{2y\sqrt{x}}\right)^{-1}+y\left(\frac{\sqrt{x}+\sqrt{y}}{2y\sqrt{y}}\right)^{-1}}{\left(\frac{\sqrt{x}+\sqrt{xy}}{2xy}\right)^{-1}+\left(\frac{y+\sqrt{xy}}{2xy}\right)^{-1}}$$ reduces to

a) $\sqrt{xy}$ b) $\frac{\sqrt{x}+\sqrt{y}}{2}$ c) $\frac{2}{\sqrt{x}+\sqrt{y}}$ d) $\frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$

Ans. **(a)**

Sol.

$$\frac{x\left(\frac{\sqrt{x}+\sqrt{y}}{2y\sqrt{x}}\right)^{-1}+y\left(\frac{\sqrt{x}+\sqrt{y}}{2x\sqrt{y}}\right)^{-1}}{\left(\frac{x+\sqrt{xy}}{2xy}\right)^{-1}+\left(\frac{y+\sqrt{xy}}{2xy}\right)^{-1}}$$

$$=\frac{x\left(\frac{2y\sqrt{x}}{\sqrt{x}+\sqrt{y}}\right)+y\left(\frac{2x\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)}{\left(\frac{2xy}{x+\sqrt{xy}}\right)+\left(\frac{2xy}{y+\sqrt{xy}}\right)}$$

$$=\frac{\frac{2xy\left(\sqrt{x}+\sqrt{y}\right)}{\sqrt{x}+\sqrt{y}}}{2xy\left(\frac{1}{\sqrt{x}\left(\sqrt{x}+\sqrt{y}\right)}+\frac{1}{\sqrt{y}\left(\sqrt{y}+\sqrt{x}\right)}\right)}$$

$$=\frac{1}{\frac{1}{\sqrt{x}+\sqrt{y}\left(\frac{\sqrt{x}+\sqrt{y}}{\left(\sqrt{x}-\sqrt{y}\right)}\right)}}=\frac{1}{\frac{1}{\sqrt{xy}}}=\sqrt{xy}$$

4. The sum of the digits of a two-digit number is multiplied by 8 and the result is found to be 13 more than the number. Then the two digit number is

a) A prime number
b) An even number
c) Such that the difference of its digits is 2.
d) Such that the sum of its digits is a composite number.

Ans. **(a)**

Sol. Let two digit no. is xy
xy
(a)
$8(x + y) = xy + 13$
$8x + 8y = 10x + y + 13$
$7y = 2x + 13$
$x = 4$
then we get
$7y = 8 + 13$
$7y = 21$
$y = 3$
$\therefore$ No. is 43 (A prime no.)

5. A water tank is fitted with four different taps as outlets. If the tank is full,
it takes 1 hour to empty the tank when the first tap alone is opened;
it takes 2 hours to empty the tank when the second tap alone is opened;
it takes 3 hours to empty the tank when the third tap alone is opened;
it takes 4 hours to empty the tank when the fourth tap alone is opened.
When all the taps are opened simultaneously, the full tank will be emptied in

a) More than 29 minutes b) Between 28 and 29 minutes
c) Between 29 and 30 minutes d) Less than 28 minutes.

Ans. **(b)**

Sol. Fraction of tank empty in 1 hr. = $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$= \frac{12+6+4+3}{12} = \frac{25}{12}$

Time to empty tank = $\frac{1}{\frac{25}{12}}$ hr. = $\frac{12}{25}$ hr = $\frac{12 \times 60}{25}$ min. = $\frac{144}{5}$ min. = 28.8 min

6. Two primes p, q are such that p + q is odd and q – 10p = 23. Then q – 20p equals to

a) 1 b) 3 c) 5 d) 7

Ans. **(b)**

Sol. p, q are two prime no.
p + q = odd
then one of them p, q must be even (2)
q – 10p = 23
q > p
∴ p = 2
q = 23 + 20
q = 43
q – 20 p = 43 – 20 (2)
= 43 – 40
= 3

7. Which one of the following is a false statement?

a) Diagonals of a square bisect each other at right angles.
b) Diagonals of a rectangle bisect each other.
c) Diagonals of a rhombus bisect each other at right angles.
d) If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a rectangle.

Ans. **(d)**

Sol. If the diagonal of a quadrilateral bisect each other, then the quadrilateral can be parallelogram.

8. Soham has his 23rd birthday on 1st January 2024 and he noticed that 2024 is divisible by 23. If he lives till 100 years of age, how many times other than the above, his age would be a divisor of the then year?

a) 2 b) 3 c) 4 d) 5

Ans. (b)

Sol. In 100 years there are 4 years which are divisible by 23.

One year is 2024 then 3 years more to come.

9. Consider the two figures shown here.
AB = 16 cm in both the figures.
Points P, Q, R divide AB in equal lengths in fig.1 fig.1
Similarly P,Q,R,S,T,L,M divide AB in equal length in fig 2
All the curves are semi-circles.
If [a] and [b] are the areas of fig 2 the shaded figures respectively in fig 1 and fig 2, then

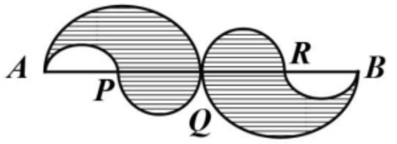


fig.1

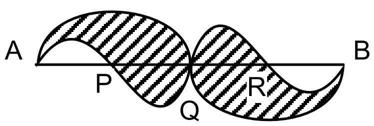
fig 2

a) [a] – [b] is a non-zero number. b) $[a] = \frac{5}{4}[b]$

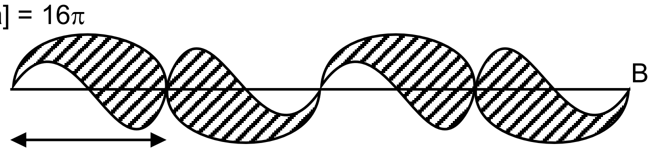
c) $[a] = \frac{4}{5}[b]$ d) [a] = [b]

Ans. (a)

Sol.



Shaded region = 1 completer circle whose radius = 4 cm
AQ = Diameter of circle
AP = Radius = 4 cm
area = $\pi R^2 = 16\pi$
[a] = 16π



diameter = 4 cm
$\therefore$ radius = 2 cm
there are 2 complete circle (whose radius) = 2 cm
[6] = $2 \times \pi(2)^2 = 8\pi$
[a] – [b] = 8π (non zero number)

10. The sum of 11 consecutive natural numbers is 121. The sum of the next three numbers is
a) 54 b) 55 c) 53 d) 57

Ans. (a)

Sol. (N) + (N + 1) + ... + (N + 10) = 121
11N + 55 = 121
11N = 121 – 55
11N = 66
N = 6
11th no. is = 6 + 10 = 16
sum of the next three no. is = 17 + 18 + 19 = 54

11. A big ship wrecked and 1000 people landed in a remote island. The food material was available for them for 60 days. After 16 days another small ship, which had no food stock, wrecked and 100 people landed in the same island. The number of days the food material for all of them available is
a) 42 b) 35 c) 40 d) 41

Ans. **(c)**

Sol. Total available food = 1000 × 60 units
food consumed by 1000 people in 16 days
= 1000 × 16 unit
Now 100 people increased so let food can last for x days so total food consumed = 1100 × x
so, 1000 × 60 = 1000 × 16 + 1100 + x
$\Rightarrow$ 1100 × x = 1000 × 60 – 1000 × 16
$\Rightarrow$ $x = \frac{1000 \times 44}{1100}$
x = 40 days

12. Two numbers are respectively 28% and 70% of a third number. The percentage of the first number to the second is
a) 40 b) 36 c) 45 d) 50

Ans. **(a)**

Sol. N, N_2, N_3 are three numbers
$N_1 = \frac{28N_3}{100}$
$N_2 = \frac{70N_3}{100}$
$N_1 = \frac{x \times N_2}{100} = \frac{x}{100} \times \frac{70N_3}{100}$
$\frac{28N_3}{100} = \frac{x}{100} \times \frac{70N_3}{100}$
40% = x

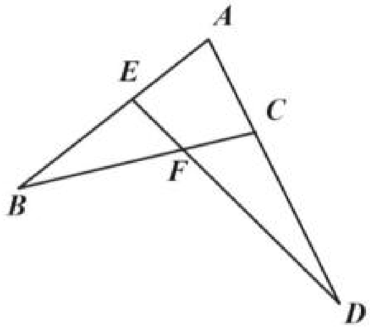
13. The sum of two natural numbers is 150. Their HCF is 15. The number of pairs of such numbers is
a) 1 b) 2 c) 3 d) 4

Ans. **(b)**

Sol. $N_1 + N_2 = 150$
$N_1 = 15x$
$N_2 = 15y$
15x + 15y = 150
x + y = 10
x = 1, y = 9
x = 7, y = 3
$N_1 = 15$ $N_1 = 15 \times 7 = 105$
$N_2 = 135$ $N_2 = 15 \times 3 = 45$
Only two such pairs.

14. ABC and ADE are isosceles triangles.

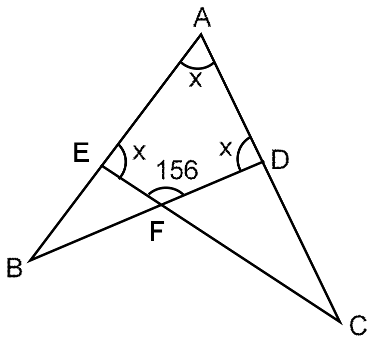
If $\angle BFD = 156^\circ$, then $\angle A =$



a) 68° b) 70° c) 66° d) 70°

Ans. **(a)**

Sol.



AB = BC

$\angle BAC = \angle BCA = x$

& CA = CE

$\angle CEA = \angle CAE$

$\angle CEA = x$

In equal ACFE

$x + x + x + 156^\circ = 360^\circ$

$3x = 360^\circ - 156^\circ$

$3x = 204^\circ = 68^\circ$

15. Some students are made to stand in rows of equal number, one behind the other. Saket is in the 3rd row from the front and 5th row from the back. He is 4th from the left and 6th from right. The total number of students is

a) 45 b) 72 c) 63 d) 81

Ans. **(c)**

Sol. No. of rows = 5 + 3 – 1 = 7

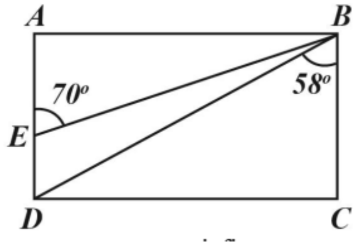
No. of column = 6 + 4 – 1 = 9

Total students = 7 × 9 = 63

SECTION-B

FILL IN THE BLANKS

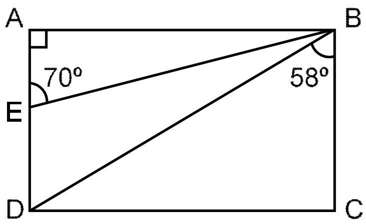
16. In the adjoining figure, ABCD is a rectangle. Then,



$\angle EBD$ = __________ degrees.

Ans. **12**

Sol.



$\angle ABE = 90^\circ - 70^\circ$

$= 20^\circ$

$\angle ABE + \angle EBD + \angle DBC = 90^\circ$

$20^\circ + \angle EBD + 58^\circ = 90^\circ$

$\angle EBD = 12^\circ$

17. An infinite sequence of positive numbers $x_1, x_2, x_3, ..., x_n, x_{n+1},...$ satisfies $x_n^2 = (3n+7) + (x-3)\,x_{n+1}$, where x_n is the nth term of the sequence. Then the numerical value of x_1 is ___________

Ans. **02**

Sol. $x_1, x_2, x_3.....x_n, x_{n+1}$ are positive numbers.

where $x_n^2 = (3n+7) + (n-3)x_{n+1}$ ...(1)

put n = 1

$x_1^2 = (3+7) + (1-3)x_{1+1}$

$x_1^2 = 10 - 2x_2$

$x_1 = \sqrt{10 - 2x_2}$

let $x_2 = 1$	let $x_2 = 2$	let $x_2 = 3$	let $x_2 = 4$
$x_1 = 2\sqrt{2}$	$x_1 = \sqrt{6}$	$x_1 = 2$	$x_1 = \sqrt{2}$

$\therefore x_1 = 2$ when $x_2 = 3$

again put n = 2 in eq. (1) $x_2^2 = 13 + (-1)x_3$

$9 = 13 - x_3$

$x_3 = 4$

$\therefore$ series is 2, 3, 4...... $\therefore x_1 = 2$

18. For $n \geq 2$ and $n \in Z$, the smallest positive integer n for which <u>none</u> of the fractions

$\frac{17}{n+17}, \frac{18}{n+18}, \frac{19}{n+17}, \ldots\ldots\ldots, \frac{100}{n+100}$ can be simplified is ________.

Ans. **101**

Sol. $n \geq 2$, $n \in z$

n cannot be even no from 2 to 100 because at least one them must be divisible by 2 (numerator & denominator)

$$\left.\begin{array}{l} n=3 \\ n=5 \\ n=7 \\ n=9 \\ n=11 \\ n=13 \\ n=15 \end{array}\right\} \text{ at least one of them also simplify.}$$

$\therefore$ n > 100

n = 101 (three digit prime no)

so non one can be simplify.

19. In triangle ABC, AB = 15 cm, BC = 20 cm and CA = 25 cm. Then the length of the shortest altitude of the triangle (in cm) is __________.

Ans. **12**

Sol. $s = \frac{15+20+25}{2} = 30$

ar $(\Delta ABC) = \sqrt{30(30-15)(30-20)(30-25)}$

$= \sqrt{30 \times 15 \times 10 \times 5}$

= 150 m^2

for smallest altitude, base should be maximum

$150 = \frac{1}{2} \times 25 \times h$

h = 12 cm

20. The units digit of $19^{2025} + 999^{2023}$ is __________.

Ans. **08**

Sol. $19^{2025} + 999^{2023}$

unit digit of 9 + 9 i.e. 18

so unit digit of $19^{2025} + 999^{2023}$ is 08.

21. *N* is a 2-digit number. When 6 is added to the tens digit and 2 is subtracted from the units digit, we get a two digit number which is equal to 3N. Then N is ________.

Ans. **29**

Sol. N = ab = 10a + b

according to question

10(a + 6) + (b – 2) = 3 (10a + b)

10a + 60 + b – 2 = 30a + 3b

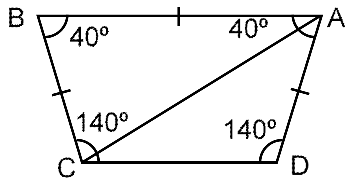
or 10a + b = 29

so N = 29

22. ABCD is a quadrilateral. AB is parallel to CD and AB > CD. If AD = AB = BC and ∠ADC = 140o, then the measure of ∠CAB is ________degrees.

Ans. **70**

Sol.



∠A + ∠D = 180º

∠A = 40º ...(1)

∵ ABCD is a isosceles trapezium

⇒ ∠A = ∠B = 40º

∠C = ∠D = 140º

In ΔABC

∵ AB = BC

∴ ∠BAC = ∠BCA = x

Now,

x + x + 40 = 180º

2x = 140 º

x = 70 º

or ∠CAB = 70º

23. The product of two positive numbers x and y is 4 times their Sum and the same product is 8 times their difference. If x ≥ y , then x = ___________.

Ans. **16**

Sol. xy = 4(x + y) ...(1)

xy = 8(x – y) ...(2)

from (1) & (2)

4(x + y) = 8(x – y)

4x + 4y = 8x – 8y

12y = 4x

x = 3y ...(3)

from (1) & (3)

3y × y = 4(3y + y)

$3y^2 = 4 \times 4y$

$3y^2 - 16y = 0$

y(3y – 16) = 0

y = 0 or y = 16/3

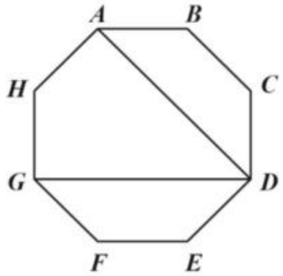
(Neglecting)

so x = 3y

= 3 × 16/3

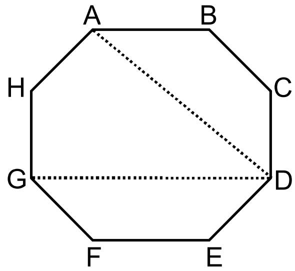
x = 16

24. In the adjoining figure, ABCDEFGH is a regular Octagon. The measure of ∠ADG (in degrees) is __________.



Ans. **45º**

Sol.



Sum of interior angle of octagon is (8 – 2)180º = 1080º

One angle is $\frac{1080^{\circ}}{8}$ = 135º

All sides are equal in regular polygon.

∵ ∠CDG = 90º, ∠HGD = 90º , ∠AHG = 135º, ∠HAD = 90º

∠HGD + ∠ADG + ∠HAD + ∠AHG = 360º

90º + ∠ADG + 90º + 135º = 360º

∠ADG = 360º – 180º – 315º

∠ADG = ∠45º

25. If $2^{3a+2} = 4^{b+7}$ and $3^{a+10} = 27^{2b+10}$ then the value of $a^2 + b^2$ is ____________.

Ans. **13**

Sol.

$2^{3a+2} = 4^{b+7}$

$2^{3a+2} = 4^{b+7}$

3a + 2 = 2b + 14

3a – 2b = 12 ……(i)

$3^{a+10} = 27^{2b+10}$

$3^{a+10} = 3^{6b+30}$

a + 10 = 6b + 30

a – 6b = 20 ……(ii)

3 × (i) – (ii)

9a – 6b = 36

a – 6b = 20

–   +   –

8a = 16

a = 2

2 – 6b = 20

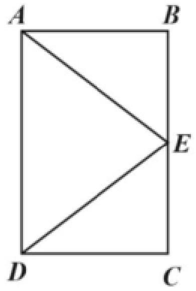
6b = – 18

b = –3

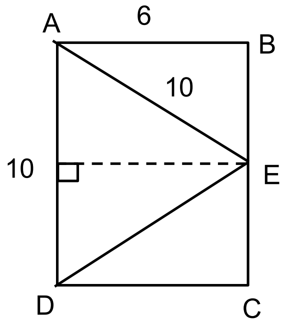
$a^2 + b^2 = (2)^2 + (–3)^2$

= 4 + 9 = 13

26. ABCD is a rectangle. AB = 6 and AD = 10.
E is a point on BC such that AE = 10.
Then area of ΔADE (in square units) is _______.



Ans. **30**
Sol.



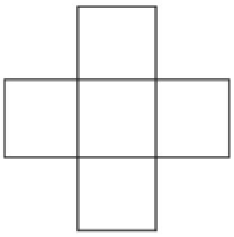
ABCD is a rectangle
Ab = 6, AD = 10, AE = 10
EF is perpendicular to AD and EF || AB
so EF = AB = 6

area of ΔADE = $\frac{1}{2}$ × AD × EF

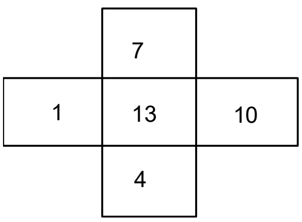
= $\frac{1}{2}$ × 10 × 6

= 30 unit square

27. The numbers 1, 4, 7, 10 and 13 are placed in each box of the figure, such that the sum of the numbers in the horizontal or vertical boxes are the same. The largest possible value of the horizontal or vertical sum is __________.



Ans. **24**
Sol.



28. The number of integer pairs (m , n) such that $m(n^2 + 1) = 48$ is _________.

Ans. **03**

Sol. $m (n^2 + 1) = 48$

$n^2 + 1 = \frac{48}{m}$

m and n both are integers

so $n^2 + 1$ and $\frac{48}{m}$ will be integers and $n^2 + 1$ is always be positive

so positive values of $\frac{48}{m}$ = 48, 24, 16, 12, 8, 6, 4, 3, 2, 1

so positive value of m = {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

so $n^2 + 1 = 1, 2$

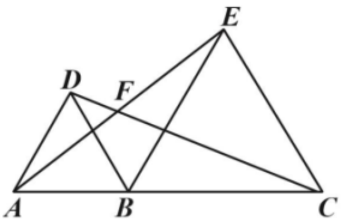
so $n = 0, -1, 1$

Pairs $(m, n) \equiv (48, 0), (24, 1), (24, -1)$

Ans. **(03)**

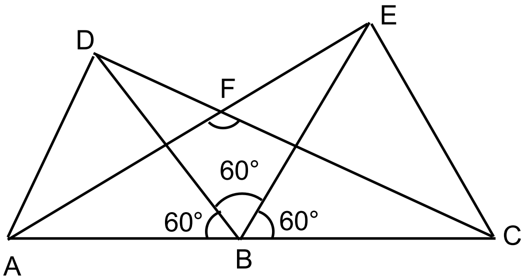
29. In the adjoining figure, ΔABD and ΔBCE are equilateral triangles.

The measure of $\angle AFC$ = ______ degrees.



Ans. **120º**

Sol.



ABD and BCE are equilateral triangle

$\because \angle DBA = 60°$

$\because \angle CBE = 60°$

$\therefore \angle DBE = 60°$

In ΔABE and ΔCBD

AB = BD

BE = BC

$\angle ABE = \angle DBC = 120º$

By SAS , $\Delta ABE \cong \Delta CBD$

By CPCT

$\therefore \angle ACD = \angle AEB = x$ (let)

$\therefore \angle FCE = 60 - x$

$\therefore \angle FEC = 60 + x$

In ΔFEC

$\angle AFC$ is the exterior angle of ΔFEC

$\angle AFC = \angle FEC + \angle ECF$

$= 60 + x + 60 - x$

$\angle AFC = 120º$

30. The value of $\frac{\sqrt[4]{27\cdot\sqrt[3]{9}}}{\sqrt[6]{9\cdot 3^3\cdot\sqrt{3}}}$ is ________.

Ans. 01

Sol. $\frac{\sqrt[4]{27.\sqrt[3]{9}}}{\sqrt[6]{9.3^3.\sqrt{3}}}$

$\Rightarrow \frac{\sqrt[4]{(3)^3.3^{2/3}}}{\sqrt[6]{3^2.3^3.3^{1/2}}}$

$\Rightarrow \frac{\sqrt[4]{3^{11/3}}}{\sqrt[6]{3^{11/2}}}$

$\Rightarrow 3^{\frac{11}{12}-\frac{11}{12}} = 3^{\circ} = 1$